MECHANISMS

DRIVES

Electromechanical
Technology Series
TERC EMT STAFF

DELMAR PUBLISHERS, MOUNTAINVIEW AVENUE, ALBANY, NEW YORK 12205

Wayne L. Shirk

MECHANISMS

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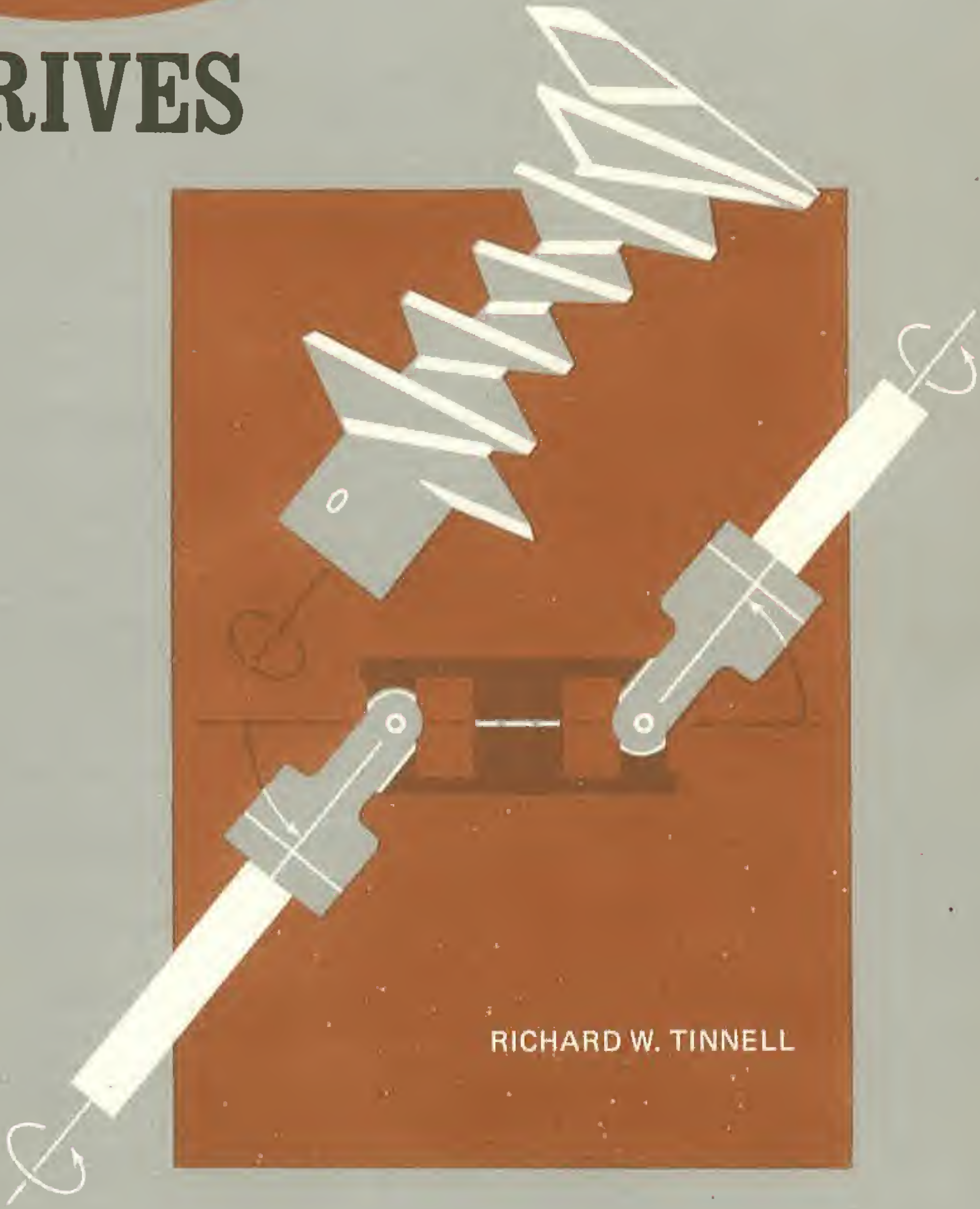

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Foreword

The marriage of electronics and technology is creating new demands for technical personnel in today's industries. New occupations have emerged with combination skill requirements well beyond the capability of many technical specialists. Increasingly, technicians who work with systems and devices of many kinds — mechanical, hydraulic, pneumatic, thermal, and optical — must be competent also in electronics. This need for combination skills is especially significant for the youngster who is preparing for a career in industrial technology.

This manual is one of a series of closely related publications designed for students who want the broadest possible introduction to technical occupations. The most effective use of these manuals is as combination textbook-laboratory guides for a full-time, post-secondary school study program that provides parallel and concurrent courses in electronics, mechanics, physics, mathematics, technical writing, and electromechanical applications.

A unique feature of the manuals in this series is the close correlation of technical laboratory study with mathematics and physics concepts. Each topic is studied by use of practical examples using modern industrial applications. The reinforcement obtained from multiple applications of the concepts has been shown to be extremely effective, especially for students with widely diverse educational backgrounds. Experience has shown that typical junior college or technical school students can make satisfactory progress in a well-coordinated program using these manuals as the primary instructional material.

School administrators will be interested in the potential of these manuals to support a common first-year core of studies for two-year programs in such fields as: instrumentation, automation, mechanical design, or quality assurance. This form of *technical core* program has the advantage of reducing instructional costs without the corresponding decrease in holding power so frequently found in general core programs.

This manual, along with the others in the series, is the result of six years of research and development by the *Technical Education Research Center, Inc.,* (TERC), a national nonprofit, public service corporation with headquarters in Cambridge, Massachusetts. It has undergone a number of revisions as a direct result of experience gained with students in technical schools and community colleges throughout the country.

Maurice W. Roney

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Preface

The study of mechanisms is one fo the oldest of the applied sciences. The early Greeks and Romans used crude pulleys and gears in a wide variety of applications, and the American industrial revolution can truly be said to have rolled on tooth gear wheels. The advent of space exploration has demanded a rebirth of interest in mechanics and mechanisms. In the past we have thought primarily of applications in the automotive, machine tool, and watchmaking fields. Today, it is more common to think of aerospace, defense weaponry, and precision instrument applications. These changes in emphasis have created subtle but important new demands upon training programs in mechanisms.

Mechanisms/Drives is an introductory treatment of modern mechanical drives, combining the elements of mechanical theory with those of practicality. The topics treated include: various gear drive configurations employing spur, bevel, and helical gears, belt drives of several types, chain drives, friction drives, and some selected special topics such as cams and universal joints. The materials are intended for use by technology students who have had little or no previous exposure to practical applied mechanics. Consequently, no attempt has been made to cover the material in the fine detail that would be appropriate for the experienced specialist in mechanical drives. An attempt has been made to expose the student to the practical skills of mechanical assembly and to the principles of operation of a variety of mechanisms.

The sequence of presentation chosen is by no means inflexible. It is expected that individual instructors may choose to use the materials in other than the given sequence.

The particular topics chosen for inclusion in this volume were selected primarily for convenience and economy of materials. Some instructors may wish to omit some of the exercises or to supplement some of them to better meet their local needs.

The materials are presented in an action oriented format combining many of the features normally found in a textbook with those usually associated with a laboratory manual. Each experiment contains:

1. An INTRODUCTION which identifies the topic to be examined and often includes a rationale for doing the exercise.
2. A DISCUSSION which presents the background, theory, or techniques needed to carry out the exercise.

3. A MATERIALS list which identifies all of the items needed in the laboratory experiment. (Items usually supplied by the student such as pencil and paper are not included in the lists.)

4. A PROCEDURE which presents step by step instructions for performing the experiment. In most instances the measurements are done before calculations so that all of the students can at least finish making the measurements before the laboratory period ends.

5. An ANALYSIS GUIDE which offers suggestions as to how the student might approach interpretation of the data in order to draw conclusions from it.

6. PROBLEMS are included for the purpose of reviewing and reinforcing the points covered in the exercise. The problems may be of the numerical solution type or simply questions about the exercise.

Students should be encouraged to study the textual material, perform the experiment, work the review problems, and submit a technical report on each topic. Following this pattern, the student can acquire an understanding of, and skill with, modern mechanisms that will be very valuable on the job. For best results, these students should be concurrently enrolled in a course in technical mathematics (algebra and trigonometry).

These materials on Mechanical Drives comprise one of a series of volumes prepared for technical students by the TERC EMT staff at Oklahoma State University, under the direction of D.S. Phillips and R.W. Tinnell. The principal author of these materials was R.W. Tinnell.

An *Instructor's Data Guide* is available for use with this volume. Mr. Arthur D. Kincannon was responsible for testing the materials and compiling the instructor's data book for them. Other members of the TERC staff made valuable contributions in the form or criticisms, corrections and suggestions.

It is sincerely hoped that this volume as well as the other volumes in the series, the instructor's data books, and the other supplementary materials will make the study of technology interesting and rewarding for both students and teachers.

THE TERC EMT STAFF

TO THE STUDENT

Duplicate data sheets for each experiment are provided in the back of the book. These are perforated to be removed and completed while performing each experiment. They may then be submitted with the experiment analysis for your instructor's examination.

Contents

experiment 1 *MECHANICAL COMPONENTS*

INTRODUCTION. Throughout this course you will be dealing with mechanical devices of various types. It is, therefore, important that you be able to identify mechanical components and be able to properly assemble them.

In this experiment we will examine some of the more common mechanical components and techniques for handling and assembling them.

DISCUSSION. There are a variety of mechanical devices which are commonly used to transmit motion from one machine part to another. Some of these are gears, belts, links, chains, and cams. Each of these devices has certain features which affect its use in specific applications.

Basically, a gear is a wheel with teeth cut into the circumference. By using toothed gears, it is possible to maintain precise angular relationships between two shafts while transmitting motion from one shaft to another.

The four primary conditions that make the application of gears advisable are:

1. *Center distances between shafts are relatively small.*
2. *Constant speed ratios between shafts must be maintained.*
3. *Shaft speeds are not appropriate for belt drives.*
4. *Relatively high torque must be transmitted.*

Gears are also used to increase or decrease rotary speeds between two shafts.

Gears are classified into three categories based on quality. These are:

1. **Commercial Gears.** Tolerances on these gears are large enough that the gear manufacturer uses high volume low-cost manufacturing methods.
2. **Precision Gears.** Allowable tolerances on these gears are less than those on commercial class gears. To maintain these closer tolerances requires additional steps in the manufacturing process; consequently, the cost of these gears is higher than commercial class gears. Precision class gears are used in instruments and other systems where high accuracy is a major consideration.
3. **Ultra-precision Gears.** This class is for gears of the highest quality instruments and special control systems and devices; therefore, relatively few of these gears are used. Costs of these gears are high because special tools and techniques are required in the manufacturing process.

Since costs are often an important factor in designing a device or system, the designer should select gears of the lowest quality which meet the design objectives.

Gears must be handled carefully and properly to avoid damage which will affect the built-in precision. Damage due to mishandling may change the performance of the device or system in which the gears are used.

When handling precision gears or other components, each part should be handled individually. Pick up one component; put it in

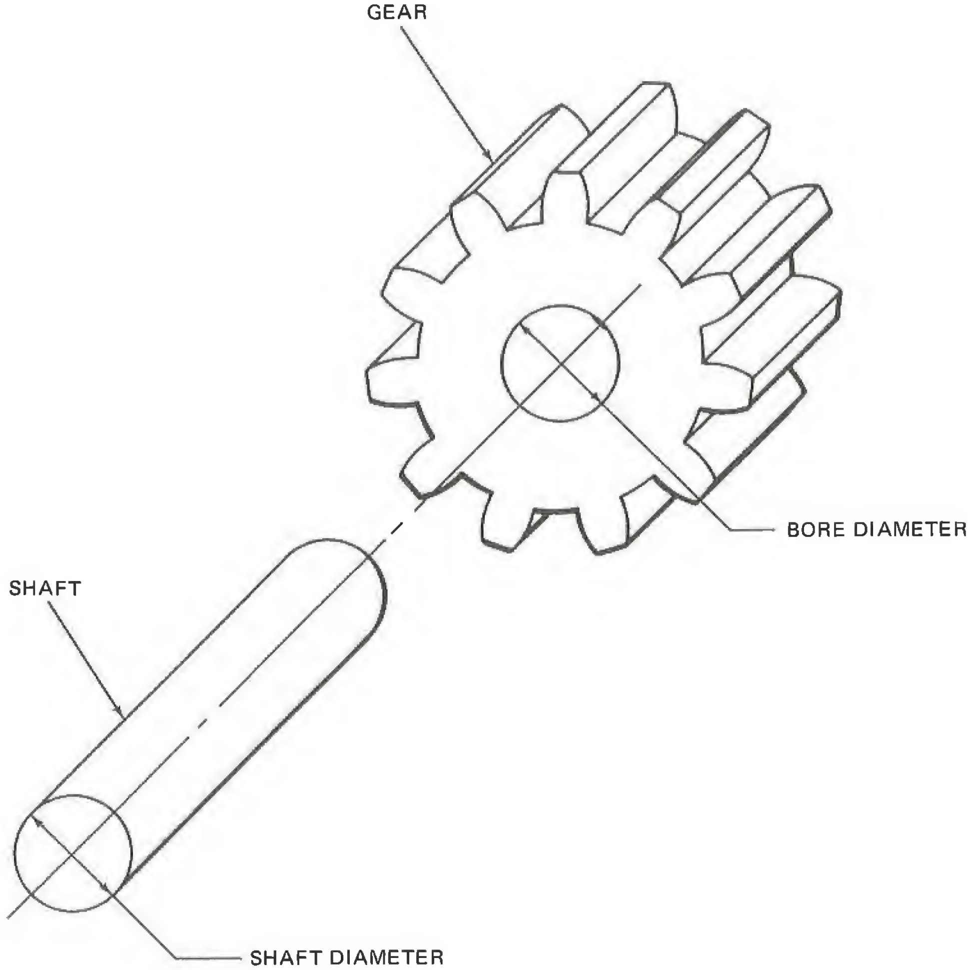


Fig. 1-1 Shaft and Bore Diameters

place; then pick up another component. Use the same procedure when placing components in trays or boxes—never throw or scoop. Be careful not to throw or drop components onto table tops or the floor. When working with components on a table or bench, be careful that they are not knocked off of the working surface. Precision components should be stored in an individual envelope or other suitable container. Do not place more than one component in a container for storage.

When working with precision components, cleanliness is a major concern. Very small particles of dust or dirt, such as cigarette ashes, may cause malfunctions. Some companies provide "cleanrooms" (sometimes called "white rooms") in which precision parts are handled and assembled. Technicians and other employees working in these rooms are required to wear lint-free clothing. Always be sure that your work space is clean and free from dust when working with precision mechanical devices and components.

Gears that are to be fitted to a shaft usually have a hole or bore diameter that is somewhat larger than the shaft diameter to which it is to be fitted, as shown in figure 1-1. For example, a gear with a 1/4 in. nominal

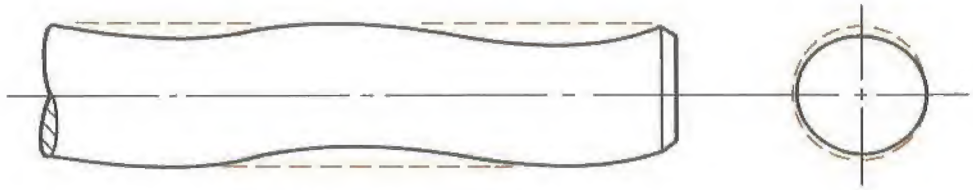


Fig. 1-2 Undesirable Effects of Reducing a Shaft Diameter

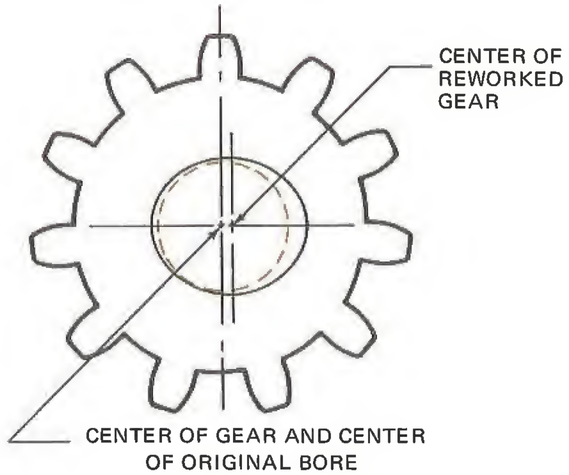


Fig. 1-3 Effect of Reworking Gear Bore

bore may have a bore diameter of .2498″ $^{+.0005''}_{-.0000''}$. A precision shaft, to fit this bore, would have a diameter of .2497″ $^{+.0000''}_{-.0002''}$. Assuming that we have a gear with the smallest possible bore, .2498″ - .0000″, and a shaft with the largest possible diameter, .2497″ + .0000″, it can be seen that we have .0001″ clearance between the shaft and bore. With this much clearance, the gear can be pushed onto the shaft without excessive effort. Do not use force to fit a shaft to a gear bore. If the shaft diameter is too large for a specific gear, check the diameter and, if necessary, choose another shaft or bore diameter.

Do not attempt to reduce the shaft diameter or enlarge the bore diameter by filing or sanding as it is not possible to retain the built-in precision during such an operation. A shaft that has been reworked by hand will have high and low places along its length and the diameter may become an oval rather than a circle, as shown in figure 1-2.

When attempting to increase the bore of a gear, it is extremely difficult to keep the bore center concentric with the gear center. This condition is shown in figure 1-3. Since the gear rotates about the bore center, it is desirable to have the bore center as near the gear center as possible to avoid problems of alignment, excessive wear, and backlash.

Once a gear has been properly fitted to a shaft, it becomes necessary to fasten it to the shaft. There are several acceptable methods for accomplishing this task. Each method has certain advantages and disadvantages which affect its use in specific applications. The choice of fastening method is dependent upon

several criteria, such as the loads to be transmitted, and the need to remove the gear from the shaft for repair and maintenance. The choice of fastening method is also determined, to some extent, by the type of gear being used.

Hubless gears, such as the one shown in figure 1-4b, are the least expensive type of gear to manufacture. Machined hub-type gears, as shown in figure 1-4a, require the removal of considerable waste material which adds to the cost of these gears.

Hubless gears may be fastened to a shaft by several methods. One method is to press or shrink-fit the gear to a hub and then attach the hub to the shaft. This method requires the use of special tools to force a hub with a diameter slightly larger than the bore diameter into the gear bore. Generally, the hub is staked to the gear. Staking simply involves using a punch to cause material from the hub to flow into notches cut into the gear bore, figure 1-5. Staking the gear to the hub pre-

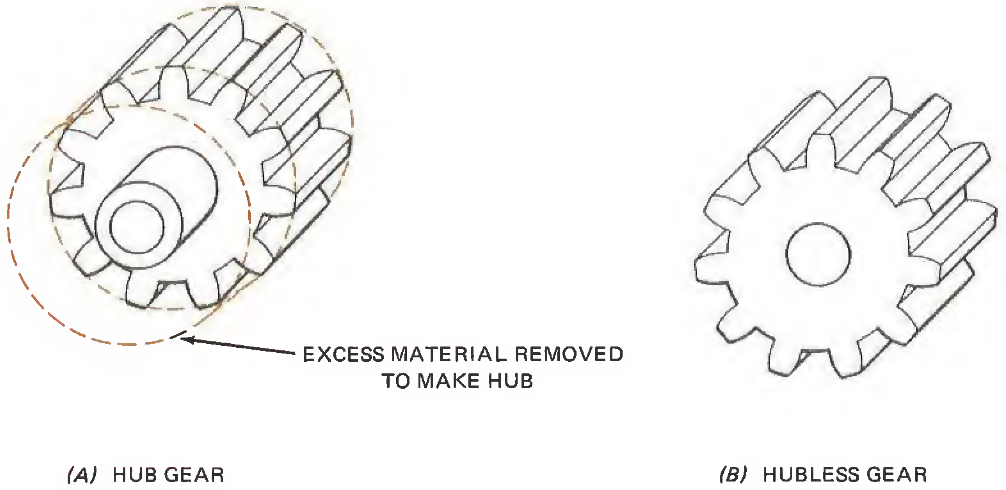


Fig. 1-4 Hub and Hubless Gears

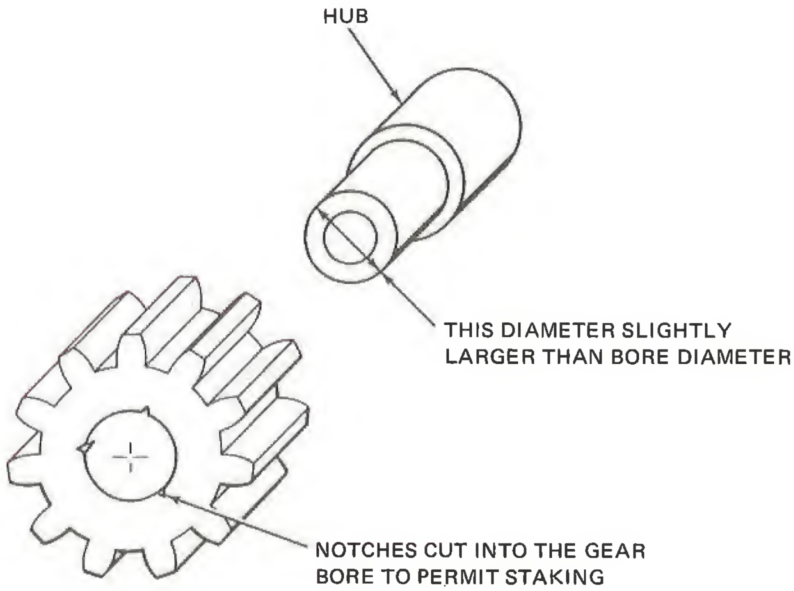


Fig. 1-5 Hubless Gear and Hub

vents slippage between the gear and hub and also prevents the gear from working off the end of the hub. This method of fastening a gear to a hub requires special tools and is usually best done by the gear manufacturer.

Once the hub is attached to a gear, we have, in effect, a hub-type gear and are faced with the problem of attaching the gear and hub to the shaft. There are a number of techniques used to accomplish this task.

When a gear is to be mounted only temporarily, a set screw may be used as shown in figure 1-6. The set screw itself may have a hard tip or it may have a soft nylon tip. Hard tip screws will frequently burr a shaft making gear removal difficult.

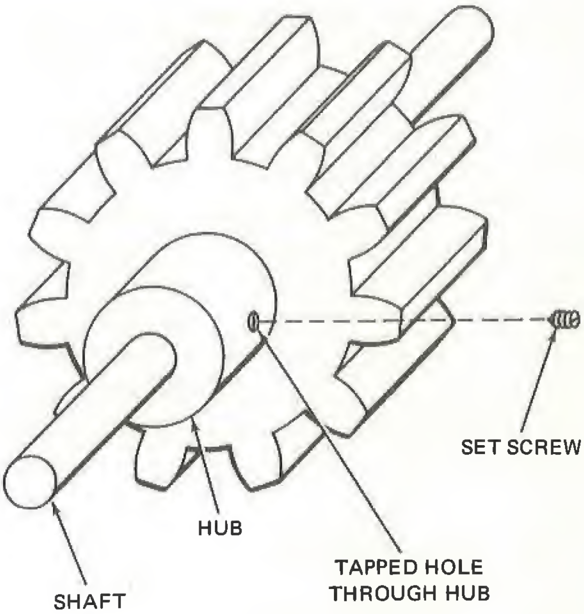


Fig. 1-6 Mounting a Gear with a Set Screw

One simple method for attaching a gear to a shaft is to pin the hub to the shaft as shown in figure 1-7.

While this method allows accurate remounting, it does have two disadvantages: (1) the hole through the shaft weakens the shaft, and (2) it is difficult to remove the hub without damaging the shaft, the hub and gear, or both.

Set screws are often used to hold gears while pin holes are being drilled. Split hub gears are attached to a shaft by means of a hub

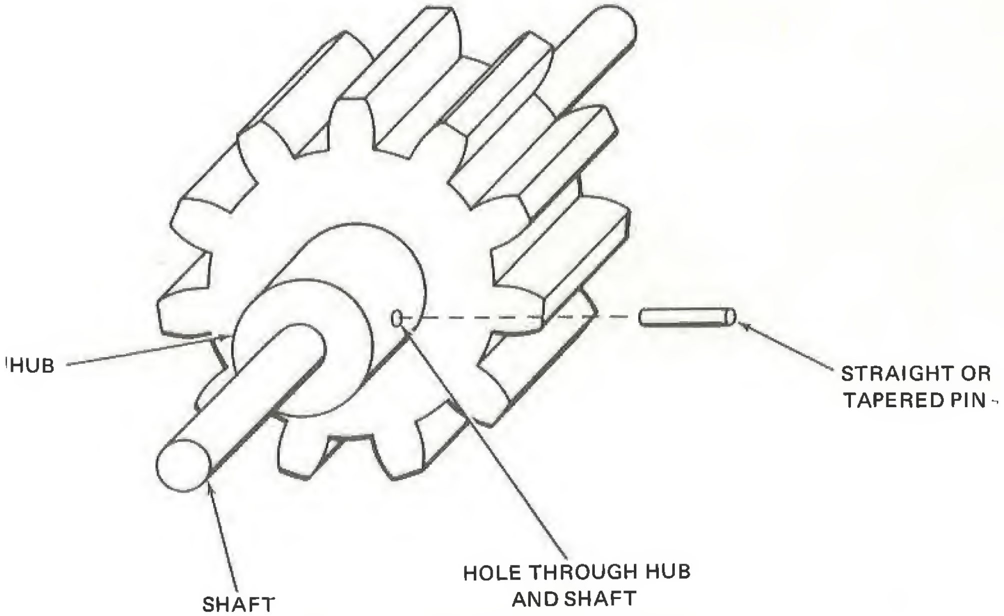


Fig. 1-7 Hub Pinned to Shaft

clamp. This type of gear is shown in figure 1-8. They are easier to assemble and disassemble than are pinned gears. Also, these gears accurately preserve concentricity. The split hub gear, however, does take up more space, requires extra parts and weight, and may cause problems of balancing. The hub itself may be split either into fourths, as shown, or into halves. Once the gear has been placed on the shaft, the hub clamp is slipped onto the hub and a screw or bolt is used to tighten the clamp which, in turn, forces the hub into contact with the shaft.

Hubless gears are sometimes attached to shafts by means of a collet chuck. As shown in figure 1-9b, the collet chuck has a tapered shoulder which fits into a sleeve with a tapered hole. When the nut is tightened, the collet chuck is pulled into the sleeve causing contact between the collet chuck and the shaft. It is important to place a washer between the gear and the nut to prevent damage to the gear.

Let us now direct our attention to some of the fundamental considerations of mount-

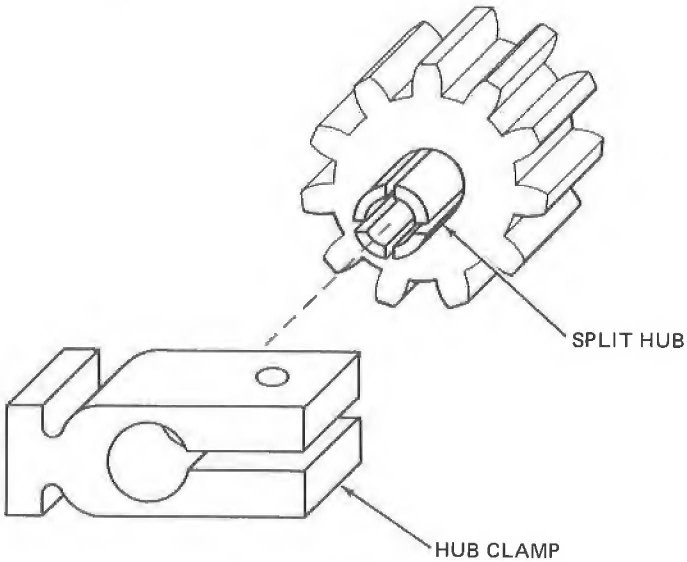


Fig. 1-8 Split Hub Gear and Hub Clamp

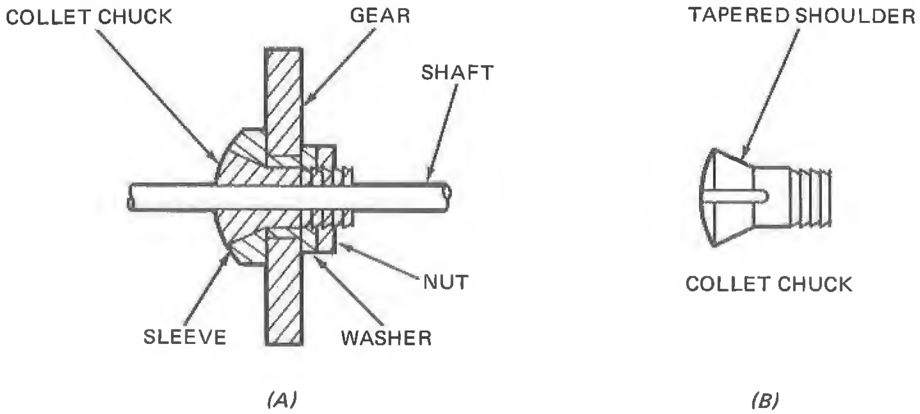


Fig. 1-9 Mounting Hubless Gears with a Collet Chuck

ing gears and shafts into a device for transmitting motion. One of the major concerns in assembling mechanical devices is the elimination of friction. The overall efficiency of a device can be reduced a great deal by frictional losses.

Contact between a shaft and its bearing surface is one of the primary sources of friction in a mechanical device. For this reason, sliding (sleeve, bushing) and rolling (ball, pin) type bearings are normally used for mounting rotating shafts. Roller type bearings may cause less friction than sliding type bearings and are used where it is important to keep frictional losses to a minimum, particularly during starting.

Contact between mating gears is also an important source of friction. Several steps may be taken to reduce frictional losses between mating gears. Proper lubrication is an important factor in reducing friction between mating components. Precise and accurate alignment of gears and shafts is also very important if a mechanical device is to function at maximum efficiency. Shafts for mounting spur gears should be parallel, and the gears should be mounted such that the angle between the shaft and the side of the gear is 90 degrees, as shown in figure 1-10. Failure to meet these conditions may seriously affect the performance of the device.

Notice that the gears shown in figure 1-10 are mounted with the hubs on opposite sides and with the faces mismeshed. This is the proper method for mounting gears which may have burrs. In manufacturing, burrs are sometimes pushed in between gear teeth by

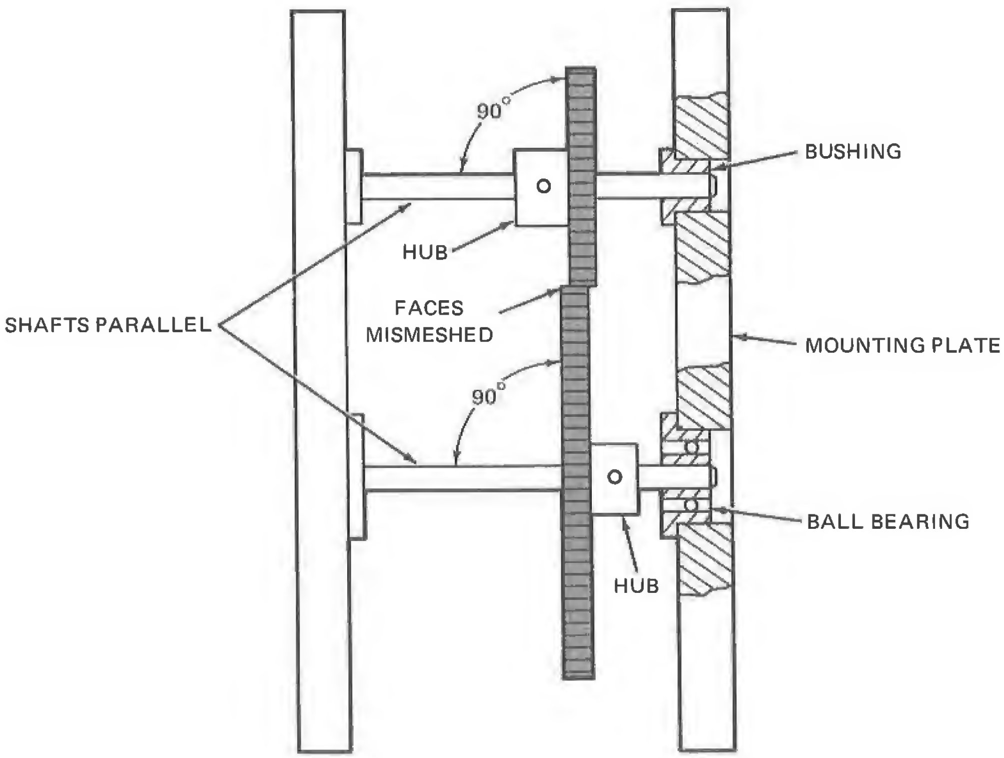


Fig. 1-10 Mounting Spur Gears

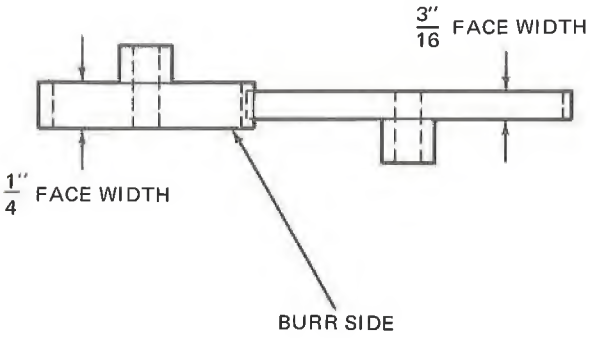


Fig. 1-11 Spur Gears of Different Face Widths

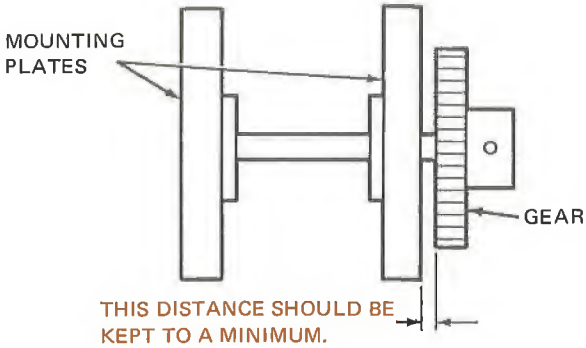


Fig. 1-12 Gear Mounted on an Extended Shaft

the gear cutting tool. On hub type gears, burrs which are formed during manufacture will be on the side opposite the hub. By alternating hubs and mismeshing the faces, we can reduce the possibility of these burrs causing trouble. Another method used to solve this problem when only one gear is likely to have burrs is to use gears of different face widths, as shown in figure 1-11. With modern machining processes, burrs are not frequently encountered. When assembling gears, always mount the gear with the face as close to the bearing as possible. This helps to eliminate flexing of the shaft. The gears in figure 1-10 have been mounted close to the right-hand mounting plate rather than being centered between the two plates.

Gear shafts should usually be supported by two bearings—rarely one or three. By using two supports, we reduce problems of bearing preload and misalignment. In those cases where it is necessary to mount a gear on an extended shaft, the overhang should be kept to a minimum. This is shown in figure 1-12.

Meshed gears in most applications should be of dissimilar materials to reduce galling and noise. For example, use a stainless steel gear in mesh with an aluminum gear, or two gears of different types of steel.

MATERIALS

1 Spur gear, approx. 2 in. OD

1 Spur gear, approx. 1 in. OD

2 Shafts, 4" x 1/4"

2 Bearing plates, with spacers

1 Breadboard with legs and clamps

4 Bearing mounts

4 Bearings

PROCEDURE

1. Assemble the bearing plates as shown in figure 1-13.
2. *Inspect each of the gears, looking for evidence of damage.* Also, look for burrs which have been pushed into the gear teeth. If burrs are found, be sure to make note of this when assembling the gears.

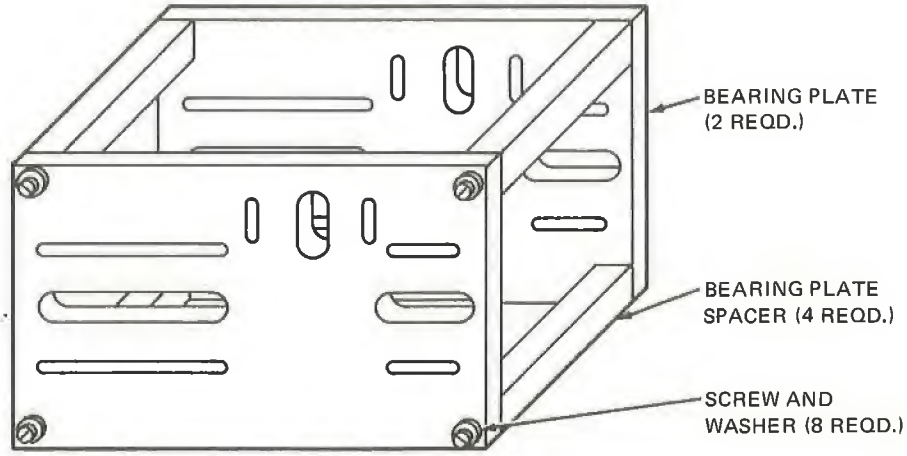


Fig. 1-13 Bearing Plate Assembly

3. Assemble the gears on the shafts and mount them in the bearing plate assembly as shown **Note:** *When fastening the gears to the shaft and fastening bearings to the plates, be sure to fasten the components snugly; but do not use excessive force.*

4. Check to see that the teeth on the two gears are fully meshed.

5. Slowly turn the gears until the larger gear has made one or two complete turns in both directions. The gears should turn smoothly and freely at all times. If a tight spot is detected, identify this spot; loosen the bearings for one of the gears and move it away from the other gear until a snug fit is obtained, and tighten the bearings. **Note:** *Be sure that the gears do not rotate while this adjustment is being made.*

6. With the large gear mismeshed to the right of the small gear, as shown in figure 1-14, turn the gears several times to get a "feel" for a normally operating set of gears.

7. Slide one of the gears until the faces are fully meshed, as shown in figure 1-15; and again turn the gears several times and observe the "feel."

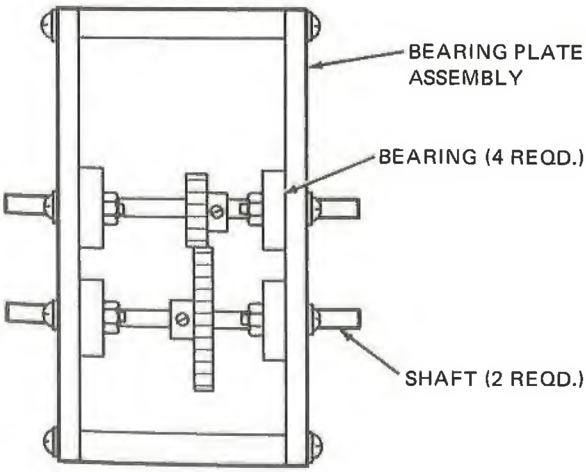


Fig. 1-14 Gear Assembly

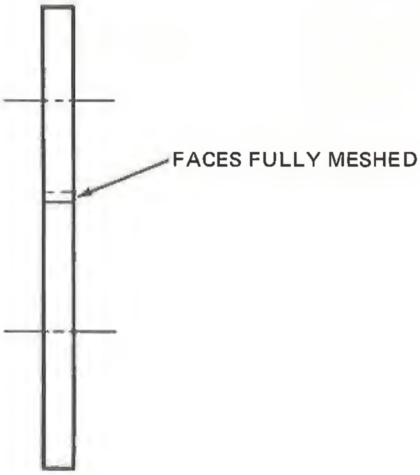


Fig. 1-15 Gear Faces Fully Meshed

8. Slide one of the gears until the large gear is mismeshed to the left of the small gear, turn the gears several times and observe the "feel."
9. Loosen one of the shafts and move it until the gears are barely engaged. Turn the gears several times in opposite directions and observe the "feel." The looseness observed is called "backlash" and only occurs when the relative rotation is changed.
10. Loosen one of the shafts and move it until the gears are not meshed. Tighten one bearing and rotate the shaft to observe the "feel." Move the loose end of the shaft slightly and tighten the bearing on that end. Rotate the shaft and again observe the "feel."

ANALYSIS GUIDE. Since the purpose of this experiment was to become familiar with selected mechanical components and proper techniques for handling and assembling these components, a written report will not be required. Keep in mind, however, that the components and techniques used in this experiment will be used in subsequent experiments.

PROBLEMS

1. Make a sketch of each component. Label each component.
2. Explain why it is possible to feel a tight spot when two gears are meshed.
3. List five important techniques to be observed when using gears.
4. Why is it considered good practice to mount gears close to the bearing supports?
5. If the large gear in the experiment were rotated clockwise, in which direction would the small gear turn?
6. Which of the two gears would turn faster? Why?
7. Why is it important to have the gears and shafts properly mounted and aligned?
8. What might be some of the undesirable effects of using gears that are not properly meshed?

experiment 2 GEAR DIAMETERS

INTRODUCTION. Mechanical laboratory work will frequently involve the construction of a mechanism and the measurement of physical quantities associated with it. In this experiment we shall concentrate on determining the diameter of a gear wheel.

DISCUSSION. Toothed wheels or *gears* have been used to transmit mechanical motion since about the third century B.C. But it was not until the Rennaisance period that the gear wheel was analyzed geometrically in an attempt to optimize its operation. During World War II, precision gearing became very important in aircraft autopilots and other mechanical "computing" applications. Today, with the even greater precision requirements of the space industries, gears are becoming of vital importance to the technician.

Basically, a gear is a wheel with teeth cut into its edge. Figure 2-1 shows a gear wheel with several of its parameters identified.

The *outside circle* of the gear wheel is, as the name implies, a circle drawn about the tips of the teeth. The distance from the center of the gear to the outside circle is called the *outside radius, R_o*. Completely across the outside circle is a distance called the *outside diameter, D_o*. The outside diameter is, of course, twice the outside circle radius.

$$D_o = 2R_o \tag{2.1}$$

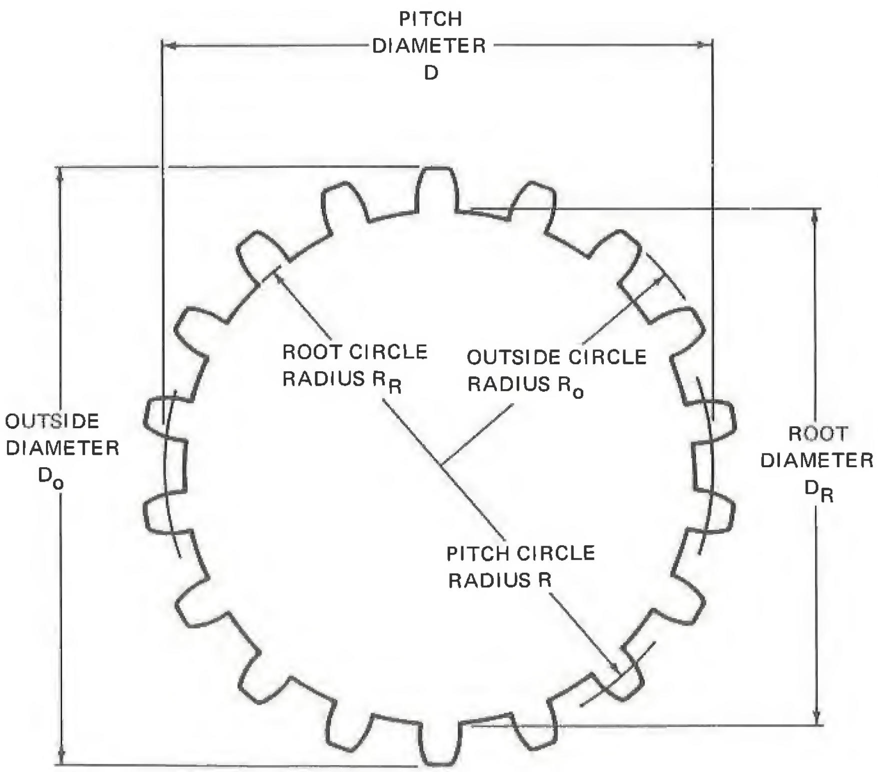


Fig. 2-1 Profile of a Gear Wheel

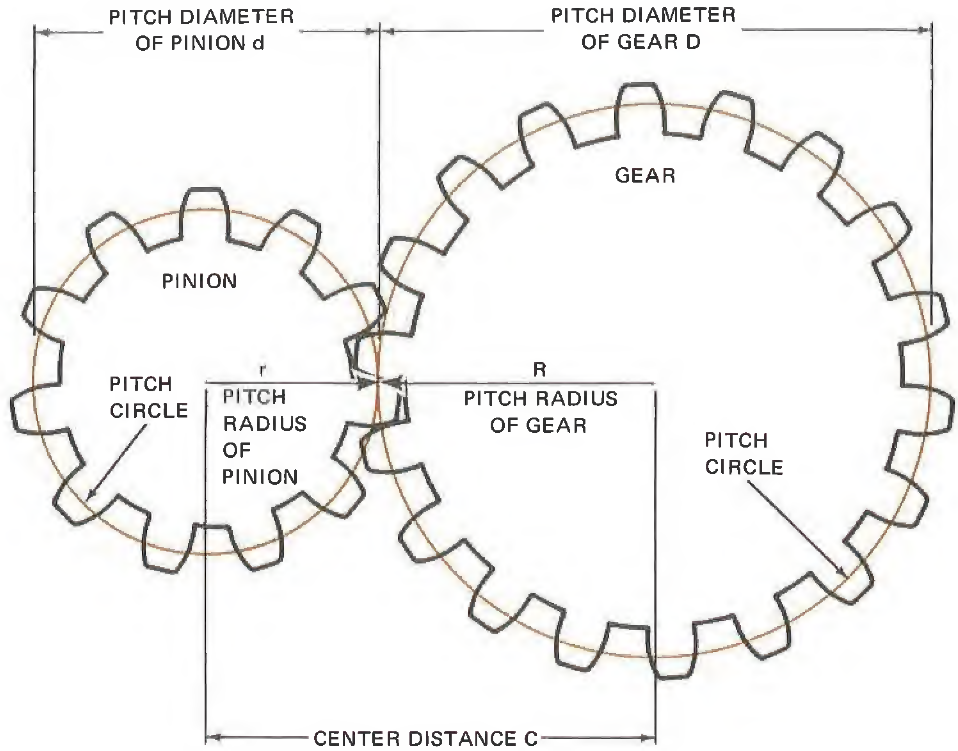


Fig. 2-2 Meshed Gears

The circle drawn around the base (or roots) of the teeth is called the root circle, and its radius and diameter are referred to as the *root radius,* R_R, and the *root diameter,* D_R, respectively. The relationship between these two quantities is, of course,

$$D_R = 2R_R \tag{2.2}$$

Between the outside and root circles is the *pitch circle.* This circle is very important because it is the effective size of the gear. To better understand the significance of this circle, consider the two *meshed* gears shown in figure 2-2. Notice that when the teeth of the two gears mesh they overlap each other. Consequently, the gears have an effective diameter a little less than the outside diameter. We call this diameter the *pitch diameter* of the gear. The pitch diameter (D) and the pitch radius (R) are related by

$$D = 2R \tag{2.3}$$

When two gears are meshed, as in figure 2-2, the smaller of the pair is called the *pinion* and the larger one is called the *gear.* Normally, we use small letters to represent quantities associated with the pinion and capital letters for the gear. Equation 2.3, then, is the relationship for the gear in figure 2-2, and the corresponding relationship for the pinion is

$$d = 2r$$

In figure 2-2 we can observe another important fact dealing with the pitch diameters of two meshed gears; that is, the distance

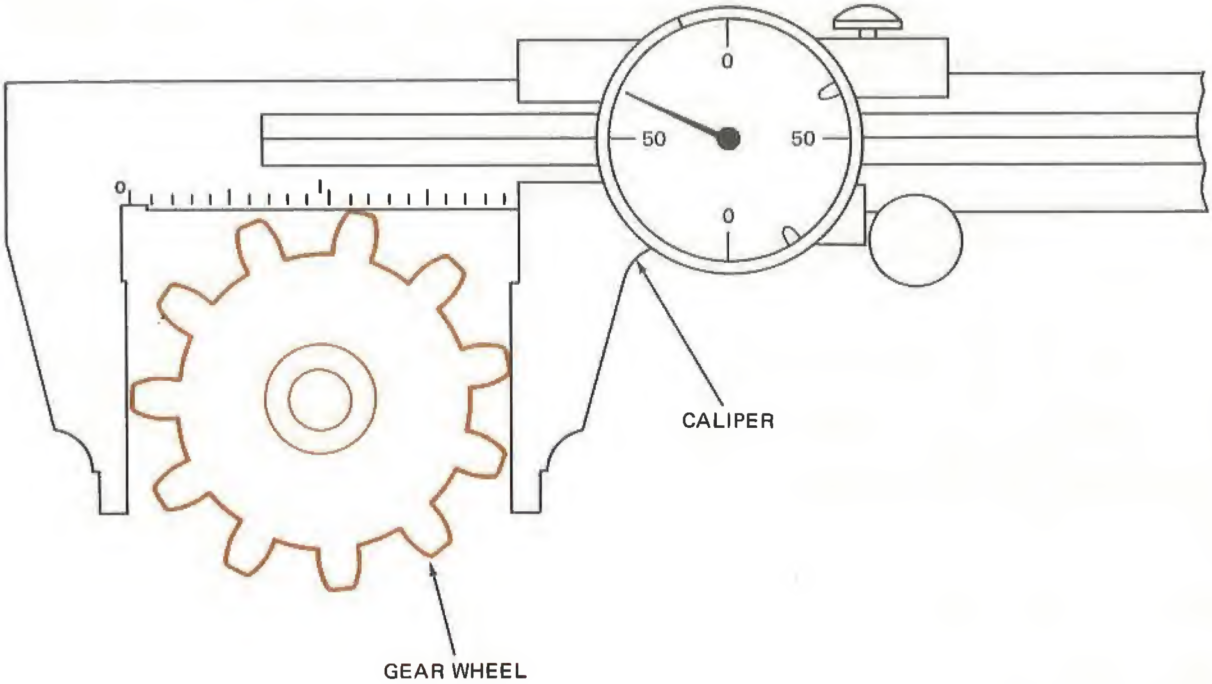


Fig. 2-3 Measuring the Outside Diameter

between the centers of the two gears is equal to the sum of their pitch radiuses (radii). We may therefore express this *center distance*, C, as

$$C = R + r \tag{2.4}$$

And since the pitch radius is always one-half the pitch diameter, we have

$$\boxed{C = \frac{D}{2} + \frac{d}{2} = \frac{D + d}{2} = 1/2\,(D + d)} \tag{2.5}$$

The relationship between the pitch diameter (D) and the outside diameter (D_o) has been standardized such that

$$\boxed{D = D_o \frac{N}{N + 2}} \tag{2.6}$$

is true for most standard size gear wheels. *N* in this equation is the number of teeth on the gear.

In working with gear assemblies, there are three quantities that we can determine very readily. They are:

1. The number of teeth on each gear (N)
2. The outside diameter of each gear (D_o)
3. The center distance between two gears (C)

From these quantities we can determine the approximate pitch diameter of a gear.

The number of teeth on a gear can be found by marking one tooth with a pencil; then starting with the marked tooth, simply count the total number of teeth.

The outside diameter of a gear can be measured with a caliper as shown in figure 2-3. In making the measurement, one should exercise some care to insure that the caliper setting is read accurately. The size of the gear teeth is exaggerated in this drawing to illustrate how inaccuracies can occur.

To determine the center distance, the gear and pinion are mounted on shafts and the outside shaft spacing (X) is measured as shown in figure 2-4. Then the diameter of each shaft (D_1 and D_2) is measured. The center distance is then equal to the outside shaft spacing *less* half of each shaft diameter. That is,

$$C = X - \frac{D_1}{2} - \frac{D_2}{2} = X - 1/2(D_1 + D_2) \quad \textbf{(2.7)}$$

If the two shafts happen to be the same diameter (d_s), then the center distance is

$$C = X - d_s \quad \textbf{(2.8)}$$

For example, if we measure an outside shaft spacing of 2.6 in. and both shafts have a diameter of 0.25 in., then the center distance is

$$C = 2.6 - 0.25 = 2.35 \text{ in.}$$

If we have a dependable value for the outside diameter (D_o) and the number of teeth (N) that a gear has, then we can effectively approximate the pitch diameter. For example, if D_o = 1.333 in. and N = 30, we can calculate

$$D = D_o \frac{N}{N+2} = 1.333 \times \frac{30}{32} = 1.250 \text{ inches}$$

This method of determining D is not extremely accurate. However, for many applications it is adequately accurate. It does have the advantage of being simple and direct. For these reasons we shall use it in this experiment.

When we know the pitch diameters of two mated gears, we can determine the center distance using equation 2.7.

You should remember that the method of determining pitch diameter that we will use is neither standard nor precise. It is, however, easy and direct.

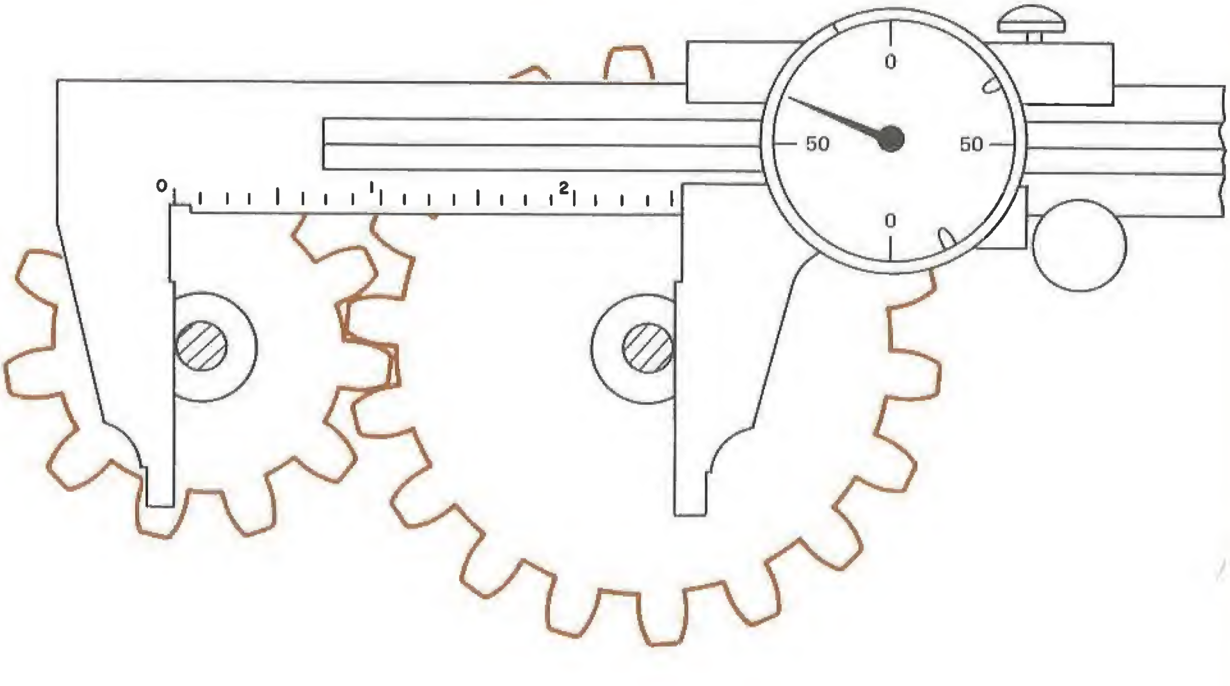


Fig. 2-4 Measuring Outside Shaft Spacing

MATERIALS

1 Dial caliper (0-4 in.)

1 Gear, approx. 2 in. OD

1 Pinion, approx. 1 in. OD

2 Shafts 4″ X 1/4″

2 Bearing plates, with spacers

1 Breadboard with legs and clamps

4 Bearing mounts

4 Bearings

4 Collars

PROCEDURE

1. Measure the outside diameter of the pinion and determine the number of teeth on it. Record these quantities as d_o and n in the Data Table.
2. In the same way, determine and record the diameter (D_o) and number of teeth (N) for the gear.
3. Measure and record the diameter of the shafts (d_s).
4. Assemble the gears on the shafts and mount them between the bearing plates as shown in figure 2-5.
5. Be very sure that the pinion and gear are *fully* meshed. Measure and record the outside shaft spacing (X in the Data Table).
6. Using the appropriate equation from the discussion and the values from step 1, compute and record the pitch diameter of the pinion (d).

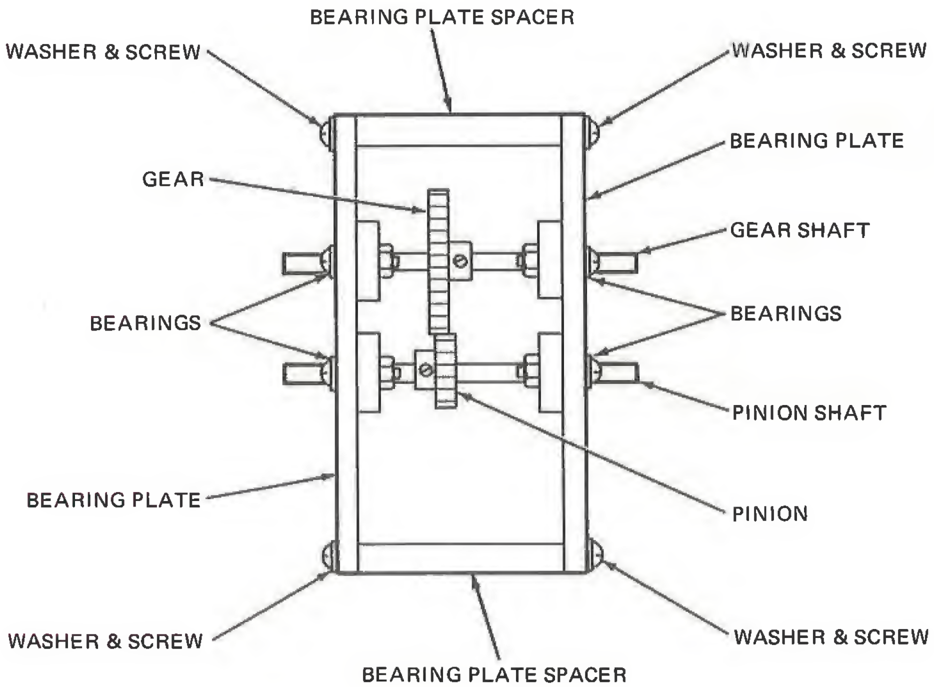


Fig. 2-5 The Experimental Assembly

7. Similarly compute and record the pitch diameter of the gear (D).
8. Compute the pitch radius of the pinion and gear. Record the values as r and R in the Data Table.
9. Using only the pitch radius values, determine the center distance (C) and record it in the Data Table.
10. Using the outside shaft spacing (X) and the shaft diameter (d_s), compute and record the center distance again.

Qty	d_o	n	D_o	N	d_s	X	d	D	r	R
Value										

C

C

Fig. 2-6 The Data Table

ANALYSIS GUIDE. In analyzing your results, consider which method of determining the center distance is most accurate. Under what circumstances would the pitch diameter method be most useful? Which of your values was the largest? Why do you think this was so? Compare your value of D with the appropriate value from a catalog or handbook.

PROBLEMS

1. A gear and pinion have 42 and 12 teeth respectively, and OD's of 11 and 3.5 inches. What is the pitch diameter of each gear?
2. If you were going to mount the gear and pinion in Problem 1 using 0.75 inch shafts, what center distance would you use?
3. A certain gear measures 3.0625 inches OD and has 96 teeth. What is its pitch diameter?
4. A 68-tooth gear is meshed with a 30-tooth pinion. What is the center distance if the gear has an OD of 2.1875 inches and the pinion has an OD of 1.0 inch?

experiment 3 GEAR TEETH

INTRODUCTION. Although gear wheels were used by the ancient Greeks and Romans, relatively little progress in gear design was achieved until Galileo, Huygens, and others started experimenting with different tooth shapes. Since then, most gear applications have developed around a relatively few *standard* tooth shapes. In this experiment we shall consider only the more common gear tooth configurations.

DISCUSSION. Almost everyone realizes that two surfaces that are in rolling contact transmit energy more efficiently than do the same two surfaces in sliding contact. Perhaps the most familiar example of this fact is the automobile tire in contact with the road surface. Most of us have experienced the difference between pushing a car with locked wheels and one with rolling wheels.

The same principle applies to gear teeth. When the teeth of two gears are rolling across each other, energy can be transmitted very effectively. On the other hand, if the gear teeth slip against each other, there is a lot of wasted motion and energy is not so effectively transmitted.

The early Greeks discovered that some shapes of gear teeth produce mainly rolling contact while others result mostly in a sliding contact. By far, the most commonly encountered tooth shape used today is the *involute.* An involute shaped tooth results in a contact point that rolls with little slippage.

You can make an involute curve by taking a cardboard cylinder and winding a piece of string around it. Then tie your pencil to the end of the string and draw the line inscribed by the pencil as the string is unwound. Figure 3-1 shows an involute curve drawn in this way.

As you can imagine, if we continue unwinding the string, we will have a spiral-shaped

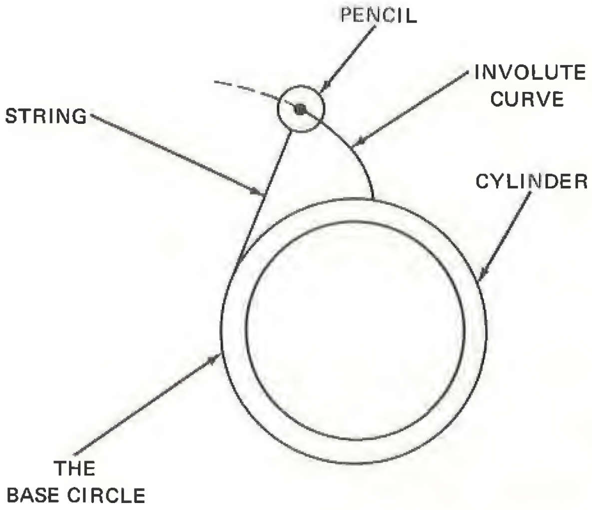


Fig. 3-1 Drawing an Involute Curve

curve around the cylinder. Only the first small portion of the curve is used in shaping modern gear teeth. Each tooth on a gear wheel is involute-shaped on each of its two *faces.* The face of a gear tooth is understood to mean the surface of the tooth that contacts a mating gear.

The circle from which the involute teeth are generated is called the *base circle* of the gear. In larger gears, the base circle will be inside the root circle. However, as we make the diameter of the gear smaller, it becomes necessary to undercut the teeth. That is, the involute curve must be extended inside the base circle. As a result, the base circle will fall outside the root circle. Figure 3-2 shows a gear with undercut teeth. Actually, undercut teeth are very rarely encountered today.

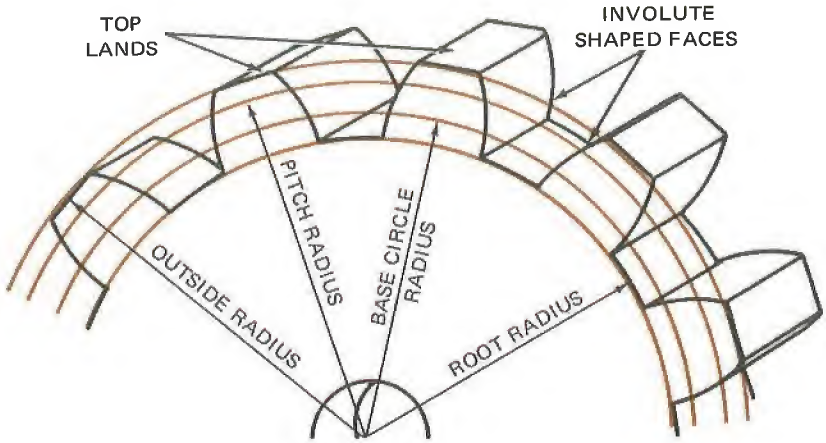


Fig. 3-2 Tooth Shape and Gear Radii

Undercutting the teeth of a gear wheel tends to make the individual tooth weaker. Consequently, there is a practical limit to the amount of undercutting that can be used and still produce a serviceable gear.

Since the amount of undercut necessary is dependent on the number of teeth and the size of the gear, there is a minimum number of teeth that can be put on a particular size gear without resorting to undercutting.

When two gear wheels are meshed, their pitch circles are tangent (touching), as shown at point A in figure 3-3. A portion of each gear tooth extends beyond the pitch circle of the gear. This part of the tooth is called the *addendum* (a), and it is equal to the difference between the gear's outside radius (R_O) and its pitch radius (R). That is

$$a = R_O - R \tag{3.1}$$

And since the radii are one-half of the respective diameters, we have

$$a = \frac{D_O}{2} - \frac{D}{2} = \frac{D_O - D}{2} = 1/2\,(D_O - D) \tag{3.2}$$

In other words, the addendum is one half the difference between the outside and pitch diameters.

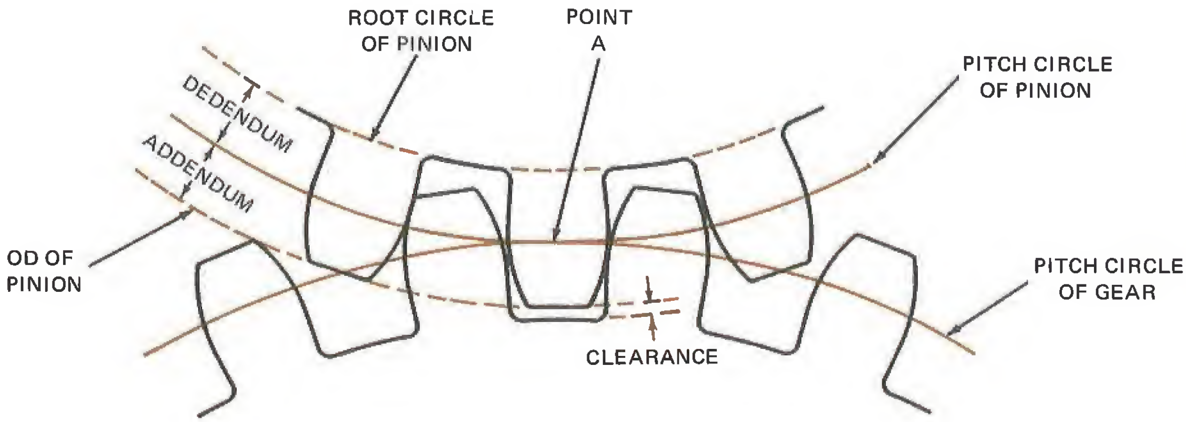


Fig. 3-3 Gear Tooth Nomenclature

Also, we know that the outside and pitch diameters are related by

$$D = D_o \frac{N}{N+2}$$

where N is the number of teeth on the gear. If we substitute this quantity into equation 3-2 for D we have

$$a = 1/2\,(D_o - D_o \frac{N}{N+2}) = \frac{D_o}{2}(1 - \frac{N}{N+2})$$

We can simplify this equation by using the common denominator and adding the right-hand quantities.

$$a = \frac{D_o}{2}(\frac{N+2}{N+2} - \frac{N}{N+2})$$

$$= \frac{D_o}{2}(\frac{2}{N+2}) = \frac{D_o}{N+2} \quad (3.3)$$

This relationship is very convenient because we can determine D_o and N easily.

Returning to figure 3-3 we observe that there is also a portion of each tooth which extends from the root circle to the pitch circle. This part of a gear tooth is called the *dedendum.* When two gears are meshed, the addendum of one protrudes into the dedendum of the other. If the gears are to work smoothly, there must be some *clearance* between the top land of the protruding addendum and the root of the dedendum.

The size of the clearance (c) for American standard full-depth involute teeth has been standardized at

$$c = 0.157\,a \quad (3.4)$$

And since a is D_o/N + 2), we see that

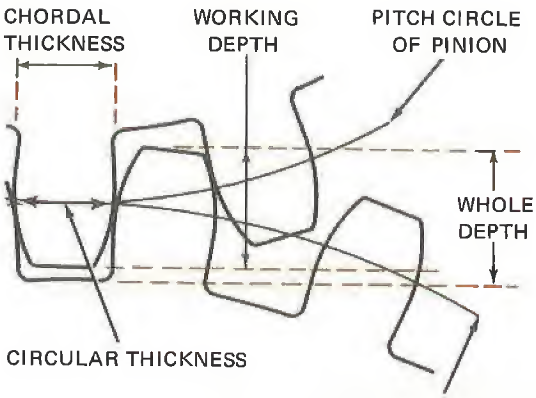


Fig. 3-4 Tooth Depths and Thicknesses

$$c = 0.157 \frac{D_o}{N+2} \quad (3.5)$$

is also valid.

Once again referring to figure 3-3, we can see that the dedendum (b) is equal to the sum of the addendum and the clearance.

$$b = a + c \quad (3.6)$$

or

$$b = \frac{D_o}{N+2} + 0.157 \frac{D_o}{N+2} = 1.157 \frac{D_o}{N+2} \quad (3.7)$$

The total radial distance from the top land of a tooth to its root is called the *whole depth* of the tooth and is equal to the sum of the addendum and dedendum. Figure 3-4 shows a sketch of the whole depth as well as the *working depth* of a pair of gears. Working depth is also a radial distance. For teeth with equal addendums, the working depth is the amount of overlap of two mating teeth and is equal to twice the individual tooth addendum.

Tooth *thickness* (T) is the distance across a tooth along the pitch circle. The *chordal thickness* of a gear tooth is defined as the straight line distance across the tooth at the pitch circle.

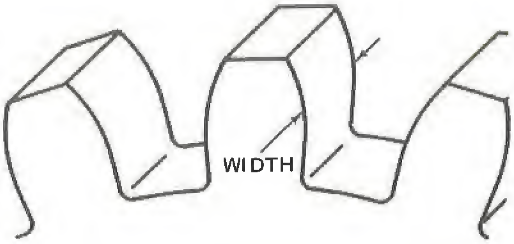


Fig. 3-5 Width of a Gear Tooth

The *width* of a gear tooth is the distance across the face of the tooth. Figure 3-5 shows this distance. It seems sensible that the wider gear teeth are, the stronger they are.

When two gears are rolling together, as seen in figure 3-6, they first come into contact at Point A; then as they rotate, contact is finally broken at Point A′. The path from A to A′ is called the *path of contact*. This path lies along a line which is called the *line of action.* This line passes through the pitch point (P) and is tangent to both base circles.

If we draw the center line through the gears (XX′) and a perpendicular (YY′) through the pitch point (P), then the angle between the perpendicular and the line of action is called the *pressure angle* (ϕ). This angle is one of the characteristics designed into gear teeth. Almost all spur gears used today have pressure angles of either 14 1/2° or 20°. In precision gearing and instrument gearing, the 20° pressure angle gears are currently most widely used. *Gears must have the same pressure angle if they are to be meshed.*

In addition to having the same pressure angle, mating gears must have teeth of the same size. The size of a gear tooth is normally expressed in terms of its *pitch.* The pitch of a gear (or its teeth) may be expressed in several ways. Perhaps the most common method of expressing pitch is the system called *diametral pitch.* The diametral pitch (P_d) of a gear is the ratio of the number of teeth to the pitch diameter.

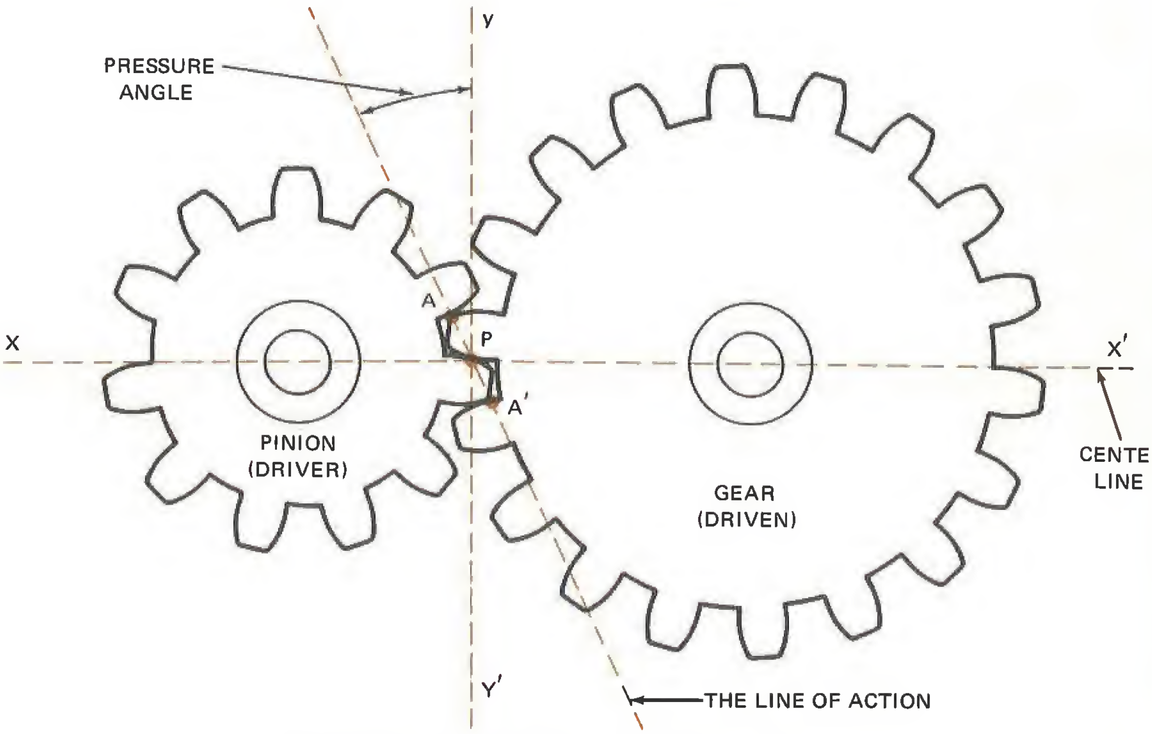


Fig. 3-6 Line of Action and Pressure Angle

That is,

$$\boxed{P_d = \frac{N}{D}} \qquad (3.8)$$

The diametral pitch of an American standard gear is usually a whole number (i.e. 32, 48, etc.).

A system of measuring pitch which is only rarely encountered is the *circular-pitch* system. The circular pitch (P_c) of a gear is the ratio of the pitch circumference to the number of teeth. Since pitch circumference is π times the pitch diameter, we have

$$P_c = \frac{C_p}{N} = \frac{\pi D}{N} = \frac{\pi}{P_d} \qquad (3.9)$$

The circular pitch of American standard gears will normally not be a whole number (i.e. circular pitches of 0.098 and 0.065 are equivalent to diametral pitches of 32 and 48 respectively). Circular pitch is actually the distance per tooth along the pitch circle.

In the United Kingdom, the (British) *Module* system is frequently encountered. The module of a gear is simply the reciprocal of the diametral pitch.

$$\text{Module} = \frac{1}{P_d} \qquad (3.10)$$

The diametral pitch of a gear is actually very handy to know because it is related to other gear parameters. For example, the pitch diameter of a gear is

$$D = D_o \frac{N}{N+2}$$

or

$$\frac{N}{D} = \frac{N+2}{D_o} = P_d \qquad (3.11)$$

Similarly, from equation 3.3, we know that the addendum is

$$a = \frac{D_o}{N+2} = \frac{1}{P_d} \qquad (3.12)$$

The dedendum and clearance are therefore (see equations 3.5 and 3.7)

$$b = 1.157 \frac{D_o}{N+2} = \frac{1.157}{P_d} \qquad (3.13)$$

and

$$c = 0.157 \frac{D_o}{N+2} = \frac{0.157}{P_d} \qquad (3.14)$$

Up to this point we have considered only the American standard full-depth involute tooth. As was mentioned previously, gears having only a few teeth must be undercut, thereby weakening the teeth. When a pinion with very few teeth is essential, it is common practice to use other tooth shapes to avoid excessive undercutting. One such modified tooth system is the American Standard 20-Deg. Stub Involute Tooth. In this system the tooth geometry is such as to produce an addendum, dedendum and clearance of

$$\boxed{a = \frac{0.8}{P_d},\ b = \frac{1}{P_d},\ \text{and } c = \frac{0.2}{P_d}} \qquad (3.15)$$

A second such tooth arrangement is the *Fellows Stub Tooth.* In this system the diametral pitch is always expressed as a common fraction (i.e. 6/8, 7/9, 10/12, etc.). Calculations based on a Fellows Stub Tooth gear are carried out using the equations for a full-depth involute. However, the addendum, dedendum, and clearance values are computed using the *denominator* of the pitch fraction (8, 9, 12 in the example above). All other calculations

are performed using the numerator (6, 7, 10 above). When making the calculations for *a, b,* and *c* for a Fellows Stub Tooth gear, the appropriate relationships are:

$$\boxed{a = \frac{1}{P} \qquad b = \frac{1.25}{P} \qquad c = \frac{0.25}{P}}$$

Notice that these are not quite the same as for either the full-depth or stub involute. Also notice that because of the different tooth shapes, *we cannot use the same relationship between pitch and outside diameters.*

When two spur gears are meshed, as in figure 3-7, we can determine the value of the addendum (a) by subtracting the center distance (C) from the sum of the outside radius of one gear and the pitch radius of the other. Algebraically, that is

$$a = R + r_o - C \qquad \textbf{(3.16)}$$

This relationship will allow us to determine *a* experimentally, provided that we can determine outside and pitch diameter dependably.

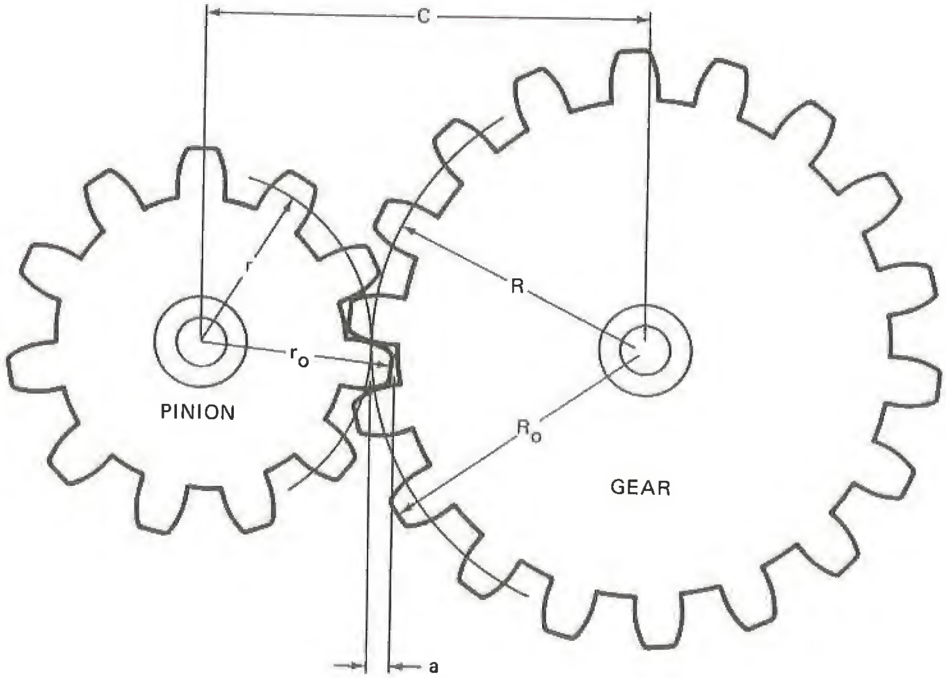


Fig. 3-7 Meshed Gears

MATERIALS

1 Spur gear, approx. 2 in. OD
1 Pinion, approx. 1 in. OD
2 Shafts 4" X 1/4"
2 Bearing Plates with spacers
1 Breadboard with legs and clamps

4 Bearing mounts
4 Bearings
4 Collars
1 Dial caliper

PROCEDURE

1. Inspect all of the components to insure that they are undamaged, and assemble the mechanism shown in figure 3-8.

EACH BEARING ASSEMBLY WILL BE TYPICALLY MOUNTED WITH TWO SCREWS, WASHERS, AND NUTS.

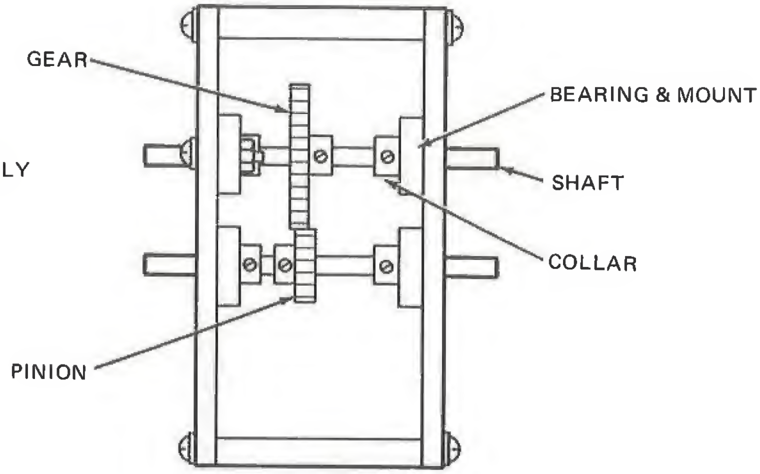


Fig. 3-8 The Experimental Mechanism

2. Measure and record the OD of each gear wheel, (D_o and d_o).
3. Count and record the number of teeth on each gear wheel, (N and n).
4. Measure and record the outside shaft spacing (X) and the shaft diameter (d_s).
5. Compute the value of the center spacing (c) and record it in the Data Table.
6. Compute and record the diametral pitch of each gear wheel, (P_d and p_d).
7. Compute and record the pitch diameter of each gear wheel, (D and d).
8. Using your values of D_o and d, determine the value of *a* for the gear using equation 3.16.
9. Determine *a* for the gear using equation 3.12.
10. Similarly, determine the values of *b* and *c*.
11. Measure and record the tooth width of the gears, (W).
12. With your values of d_o and D, determine the value of *a* for the pinion. Record this value as *a′* in the Data Table.

Qty	D_o	d_o	N	n	X	d_s	C	P_d	p_d
Value									

Qty	D	d	a (3-16)	a (3-12)	b	c	w	a′
Value								

Fig. 3-9 The Data Table

ANALYSIS GUIDE. In analyzing the results from this experiment, you should be primarily concerned with whether or not the relationships discussed in the experiment agreed with your measured quantities.

In particular, did the two gears have the same pitch? Did all of the values of addendum agree?

Check your values of pitch diameter with those given in a handbook or catalog. How well do they agree?

PROBLEMS

1. A certain gear has a pitch diameter of 5.0 in. and a diametral pitch of 8.
   (a) How many teeth does the gear have?
   (b) What is the size of the addendum, dedendum, and clearance?
   (c) What is the OD of the gear?
2. Two 32-pitch gears have 60 and 85 teeth respectively. What are the pitch diameters, outside diameters, and whole tooth depth?
3. How would you figure the circular tooth thickness for a 42-tooth, 24-pitch gear? What is the thickness?
4. A Fellows Stub Tooth gear has a pitch of 5/7 and 24 teeth. What is the pitch diameter, outside diameter, addendum, dedendum, and clearance?
5. What would be the results in problem 1 if the gears were American standard stub involute types?

experiment 4 *DISPLACEMENT RATIO*

INTRODUCTION. One of the basic purposes of a set of instrument gears is to transmit motion from one shaft to another. In this experiment we shall examine some of the factors that determine the effectiveness with which a gear pair performs this function.

DISCUSSION. Let us consider a pair of meshed gears such as shown in figure 4-1. Suppose that we rotate the pinion until point 2 is exactly where point 1 is shown. The pinion has now rotated clockwise through an angle marked θ_p in the figure. Also, we could say that the *pitch point* (that point at which the two pitch circles touch, point 1 in the figure) has moved along the pitch circle of the pinion a distance that we will call the *pitch circle displacement, S_p.*

As the pinion is rotated, the gear also rotates. If the gears are perfectly mated, the pitch point will move along the pitch circle of the gear exactly the same distance as it did along the pitch circle of the pinion. Let's call this pitch circle displacement, S_g.

As mentioned above, the pitch circle displacements of the two gears will be equal if the gears are perfectly mated. However, the pinion rotates clockwise (CW), and the gear rotates counter-clockwise (CCW). It is customary to assign a positive algebraic sign to clockwise motion and a negative sign to counter-clockwise motion; therefore, we may write

$$S_p = -S_g \qquad \textbf{(4.1)}$$

In summary, then, we can say that *meshed gears have the same pitch circle displacement but the direction of displacement is reversed.*

In order to produce the pitch circle dis-

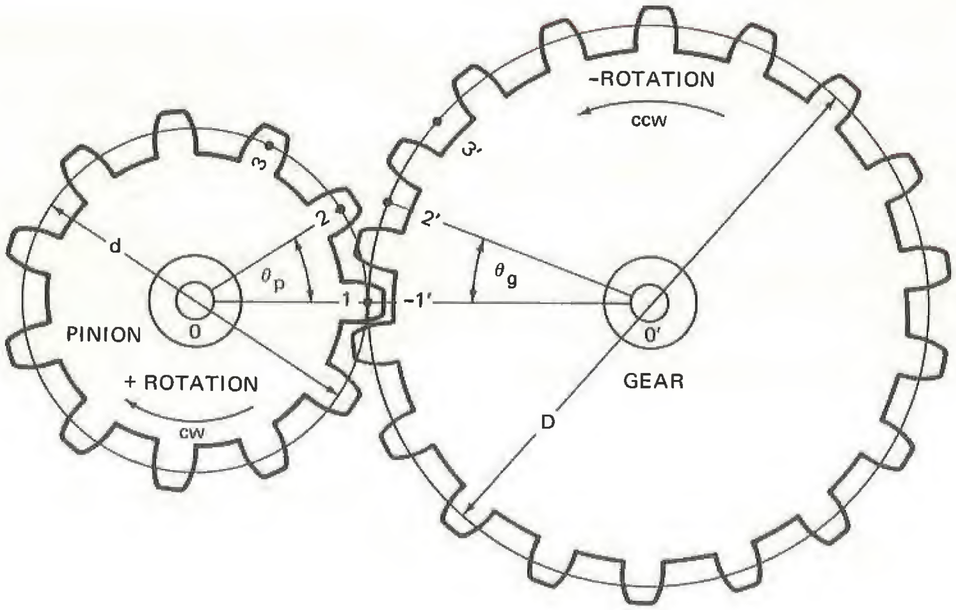


Fig. 4-1 A Pair of Meshed Gears

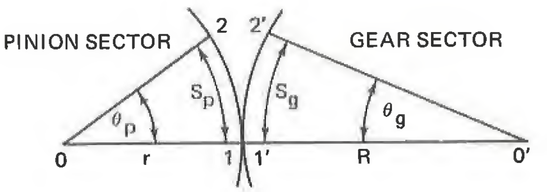


Fig. 4-2 The Rotational Sectors

placement described above, each of the gears must rotate through angles of θ_p and $-\theta_g$, respectively. If we focus our attention on the sector of the pinion pitch circle, we see that it appears as shown in figure 4-2. It is possible to evaluate the angle through which the pinion rotates by observing that the ratio of the displacement to the pitch circle circumference is proportional to the ratio of the angle (θ_p) to the whole circle angle. That is,

$$\frac{S_p}{2\pi r} = \frac{\theta_p}{2\pi}$$

where $2\pi r$ is the circumference of the pitch circle and 2π (on right) is the angle enclosed in the whole pitch circle. The angle θ_p is in radians (360° = 2π radians). Multiplying both sides by $2\pi r$, we have

$$S_p = r\theta_p$$

In exactly the same way we can evaluate S_g as

$$S_g = R\theta_g$$

However, we have seen in equation 4-1 that

$$S_p = -S_g$$

Consequently, we can substitute $r\theta_p$ and $R\theta_g$ for S_p and S_g, giving us

$$r\theta_p = -R\theta_g$$

Then, dividing both sides by R and θ_p we have

$$\frac{r}{R} = -\frac{\theta_g}{\theta_p} \qquad \textbf{(4.2)}$$

If we multiply both the numerator (r) and denominator (R) on the left by 2 we have

$$\frac{2r}{2R} = -\frac{\theta_g}{\theta_p}$$

and we recognize 2r as the pitch diameter of the pinion (d). Similarly, 2R is the pitch diameter of the gear (D). Consequently we see that

$$\boxed{\frac{d}{D} = -\frac{\theta_g}{\theta_p}} \qquad \textbf{(4.3)}$$

which tells us that the ratio of the pitch diameters is equal to the ratio of the *angular displacements.* The negative sign indicates the direction reversal between θ_p and θ_g. The negative sign applies to external tooth gears only.

Moreover, since the pitch of the two gears is the same and equal to N/D,

$$P_d = \frac{n}{d} = \frac{N}{D}$$

we observe that the diameters are related to the turns by

$$d = \frac{n}{P_d} \text{ and } D = \frac{N}{P_d}$$

Substituting these relationships into equation 4.3 gives us

$$\boxed{\frac{n}{N} = -\frac{\theta_g}{\theta_p}} \qquad \textbf{(4.4)}$$

In other words, the angular displacement ratio is related to the *Tooth Ratio* of the gears.

The angular displacement of a gear may be measured in a number of ways. Perhaps the most common way is to use an angular *dial* connected to the shaft on which the gear is mounted. Dials are available in two basic forms: the disk dial and the drum dial. Figure 4-3 shows a sketch of each type.

Dials are available with a great variety of calibration marks. They may be marked for either clockwise or counterclockwise rotation. They may be marked for a full 360 degrees or they may not. The *index* may simply be a single line, or it may be a vernier scale.

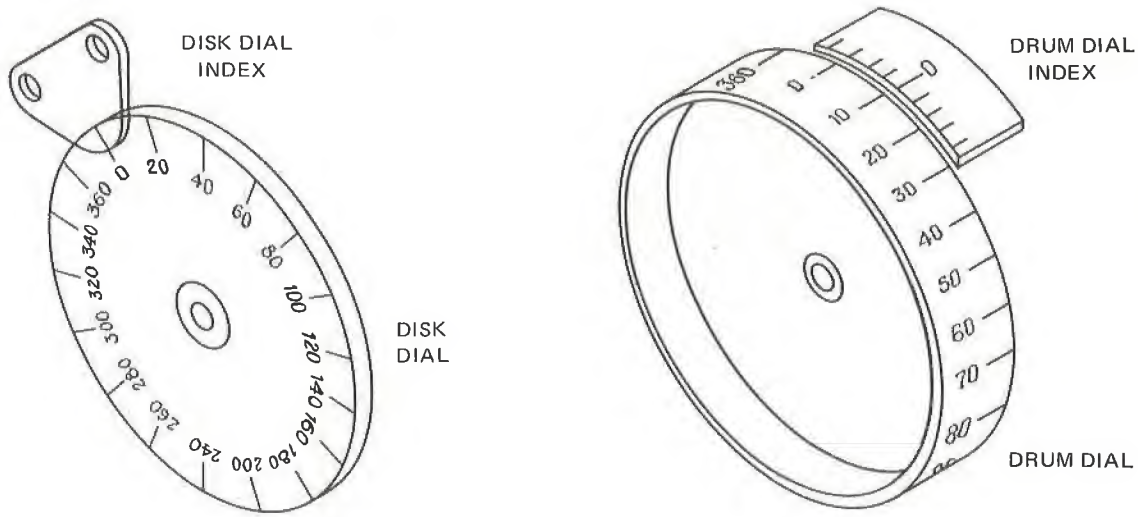


Fig. 4-3 Disk and Drum Type Dials

MATERIALS

- 1 Spur gear, approx. 2 in. OD
- 1 Pinion, approx. 1 in. OD
- 2 Shafts 4" X 1/4"
- 2 Bearing plates with spacers
- 1 Breadboard with legs and clamps
- 4 Bearing mounts
- 4 Bearings
- 4 Collars
- 2 Dials with 1/4 in. bore hubs
- 2 Dial indexes with mounting hardware
- 1 Dial caliper

PROCEDURE

1. Inspect all of the components to insure that they are undamaged and assemble the mechanism shown in figure 4-4.
2. Measure and record the OD of the gear and pinion (D_o and d_o).
3. Record the number of teeth on each gear (N and n)

4. Compute and record the pitch diameter of each gear (D and d).

5. Compute the ratio of the pitch diameters and record it in the Data Table.

6. Compute the tooth ratio and record it in the Data Table.

7. Carefully adjust the gear and pinion dials *so that they both read zero.*

8. Turn the pinion to a dial reading between 20° and 30°. Record the pinion dial setting (θ_p).

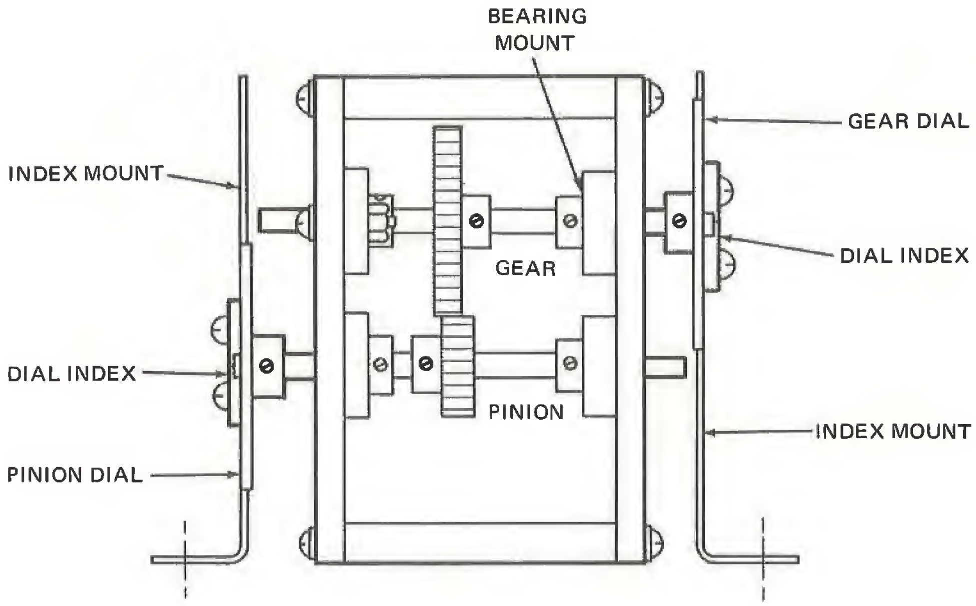


Fig. 4-4 The Experimental Mechanism

9. Read and record the gear dial value (θ_g).

10. Compute the angular displacement ratio (θ_g/θ_p).

11. Using the equation

$$\boxed{\% \text{ Diff.} = 100 \frac{\left(\frac{d}{D} - \frac{\theta_g}{\theta_p}\right)}{\frac{d}{D}}}$$

compute the percent difference between the pitch diameter ratio and the angular displacement ratio. Record this percentage beside θ_g and θ_p in the Data Table.

12. Repeat steps 8, 9, 10, and 11 for pinion dial settings between:
(a) 160° and 170°
(b) 300° and 310°
(c) 390° and 400°
Record these data as measurements numbers 2, 3, and 4 in the Data Table.

D_o	d_o	N	n	D	d	d/D	n/N	% Diff.

MEAS. NO.	θ_p	θ_g	% DIFF.
1			
2			
3			
4			
5			
6			
7			

Fig. 4-5 The Data Table

13. In the same manner, record data for *gear dial settings* between:
    (a) 80° and 90°
    (b) 250° and 260°
    (c) 410° and 420°
14. Compute the percent difference between the pitch diameter ratio and the tooth ratio using the equation

$$\% \text{ Diff.} = 100 \frac{\frac{n}{N} - \frac{d}{D}}{\frac{n}{N}}$$

Record this value in the appropriate Data Table space.

ANALYSIS GUIDE. In evaluating the results that you achieved in this experiment there are a number of points that you should consider. For example, did your results agree in general with the information in the discussion? Was the percent difference between each angular displacement ratio and the pitch diameter ratio the same? Why? What does the value in step 14 tell you? What was the percent difference between your values of pitch diameters and those given in a handbook or catalog?

In preparing your report for this experiment you should cover such points in the analysis of your results.

PROBLEMS

1. Two gears have pitch diameters of 3.50 and 1.40 in. respectively. If the larger gear is displaced 90°, how much will the smaller gear be displaced?
2. One of the gears in problem 1 had 20 teeth. What are the two possible numbers of teeth on the other gear?
3. A certain pinion rotates 215° when its mating gear rotates 123°. How many teeth does the pinion have if the gear has 70 teeth?
4. What are the pitch diameters of the two gears in problem 3 if they are 32-pitch, 20° gears?
5. What are the ODs of the gears in problem 3?

experiment 5 VELOCITY RATIO

INTRODUCTION. In a practical application a gear pair is used to transmit motion from one shaft to another. As a set of gears performs this function, they may also provide a change in rotational speed. In this experiment we shall consider the factors which determine the relative velocities of a gear pair.

DISCUSSION. When the pinion in figure 5-1 is rotated through an angle θ_p, the gear rotates through a corresponding angle θ_g. The relationship between these two angles and the gears' parameters can be summarized as

$$\boxed{-\frac{\theta_g}{\theta_p} = \frac{n}{N} = \frac{d}{D}} \qquad (5.1)$$

where n and N are the tooth counts of the pinion and gear respectively. Also, d and D are the pitch diameters of the pinion and gear.

Since the two gears are meshed, the teeth on the gear move the same distance (S_g) as do those on the pinion (S_p), but in opposite directions.

$$S_p = - S_g$$

Moreover, since the two gears move for exactly the same length of time, we can write

$$\frac{S_p}{T} = - \frac{S_g}{T}$$

Since distance traveled divided by the time of travel is, by definition, what we term *velocity,* we see that the pitch circle velocities of the two gears are equal and in opposite directions. Using v as the symbol for pitch circle velocity, we have

$$v_p = - v_g \qquad (5.2)$$

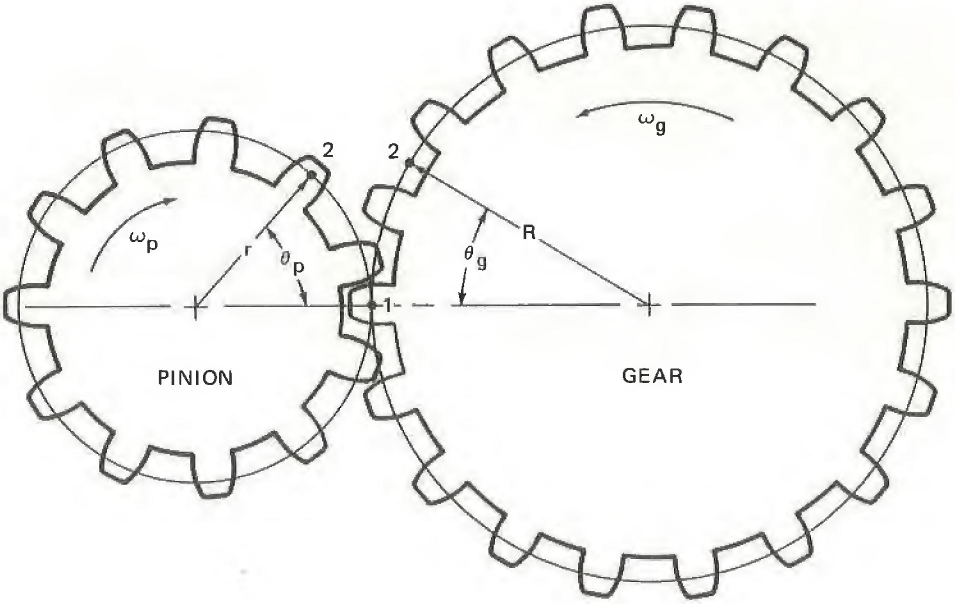


Fig. 5-1 Meshed Gears

The pitch circle velocity is often expressed in teeth per second and may be called the *tooth velocity* of the gear. Tooth velocity may also be expressed in inches-per-second or even feet-per-second, depending on convenience.

The *angular velocity* (ω) of a rotating body is an expression of how fast it is turning. Angular velocity may be expressed in degrees-per-second, radians-per-second, or revolutions-per-minute. This last one (RPM) is perhaps the most commonly encountered unit system. In any event, angular velocity is always an expression of an angle of rotation per unit of time. (A revolution is, of course, 360° of rotation.) We may, therefore, write an equation for angular velocity in the form,

$$\omega = \frac{\theta}{t} \tag{5.3}$$

At this point, let's return our attention to equation 5-1 which we may write in the form,

$$\frac{\theta_g}{\theta_p} = -\frac{d}{D}$$

Then if we divide both the numerator and denominator on the left by t (time), we have

$$\frac{\theta_{g/t}}{\theta_{p/t}} = -\frac{d}{D}$$

And we understand that $\theta_{g/t}$ is the angular velocity of the gear (ω_g) while $\theta_{p/t}$ is the angular velocity of the pinion (ω_p).

Consequently, we find that the ratio of the angular velocities is equal to the negative ratio of the pitch diameters:

$$\boxed{\frac{\omega_g}{\omega_p} = -\frac{d}{D}} \tag{5.4}$$

Also, the pitch diameter ratio and the tooth ratio are equal,

$$\boxed{\frac{\omega_g}{\omega_p} = -\frac{n}{N}} \tag{5.5}$$

These two relationships involving the angular velocities of meshed gears are very useful. In this case, ω may be either radians per second or revolutions per minute. Consider the following example of their application.

Example: A 36-tooth 32-pitch pinion turning 500 RPM is meshed with a 60-tooth gear. What is the angular velocity of the gear?

Solution:

$$\frac{\omega_g}{\omega_p} = -\frac{n}{N}$$

$$\frac{\omega_g}{500\ \text{RPM}} = -\frac{36}{60}$$

$$\omega_g = -500\,\frac{36}{60}\ \text{RPM}$$

$$\omega_g = -300\ \text{RPM}$$

Note that the negative sign indicates rotation in a direction opposite to that of the pinion.

Equation 5.4 is applied in exactly the same way when the pitch diameters are known. In the above example, we could have found the pitch diameters using

$$d = \frac{n}{P_d} \text{ and } D = \frac{N}{P_d} \tag{5.6}$$

When the angular velocity and the number of teeth are known for a given gear, we can find the tooth velocity very easily. Since N is the number of teeth-per-gear revolution and ω is the revolutions-per-minute, we have

$$v = N\omega \text{ teeth-per-minute}$$

or

$$v = \frac{N\omega}{60} \text{ teeth-per-second} \qquad \textbf{(5.7)}$$

Similarly, if the pitch diameter and RPM are known, we may find the pitch circle velocity as follows:

1. The pitch circumference is the distance that a point on the pitch circle travels in one revolution and equals πD.
2. The angular velocity, ω, is the number of revolutions-per-minute.
3. Therefore, the pitch circle velocity is

$$v = \pi D\omega \text{ inches-per-minute}$$

or

$$v = \frac{\pi D\omega}{60} \text{ inches-per-second} \qquad \textbf{(5.8)}$$

Equations 5.7 and 5.8 only produce the units shown when ω is in RPM and D is in inches.

Any practical encounter with gear wheel velocities eventually raises the question of measurement. How do you measure angular velocity? The answer is, with a *tachometer.*

Tachometers come in a great variety of forms. There are mechanical tachometers, electrical tachometers, and electronic tachometers, as well as optical tachometers, nuclear tachometers and a good many others. In this experiment we will use a device called a *stroboscope.*

A stroboscope or *strobe,* as it is commonly called, is a light that flashes at a controlled rate. To see how we can use a controlled flashing light to measure speed, let's consider the fan blade shown in figure 5-2. Suppose that the fan is running and the blade is revolving at 1800 RPM. Looking at the blade we will see just a blur because the fan is turning so fast.

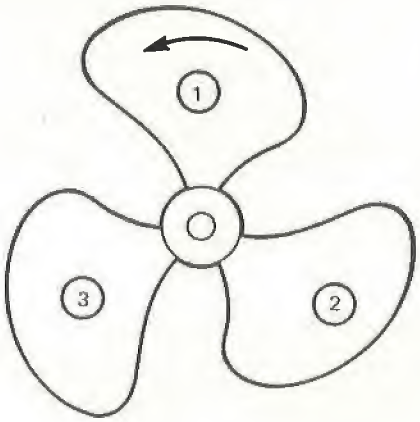


Fig. 5-2 Strobing A Fan Blade

Now, let's set the strobe light in front of the fan and set it to flash 1800 times a minute. The fan blades will *appear* to stand still. What is happening is that even though the blades are turning, they are in the same position every time the light flashes. During the time that the light is off, the blades turn one revolution, then end up back where they were when the light flashes again.

Consequently, it would seem that all we have to do is adjust the strobe rate until the fan appears to stand still. Then read the speed in RPM on the scale of the stroboscope.

There are, however, a couple of things we had better consider. The fan in figure 5-2 has three blades which look alike, so if we

flash the strobe 5400 times a minute (3 X 1800), the blades will again appear to stand still. This happens because we "see" the blades every 1/3 of a revolution and since the blades are alike, we can't tell that we are seeing a different blade each time. This difficulty can easily be cured by marking one of the blades with a scratch or a piece of tape. Then if the mark stands still, we know the strobe isn't flashing too fast.

In an opposite manner, if we set the strobe to flash 900 times a minute (1800 X 1/2), the marked blade will turn two revolutions between flashes and will also appear to stand still. Similarly, it will appear to stand still at 600 flashes (1800 X 1/3), 450 flashes (1800 X 1/4), 360 flashes (1800 X 1/3), 300 flashes, etc. per minute.

To avoid this problem we must be sure that the flash rate of the stroboscope is set to the *highest possible value* which will cause the marked blade to stand still.

Once in a while it is difficult to be sure that we have the highest possible flash rate that will make the mark stand still. In such a case we take the *highest available reading* which makes the mark stand still. Since we know that this reading is some angular velocity, we call this reading ω/x. Then we slowly and carefully reduce the flash rate to the *next lower reading* which is the next smaller integral fraction of ω and is equal to $\omega/(x+1)$. We then take the product of these two readings and divide it by their difference: *See box below.*

And we have algebraically,

$$\frac{\left(\frac{\omega}{x}\right)\left(\frac{\omega}{x+1}\right)}{\frac{\omega}{x}-\frac{\omega}{x+1}} = \frac{\frac{\omega^2}{x^2+x}}{\frac{\omega x+\omega-\omega x}{x^2+x}}$$

$$= \frac{\omega^2}{\omega} = \omega$$

which is, of course, the actual value of the angular velocity.

For example, suppose that 900 flashes per minute was the highest reading we could get with our stroboscope and the fan discussed above. We would reduce the strobe rate carefully until we found the next lower reading which caused the marked blade to stand still. This reading would be 600 flashes per minute. We then calculate the angular velocity.

$$\omega = \frac{(900)(600)}{(900)-(600)}$$

$$= \frac{540{,}000}{300} = 1800 \text{ RPM}$$

This technique is quite useful, but since it is rather involved it should only be used when the *highest possible value* of flash rate cannot be found.

Finally a word of caution — **DON'T FORGET THAT A STROBOSCOPE DOES NOT STOP ROTATION** — it only makes it look stopped. **Don't try to touch a rotating mechanism when it is being strobed.**

$$\frac{\text{(highest available reading)} \times \text{(next lower reading)}}{\text{(highest available reading)} - \text{(next lower reading)}}$$

MATERIALS

1 Spur gear, approx. 1-1/2 in. OD
1 Pinion, approx. 1 in. OD
2 Shafts 4" X 1/4"
2 Bearing plates with spacers
1 Breadboard with legs and clamps
4 Bearing mounts
4 Bearings
4 Collars
1 Motor and mount
1 Shaft coupling
1 Power supply
2 Dials with 1/4 in. bore hubs
1 Dial caliper
1 Stroboscope

PROCEDURE.

1. Inspect each of the components to insure that they are undamaged.
2. Measure and record the outside diameters of the two gear wheels, (d_o and D_o).
3. Count the number of teeth on each gear wheel and record your results, (n and N).
4. Compute and record the pitch diameters of the two gear wheels, (d and D).
5. Assemble the mechanism shown in figure 5-3. **Be very sure that the bearings for each shaft are aligned well enough to allow free rotation. Check the entire mechanism for free rotation before connecting the motor wires to the power supply.**

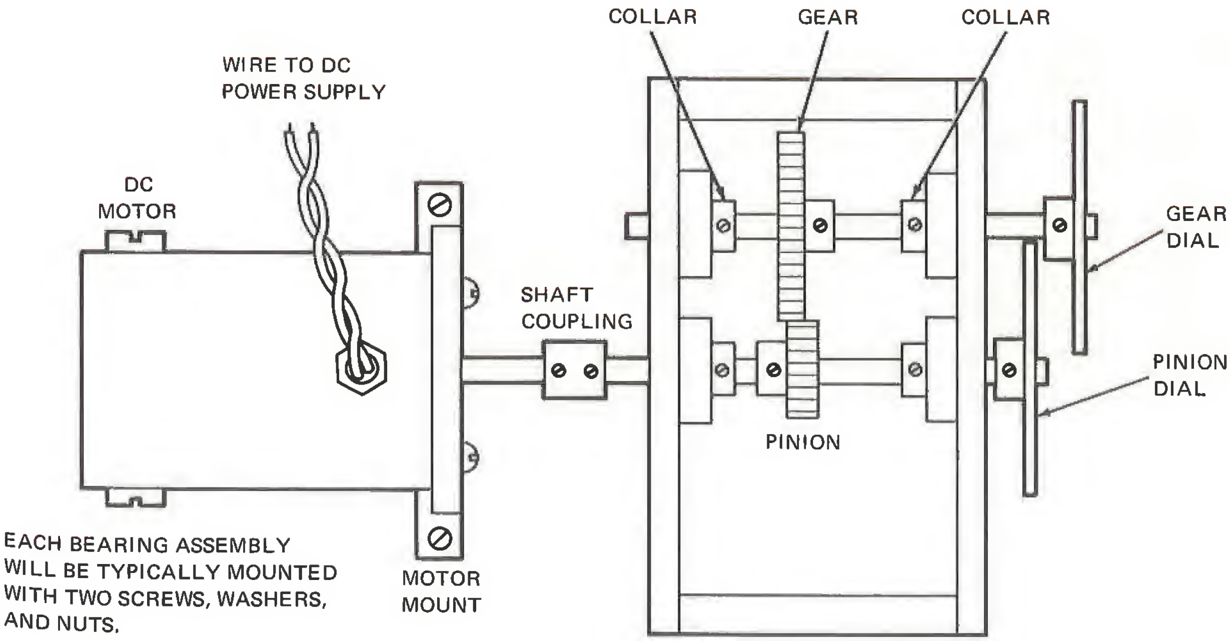


Fig. 5-3 The Experimental Mechanism

6. With the power off, connect the motor wires and set the voltage to zero. Then turn on the power supply. Set the power supply voltage to about 24 volts. Allow the mechanism to run for several minutes. During this time turn on the stroboscope and allow it to warm up.
7. Starting at a strobe setting of about 5000 RPM, slowly decrease the strobe rate until you locate the highest possible value which will make the numbers on the pinion dial appear to stand still. Record this value as ω_p in the Data Table.

   While you have the stroboscope at this setting, take particular note of the appearance of the portion of the pinion dial which is in the shadow of the gear dial.
8. Slowly decrease the strobe rate until the gear dial appears to stand still. Record this reading as ω_g in the Data Table.
9. Compute the ratio ω_g/ω_p and record it in the Data Table.
10. Compute n/N and record it in the Data Table.
11. Compute the percent difference between n/N and ω_g/ω_p.
12. Compute the pitch diameter ratio d/D and record it in the Data Table.
13. Compute the percent difference between d/D and ω_g/ω_p.
14. Using ω_p and n compute the tooth speed of the pinion in teeth per second, (v_p).
15. Using ω_g and N compute the tooth speed of the gear in teeth per second, (v_g).
16. Compute the percent difference between the two values of tooth speed.
17. *Decrease* the strobe rate again until the pinion dial appears to stand still. Record this value as ω_1 in the Data Table.
18. Again decrease the strobe rate until the pinion dial appears to stand still. Record this value as ω_2 in the Data Table.
19. Using the method of the largest available reading and the values of ω_1 and ω_2, compute ω'_p.
20. Compute the percent difference between ω_p and ω'_p.
21. Reduce the power supply voltage to about 20 volts and repeat steps 7, 8, 9, 11, and 13 through 20.

ANALYSIS GUIDE. In analyzing your results for this experiment you should cover such points as:

1. What did you see in step 7 and why?
2. Why is the percent difference between the angular velocity ratio and the other two ratios so large? Does it mean that measurements were inaccurate?
3. Which method of measuring ω_p was most convenient? Why?

In addition to these specific kinds of things, you should consider the general importance of the exercise. In other words, was the material presented in the discussion verified?

Measured Values

d_o	D_o	n	N	ω_p	ω_g	ω_1	ω_2

Computed Values

d	D	$\frac{\omega_g}{\omega_p}$	$\frac{n}{N}$	$\frac{d}{D}$	v_p	v_g	ω'_p

Percent Difference

between $\frac{\omega_g}{\omega_p}$ and $\frac{n}{N}$	between $\frac{\omega_g}{\omega_p}$ and $\frac{d}{D}$	between v_p and v_g	between ω_p and ω'_p

Fig. 5-4 The Data Tables

PROBLEMS

1. Two 32-pitch gears are meshed and the gear is running at 1450 RPM. What is the speed of the pinion if the gear wheels have 24 and 60 teeth?
2. What are the pitch diameters of the gear wheels in Problem 1?
3. What is the tooth speed in Problem 1? Give your answer in teeth-per-second.
4. What is the pitch circle velocity in Problem 1? Give your answer in inches-per-second.
5. What is the center distance between the two shafts in Problem 1?
6. What are the OD's of the gears in Problem 1?
7. A rotating gear is strobed and appears to stand still at 2800 and 2240 flashes-per-minute but not in between these two rates. What is the angular velocity of the gear?
8. If the gear in Problem 7 is 72 pitch and has 40 teeth, how fast will a 120-tooth gear turn if they are meshed?
9. What is the pitch circle velocity of gears in Problem 8?

INTRODUCTION. Meshed gear wheels are capable of transmitting mechanical energy from one shaft to another. In this experiment we shall examine the relationships between energy transfer and the physical parameters of a gear pair.

DISCUSSION. In mechanics, *work* can be defined as a constant force acting through a distance. That is,

$$\boxed{W = fs} \qquad (6.1)$$

So if a man raises a 350-pound automobile engine 2 feet with a hoist, the amount of work he has done is

$$W = fs = (350 \text{ lb})\ (2 \text{ ft}) = 700 \text{ foot-pounds}$$

Mechanical *power,* on the other hand, is defined as the rate at which work is performed, or

$$\boxed{P = \frac{W}{t} = \frac{fs}{t}} \qquad (6.2)$$

Therefore, if the man in the above example took two minutes to raise the engine, the power required to do the job was

$$P = \frac{W}{t} = \frac{700 \text{ ft-lb}}{2 \text{ min.}} = 350 \text{ ft-lb per minute}$$

James Watt, in working with his steam engine, found it convenient to use a unit of mechanical power that he called a *horse power.* He defined one horse power as being the same as 33,000 ft-lb per min. We can, therefore, always convert ft-lb/min. to horse power using

$$\boxed{HP = \frac{\text{ft-lb/min}}{33000}} \qquad (6.3)$$

In the example above, the horse power required to raise the automobile engine was:

$$HP = \frac{\text{ft-lb/min}}{33000} = \frac{350}{33000} = 0.0106 \text{ horse power}$$

When the man in the example raised the auto engine, he did 700 ft-lb of work. This amount of work is by no means lost. It is, in fact, *stored* in the engine itself. That is, the engine is now *capable* of doing 700 ft-lb of work as it returns to its original position. We call this capability to do work **Energy.** There are two ways in which energy may be stored by an object. One way for an object to store energy is by virtue of its position. This is the case with the engine in the previous example. We call this type of energy *Potential Energy* and it is frequently the product of the object's weight and its height. Therefore, we have

$$\boxed{PE = wh} \qquad (6.4)$$

Consequently, the potential energy of the engine in the example is

$$PE = wh = (350 \text{ lb.})\ (2 \text{ ft.}) = 700 \text{ ft-lb}$$

The second way in which an object can store energy is by virtue of its velocity. This type of energy is called *Kinetic Energy.* The kinetic energy of a translating object is one-half its mass times its velocity squared. That is

$$\boxed{KE = 1/2\ mv^2} \qquad (6.5)$$

Suppose the engine in the example falls. As it travels downward it picks up speed until just before it touches ground. It has reached its maximum velocity. Now since it has not

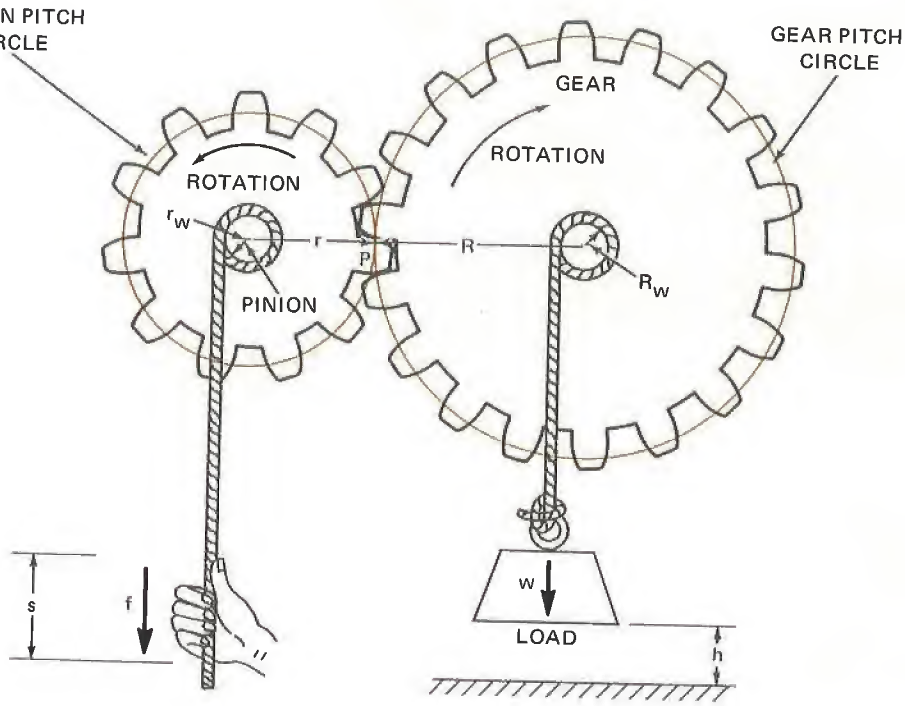


Fig. 6-1 Work Transmission

as yet done any work, it still has 700-ft-lb of energy stored. However, since its height is now virtually zero, it has no potential energy. Its energy is now due to its velocity, or in other words, it has a kinetic energy of 700 ft-lb.

With these ideas of work, power, and energy in mind, let's turn our attention to the gear system shown in figure 6-1.

In this mechanism, the string on the left is wrapped around the pinion shaft. When the string is pulled down, it rotates the pinion counterclockwise. The pinion, in turn, rotates the gear clockwise which winds up the string on the gear shaft, raising the weight.

The amount of potential energy that is stored in the weight after it has been raised a distance h, is

$$PE = wh \text{ ft-lb}$$

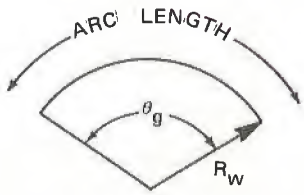


Fig. 6-2 Arc Length, Angle, and Radius

In raising the weight, the gear shaft rotates through an angle θ_g. Figure 6-2 shows such an angle of rotation.

The arc length in figure 6-2 is the distance that a point on the shaft will travel as the rotation occurs. If θ_g is measured in radians, this distance is

$$\text{arc length} = R_w \theta_g$$

Referring back to the mechanism (figure 6-1), we observe that this arc length is exactly the same as h; therefore we may write

$$h = R_w \theta_g$$

Then, multiplying both sides by the weight of the load, we have

$$wh = wR_w\theta_g \qquad \textbf{(6.6)}$$

Now, let's pause and examine what we have in this equation. On the left is the potential energy of the load. The quantity on the right is the work done by the rotation of the gear shaft. The quantity wR_w is frequently encountered in rotating objects and is called *Torque.* Torque is nothing more than the tendency of a force (weight in this case) acting to cause rotation at a distance (R_w in this case) from a pivot point (the center of the shaft). In general, we write the definition of torque in the form

$$\boxed{T = fr} \qquad \textbf{(6.7)}$$

And, since f may be in pounds or ounces while the radius may be in feet or inches, torque may be in ft.-lb., in.-lb., or in.-oz.

It must be kept in mind, however, that torque cannot be converted directly to work without knowing the distance through which the force is acting. Since the radius is already a factor of torque, it is convenient to speak of distance as the number of radii moved. This unit of measure is called a *radian.* It can be shown algebraically that there are 2π radians in the circumference of any circle.

Returning to our original mechanism (Equation 6.6), and remembering θ_g is a radian measure, the equation for the work done by the gear shaft is

$$\boxed{W_g = T_g\theta_g} \qquad \textbf{(6.8)}$$

And since the gear is rigidly fixed on the shaft, this amount of work applies to the gear as a whole (ignoring bearing friction, etc.).

At this point let's observe that the gear and shaft must turn through the same angle (θ_g). So, if both the work done (W_g) and the amount of rotation (θ_g) are the same for the gear as they are for the shaft, then their torques must also be the same. But since the shaft and gear have different radii, they must also experience different forces. To see how these different forces are related, we observe that the torques are equal and we write

$$wR_w = f_gR$$

In other words,

$$\frac{R_w}{R} = \frac{f_g}{w}$$

or, in terms of the diameters,

$$\frac{D_w}{D} = \frac{f_g}{w} \qquad \textbf{(6.9)}$$

where D_w and D are the shaft diameter and the pitch diameter respectively, f_g is the force acting on the gear teeth, and w is the load on the shaft.

The pinion and gear are meshed, see figure 6-3; therefore, the force acting on the teeth of the pinion (f_p) must be equal to the force (f_g) action on the teeth of the gear:

$$f_p = f_g \qquad \textbf{(6.10)}$$

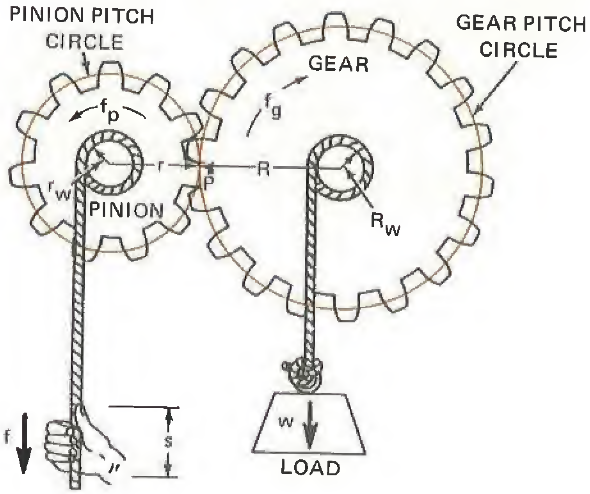


Fig. 6-3 Torque Transmission

However, since the gear and pinion have different radii, the torques acting on them are not equal. The two torques are

$$-T_p = f_p r \text{ and } T_g = f_g R$$

The negative sign with the pinion torque indicates a counterclockwise torque. Solving each of these equations for f renders

$$f_p = \frac{-T_p}{r} \text{ and } f_g = \frac{T_g}{R}$$

Since the two forces are equal, we may equate the torque/radius fractions

$$\frac{-T_p}{r} = \frac{T_g}{R}$$

or, in terms of the pitch diameters, we have

$$\frac{-T_p}{d} = \frac{T_g}{D}$$

Which can be rearranged into

$$\frac{d}{D} = -\frac{T_p}{T_g} \qquad (6.11)$$

which is a very handy relationship for determining the torque of one gear when that of its mate is known.

If the torque ratio is related to the pitch diameter ratio, then it follows that it is also related to the tooth ratio by

$$\frac{n}{N} = -\frac{T_p}{T_g} \qquad (6.12)$$

Since tooth ratios are easily determined in a practical situation, this relationship is frequently very useful.

The pinion gear and shaft are rigidly connected so the same relationship exists as did in the case of the gear and shaft. That is

$$\frac{d_w}{d} = \frac{f_p}{f}$$

where f is force pulling on the string.

The work done by the hand is given by the product of the pulling force and the distance s. Algebraically, this amount of work is

$$W = fs$$

And, if we neglect the frictional losses in the system (they will usually be *very* small), then the input work must approximate the output work.

MATERIALS

- 1 Spur gear, approx. 1-1/2 in. OD
- 1 Pinion, approx. 1 in. OD
- 2 Shafts 4" X 1/4"
- 2 Bearing plates with spacers
- 1 Breadboard with legs and clamps
- 4 Bearing Mounts
- 4 Bearings
- 4 Collars
- 2 Lever arms with 1/4 in. bore hubs
- 1 Dial caliper
- 2 Spring balances with posts and clamps

PROCEDURE

1. Inspect each component and assemble the mechanism shown in figure 6-4.
2. Check the two spring balances to be sure that they will read zero when mounted horizontally. Reset the zero adjustment if necessary.
3. Set up the experiment as shown in figure 6-5.

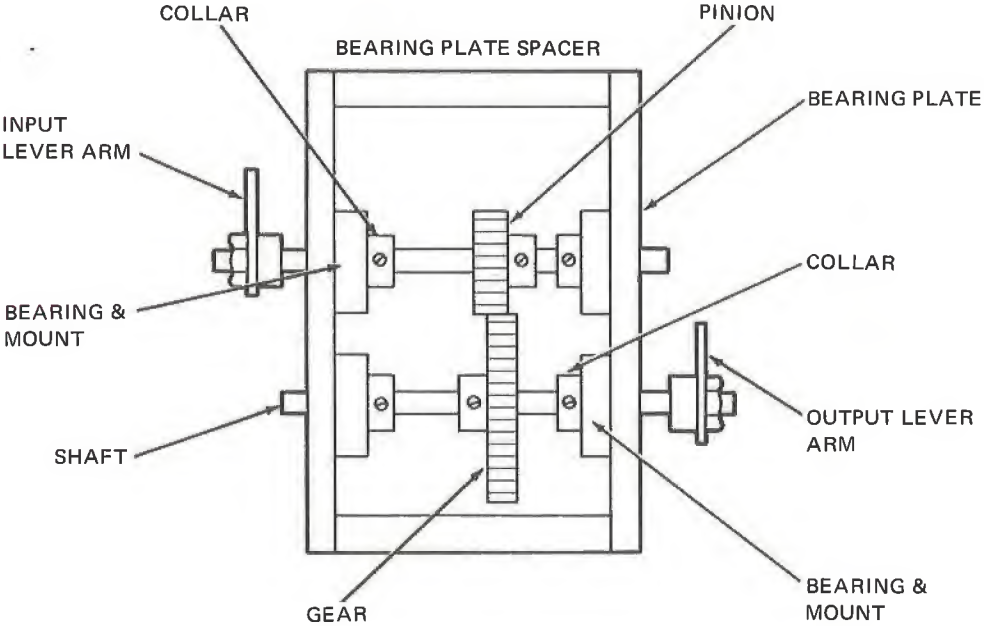


Fig. 6-4 The Experimental Mechanism

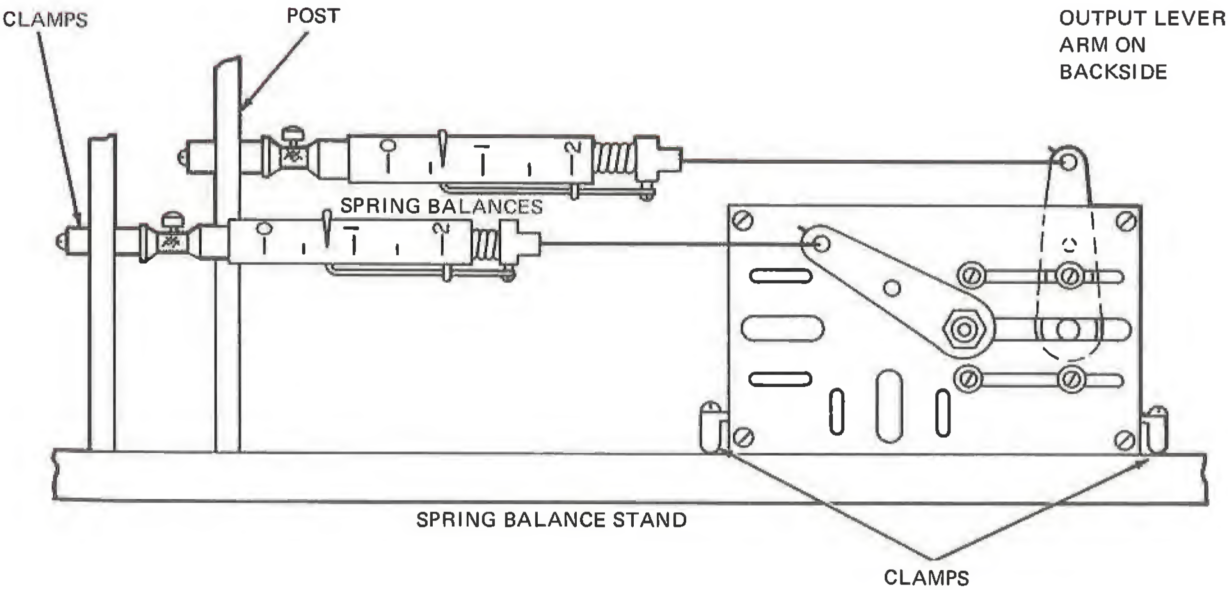


Fig. 6-5 The Measurement Setup

4. Adjust the positions of the bearing plate assembly and the spring balances until the levers are vertical and the input force is 8 oz. (The spring balances must both be horizontal when the adjustment is completed.)
5. Record the input and output forces (f_p and f_g) in the Data Table.
6. Very carefully measure the length of the input and output lever arms (r_p and r_g). These measurements should be taken from the center of the shaft to the point where the spring balance wire contacts the lever. Record your results.
7. Compute and record the input and output torque, (T_p and T_g).
8. Repeat steps 4, 5, and 7 for input forces of about 6, 4, and 2 ounces.
9. Measure and record the ODs of the pinion and gear, (d_o and D_o).
10. Count the number of teeth on each gear wheel and record the results, (n and NO).

Measured Values

f_p	f_g	r_p	r_g	d_o	D_o	n	N

Computed Values

T_p	T_g	n/N	d	D	d/D	T_p/T_g

Fig. 6-6 The Data Table

11. Compute the tooth ratio and record it in the Data Table.
12. Compute and record the pitch diameter of the pinion and gear, (d and D).
13. Compute and record the ratio of the pitch diameters.
14. Compute and record the torque ratio for each of the sets of force measurements.

ANALYSIS GUIDE. In analyzing your results for this experiment there are several important points that you should consider. Among these are:

1. How accurate were your results?
2. Do your results indicate that the torque ratios were related to the tooth ratio and pitch diameter ratio as suggested in the discussion?
3. Was friction in the bearings and gears a factor in this experiment?

In addition to these points you should also discuss any difficulties you encountered in doing the experiment.

PROBLEMS

1. Using your data, compute the force effective at the pitch circle of each gear in the experiment.
2. An automobile tire is 25 inches in diameter. What is the torque if the driving force at the road surface is 80 lb?
3. Two 32-pitch gears having 44 and 72 teeth are meshed in an adding machine. The pinion shaft is 0.25 in. diameter. What is the torque on the pinion shaft if the gear shaft has a torque of 650 in.-oz.?
4. What is the force acting on each shaft in Problem 3?
5. What is the force acting on the gear teeth in Problem 3?
6. If the gear in Problem 3 is turning at 2000 RPM, how much power is it transmitting? (Give your answer in HP.)

experiment 7 SIMPLE TRAINS

INTRODUCTION. Many gear applications employ more than a single mesh (two gears). The simplest form of multiple-gear system is the one called a *simple gear train.* In this experiment we shall examine some of the characteristics of this type of gear train.

DISCUSSION. A gear train is any system in which there are two or more meshed gears. The most elementary form of gear train is the familiar pinion-and-gear arrangement shown in figure 7-1. With a single external mesh gear train, the performance characteristics are described by

$$\boxed{\frac{n}{N} = -\frac{\theta_g}{\theta_p} = -\frac{\omega_g}{\omega_p} = -\frac{T_p}{T_g} = \frac{d}{D}} \qquad (7.1)$$

where
- d = pinion pitch diameter
- D = gear pitch diameter
- n = number of pinion teeth
- N = number of gear teeth
- θ = angular displacement (subscript p for pinion and g for gear)
- ω = angular velocity
- T = torque

The negative signs associated with the θ, ω, and T ratios indicate that the pinion and gear rotate in opposite directions.

In a single mesh gear train, both of the gear wheels have the same tooth velocity

$$v_p = v_g \qquad (7.2)$$

but in opposite directions. This is the characteristic which identifies a *simple* gear-train. Therefore, we may define a simple gear-train as follows:

A simple gear-train is a system of two or more gears arranged so that the tooth speed is a constant. It is always an arrangement with fixed centers and only one gear per shaft.

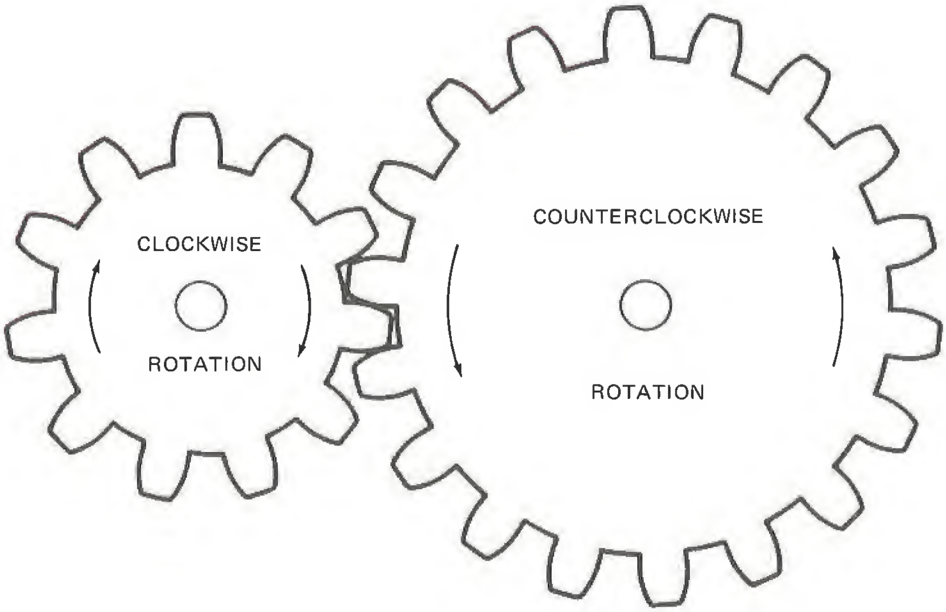


Fig. 7-1 A Simple Gear Train

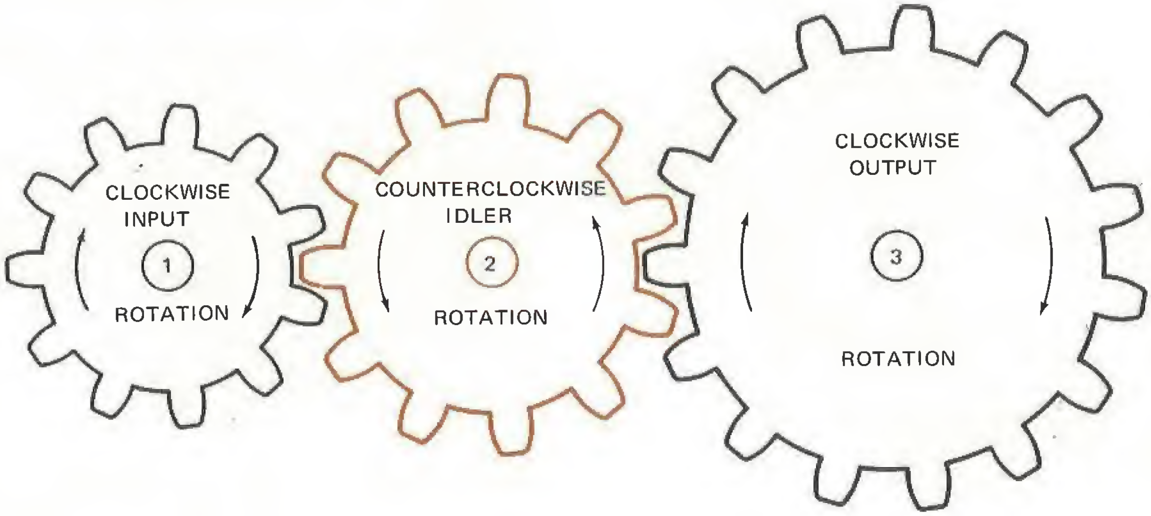


Fig. 7-2 A Simple Train with Three Gears

In some cases the output shaft must rotate in the same direction as does the input shaft. To achieve this condition we can use the simple gear train shown in figure 7-2. The input gear (no. 1) is turning in the clockwise direction. This causes gear number 2 to go counterclockwise, and the output gear (no. 3) consequently turns clockwise. In a gear train of this type, the center gear is usually called an *idler* gear.

In order to examine the relationships between the input and output parameters, let us suppose that the input gear is rotating at a speed of ω_1 RPM and has N_1 teeth. If this is the case, then the relationships between the input and idler gears are

$$\frac{N_1}{N_2} = -\frac{\theta_2}{\theta_1} = -\frac{\omega_2}{\omega_1} = -\frac{T_1}{T_2} = \frac{D_1}{D_2}$$

where the subscripts indicate the gear number and D is pitch diameter.

Similarly, the relationships between the idler and output gears are

$$\frac{N_2}{N_3} = -\frac{\theta_3}{\theta_2} = -\frac{\omega_3}{\omega_2} = -\frac{T_2}{T_3} = \frac{D_2}{D_3}$$

With those facts in mind, let us now examine the relationship between input and output speed as a function of the tooth counts. For the input and idler gear, we have

$$\frac{N_1}{N_2} = -\frac{\omega_2}{\omega_1}$$

or

$$\omega_2 = -\omega_1 \frac{N_1}{N_2}$$

Then, for the idler and output mesh there is

$$\frac{N_2}{N_3} = -\frac{\omega_3}{\omega_2}$$

We may rewrite this as

$$\omega_2 = -\omega_3 \frac{N_3}{N_2}$$

Now, since we have two expressions for ω_2, we can set them equal to each other.

$$-\omega_1 \frac{N_1}{N_2} = -\omega_3 \frac{N_3}{N_2}$$

Then, multiplying both sides by $-N_2$, we have

$$\omega_1 N_1 = \omega_3 N_3$$

or

$$\frac{N_1}{N_3} = \frac{\omega_3}{\omega_1}$$

From this equation we see that the idler gear does not affect the speed-tooth relationship between the input and output gears at all. Indeed, any simple gear train acts just like a single mesh of two gears except that the direction of rotation and the center distance between the input and output shafts are affected by the presence of idler gears.

If we work out the relationships for θ, T, and D for the three gear case using the same approach as with speed, the results are

$$\boxed{\frac{N_1}{N_3} = \frac{\theta_3}{\theta_1} = \frac{\omega_3}{\omega_1} = \frac{T_1}{T_3} = \frac{D_1}{D_3}} \qquad \textbf{(7.3)}$$

Notice that the only difference between these relationships and those for a single mesh train is the direction of rotation of the output gear.

It is worth mentioning that equations 7.1 and 7.3 cover *all simple* external gear trains. Equation 7.1 works for simple trains that have an even number of gears, and 7.2 works for simple trains with an odd number of gears.

Adding an idler gear does have a direct effect on the center distance between the input and output shafts. The center distance for the shafts-in-line arrangement shown in figure 7-2 will be

$$C = R_1 + D_2 + R_3 \qquad \textbf{(7.4)}$$

where R_1 and R_3 are the pitch radii of the input and output gears, while D_2 is the pitch diameter of the idler gear.

To physically measure the center distance, we would measure the outside shaft spacing (x), then measure the shaft diameters (d_s and D_s). The center distance is then equal to the outside shaft spacing *less* half of each shaft diameter.

$$c = x - 1/2\,(d_s + D_s) \qquad \textbf{(7.5)}$$

It is possible to use an idler gear to *adjust* the center distance between two gears over a limited range. Consider the arrangement shown in figure 7-3. Suppose that gears 1 and 3 provide a desired gear ratio, but the *required* center distance does not allow them to mesh. We can solve this type of problem by installing an idler with its center off of the center line of the two other gears. To accomplish this, we choose an idler gear of the same pitch as the other two gears and having a pitch diameter greater than the space between the two gears. Since the idler does not affect the gear ratio, the number of teeth on it is not important. The only problem is to locate the proper place for the idler center. To locate the idler center, we strike an arc about the input shaft center having a radius equal to the

sum of the pitch radii of the input and idler gears. Then we draw another arc from the output shaft center using a radius equal to the sum of the pitch radii of the idler and output gears. The intersection of these two arcs is the proper center point for the idler gear.

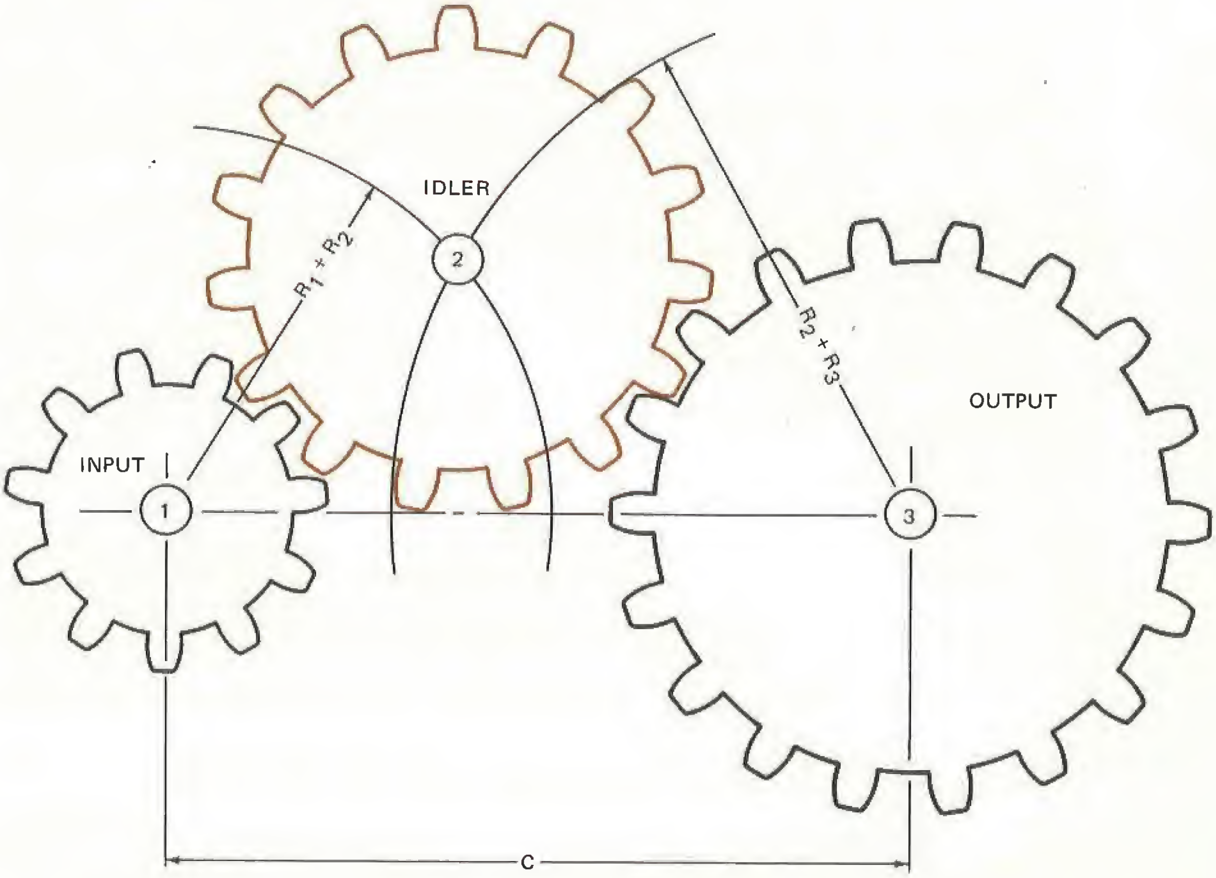


Fig. 7-3 Locating "Off-Line" Idler Gear Center

MATERIALS

1 Spur gear, approx. 3/4 in. OD

1 Spur gear, approx. 1 in. OD

1 Spur gear, approx. 1-1/2 in. OD

3 Shafts 4" X 1/4"

2 Bearing plates with spacers

6 Bearing mounts

6 Bearings

6 Collars

1 Dial caliper

1 Power Supply

2 Dials with 1/4 in. bore hubs

1 Stroboscope

1 Shaft coupling

1 Motor and mount

1 Breadboard with legs and clamps

PROCEDURE

1. Inspect each of the components to insure that they are not damaged.
2. Measure and record the OD of each gear wheel, (D_{o1}, D_{o2}, D_{o3}).
3. Count the number of teeth on each gear wheel, and record the results, (N_1, N_2, N_3).
4. Compute the pitch diameter of each gear wheel, and record it in the Data Table, (D_1, D_2, D_3).
5. Assemble the mechanism shown in figure 7-4. The largest of the three gear wheels is to be used as an "off-line" idler gear. *Be very careful in aligning the bearing mounts and bearings so that the mechanism rotates freely.*
6. Turn on the DC power supply and allow the mechanism to run for several minutes. During this time, also turn on the stroboscope and allow it to warm up.
7. Set the DC power supply for an output of about 24 volts.
8. Using the stroboscope, measure the angular velocity of the input and output shafts. Record your results in the Data Table, (ω_1, ω_2).
9. Compute the ratio of the number of teeth on the input gear to that of the output gear. Record the results.
10. Compute the ratio of the pitch diameter of the input gear to that of the output gear and record the results.
11. Compute the ratio of the output shaft speed to the input shaft speed and record your results.
12. Turn off the DC power supply and turn the input shaft by hand. Observe the direction of rotation of the output shaft.
13. Turn the DC power supply back on and repeat steps 7, 8, and 11 with a voltage of about 21 volts.
14. Repeat steps 7, 8, and 11 with a DC power supply setting of about 18 volts.

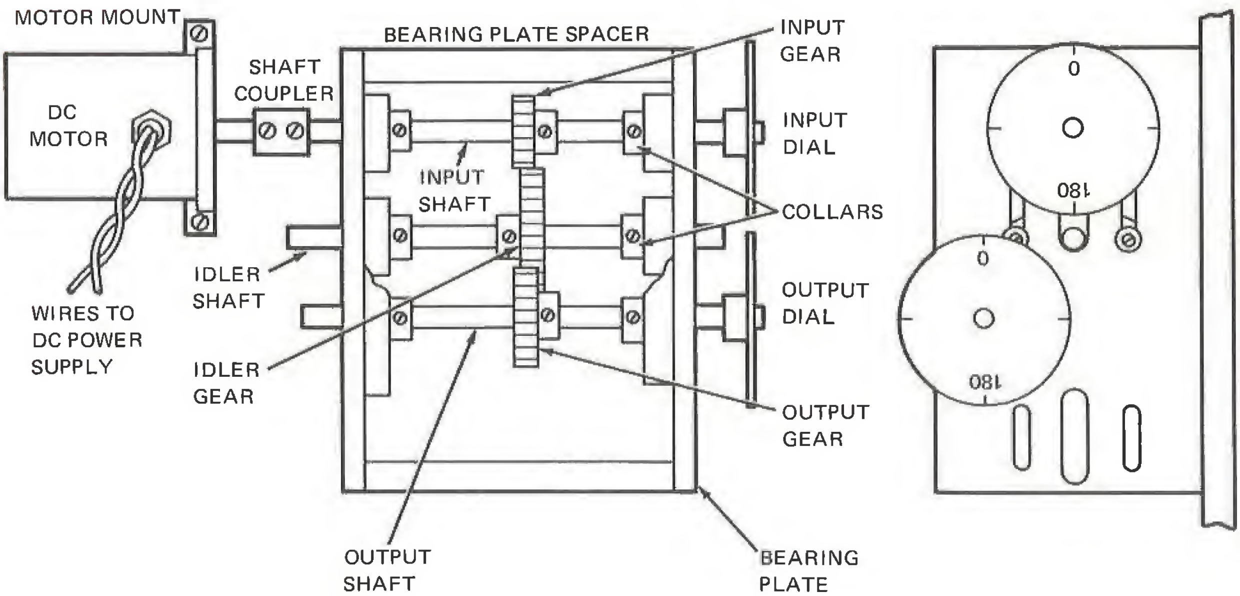


Fig. 7-4 The Experimental Mechanism

Measured Values

D_{o1}	D_{o2}	D_{o3}	N_1	N_2	N_3	ω_1	ω_3

Computed Values

D_1	D_2	D_3	N_1/N_3	D_1/D_3	ω_3/ω_1

Fig. 7-5
The Data Table

ANALYSIS GUIDE. In analyzing the results from this experiment, you should consider the extent to which the speed ratios agreed with the tooth ratios and pitch diameter ratios. Also, consider the extent to which your experience tended to confirm the points brought out in the discussion.

PROBLEMS

1. Four 48-pitch gears are arranged in a simple gear train. The input gear has 36 teeth. The two idlers have 44 each, and the output gear has 50 teeth. What is the speed and direction of the output gear if the input is turning 1800 RPM clockwise?
2. All of the gears in Problem 1 are mounted on 0.25 shafts. If the output torque is 270 in.-oz., what horsepower is being supplied to the input shaft?
3. If the train in Problem 1 is assembled "on-line," what is the center distance between the input and output shafts?
4. What is the pitch circle velocity of the idler gears in Problem 1?
5. A 40-tooth, 32-pitch pinion is to be used to drive a 72-tooth gear with a center distance of 2.50 in. On a sheet of graph paper, show how you would locate a 95-tooth idler.
6. What is the value of the addendum, dedendum, and clearance of the gears in Problem 5?
7. The input torque of the gear train in Problem 5 is 245 in.-oz. What is the force acting on the teeth of each gear?
8. If the output speed in Problem 5 is 4000 RPM and all of the gears are mounted on 0.25 in. shafts, what is the input horsepower?

experiment 8 COMPOUND TRAINS

INTRODUCTION. Simple gear trains are limited to ratios which can be achieved with a single mesh. For practical reasons a single spur gear mesh only rarely exceeds a gear ratio of about ten-to-one. In many applications, ratios of considerably above ten-to-one are required. Compound gear trains are one way to achieve larger gear ratios. In this experiment we shall examine some of the characteristics of compound trains.

DISCUSSION. A simple gear train is a system in which the magnitude of the pitch circle velocity of two or more gears *must* be the same regardless of the number of teeth on the gears. They are composed of fixed center shafts with only one gear per shaft.

> *A compound gear-train, on the other hand, is a system in which the angular velocity of two or more gears must be the same regardless of the number of teeth on the gears. Compound trains have fixed centers and more than one gear per shaft.*

Figure 8-1 shows a gear system which satisfies this definition of a compound gear train. Notice that the center gears (2 and 3) in the figure are on the same shaft. They *must*, therefore, have the same angular velocity.

The relationship between the angular velocities of the first and second gears and the tooth counts is

$$\frac{N_1}{N_2} = -\frac{\omega_2}{\omega_1}$$

Solving this equation for ω_2 gives us

$$\omega_2 = -\omega_1 \frac{N_1}{N_2}$$

Similarly, the relationship for gears three and four is

$$\frac{N_3}{N_4} = -\frac{\omega_4}{\omega_3}$$

Then, solving for ω_3 renders

$$\omega_3 = -\omega_4 \frac{N_4}{N_3}$$

Now, since gears two and three *must* have the same angular velocity, we have

$$\omega_2 = \omega_3$$

and substituting the quantities identified above,

$$-\omega_1 \frac{N_1}{N_2} = -\omega_4 \frac{N_4}{N_3}$$

As a result, the ratio of the input to output angular velocity is

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{N_2}{N_1} \cdot \frac{N_4}{N_3}} \qquad \textbf{(8.1)}$$

Comparing this result to the figure, we observe that N_2/N_1 is the tooth ratio of the first *simple* pair (1 and 2) while N_4/N_3 is the tooth ratio of the second *simple* pair (3 and 4).

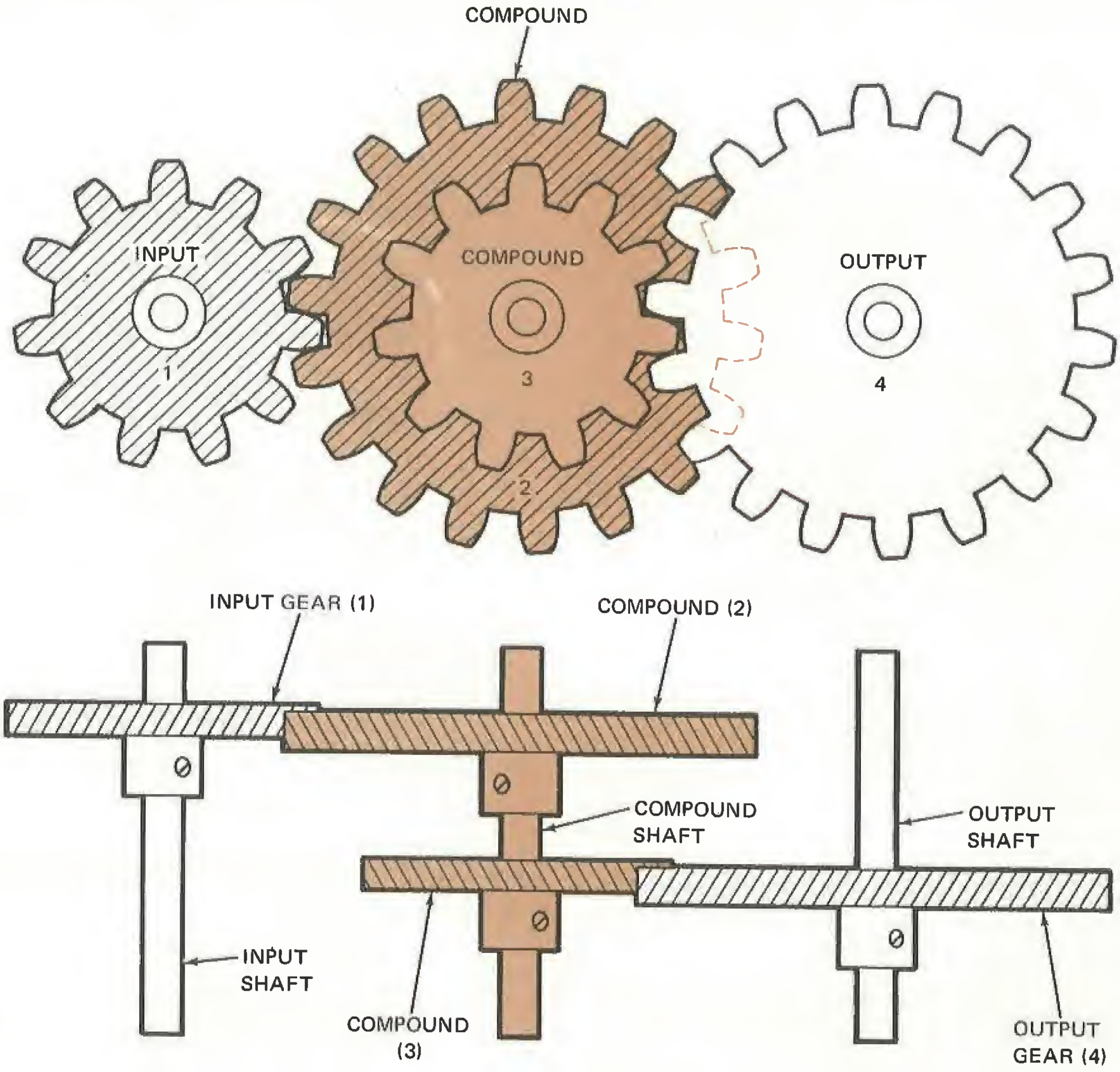


Fig. 8-1 A Compound Gear Train

From this observation we can make the following statement:

The ratio of the input-to-output speed of a compound gear train is equal to the product of tooth ratios of the simple trains included in the compound train.

This statement applies to all trains of the type shown in figure 8-1. It does not, however, tell us in what direction the output gear will turn.

Normally, the output direction can be easily determined by an inspection of the train. For example, if the input gear in figure 8-1 turns clockwise, the compound gears must both turn counterclockwise. Consequently, the output gear will turn clockwise in this case.

Another way to express the relationship given in equation 8.1 is to observe that gear one and three are *drivers*. That is, they drive

gears two and four respectively. Two and four on the other hand are *driven* gears or *followers.* Using this terminology, equation 8.1 can be described as:

> *The ratio of the input-to-output speed of a compound gear-train is equal to the product of the followers' tooth counts divided by the product of the drivers' tooth counts.*

Based on this description we can write

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{N_2 N_4}{N_1 N_3}} \qquad \textbf{(8.1)}$$

which is, of course, the same as the previous equation.

We can also use the pitch diameters of the gears to determine the speed ratio. Since the tooth ratios and pitch diameters are related by

$$\frac{N_2}{N_1} = \frac{D_2}{D_1} \text{ and } \frac{N_4}{N_3} = \frac{D_4}{D_3}$$

the speed ratio will be

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{D_2 D_4}{D_1 D_3}} \qquad \textbf{(8.2)}$$

We can analyze the torque transmission of the system by observing that torque ratios for the two *simple* pairs are

$$\frac{T_2}{T_1} = -\frac{D_2}{D_1} \text{ and } \frac{T_4}{T_3} = -\frac{D_4}{D_3}$$

Solving for T_2 and T_3 respectively, we have

$$T_2 = -T_1 \frac{D_2}{D_1} \text{ and } T_3 = -T_4 \frac{D_3}{D_4}$$

But since gears two and three are coupled rigidly together, their torques *must* be the same. That is,

$$T_2 = T_3$$

Therefore, we see that

$$-T_1 \frac{D_2}{D_1} = -T_4 \frac{D_3}{D_4}$$

and, as a result, the ratio of *output to input* is

$$\boxed{\frac{T_4}{T_1} = \frac{D_2 D_4}{D_1 D_3}} \qquad \textbf{(8.3)}$$

which is, of course, very *similar* to the speed ratio relationship.

If we observe that angular displacement (θ) is simply

$$\theta = \omega t$$

then we note that by multiplying equation 8.2 by time (t) we have

$$\boxed{\frac{\theta_1}{\theta_4} = \frac{D_2 D_4}{D_1 D_3}} \qquad \textbf{(8.4)}$$

In summary then, if we combine equations 8.1 through 8.4, the result for a compound train is

$$\boxed{\frac{\theta_1}{\theta_4} = \frac{\omega_1}{\omega_4} = \frac{T_4}{T_1} = \frac{N_2 N_4}{N_1 N_3} = \frac{D_2 D_4}{D_1 D_3}} \qquad \textbf{(8.5)}$$

Many practical trains will have more than one compound-gear pair. Figure 8-2 shows a gear box with three sets of compound gears. Let's suppose that we wish to calculate the output shaft speed (ω_8). All we need to do is apply the relationship

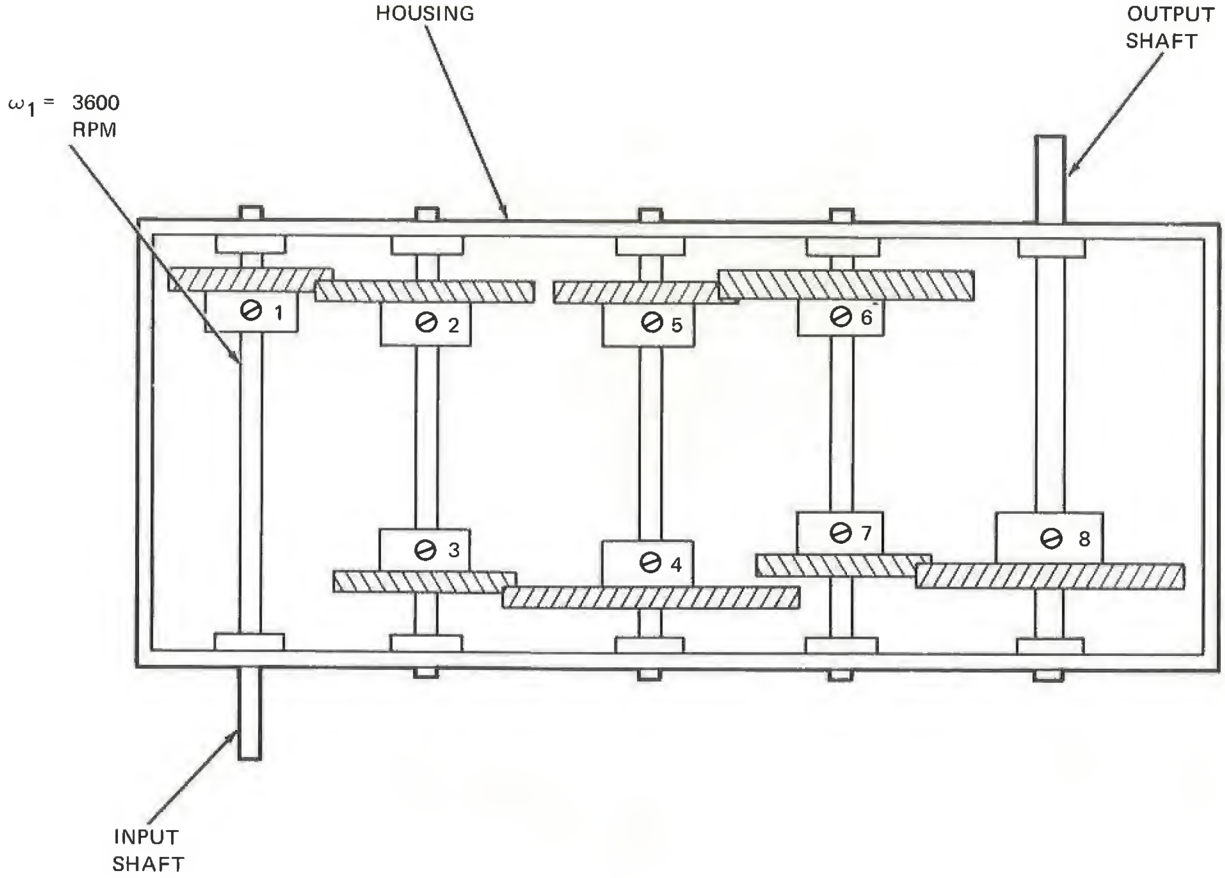


Fig. 8-2 A Speed Reducing Gear Box

$$\frac{\omega_1}{\omega_8} = \frac{\text{Product of follower teeth}}{\text{Product of driver teeth}}$$

which, for this case, is

$$\frac{\omega_1}{\omega_8} = \frac{N_2 N_4 N_6 N_8}{N_1 N_3 N_5 N_7}$$

Substituting the numbers from figure 8-2,

$$\frac{3600}{\omega_8} = \frac{48 \times 60 \times 54 \times 50}{25 \times 26 \times 36 \times 24}$$

or

$$\omega_8 = \frac{3600 \times 0.562 \times 10^6}{7.776 \times 10^6} \approx 260 \text{ RPM}$$

Finally we observe from the figure that the output turns in the same direction as the input shaft.

The compound trains shown in figures 8-1 and 8-2 are arranged so that the compound gear shafts are on the same center line as the input and output shafts. While this is a very common arrangement, the compound gear shaft is sometimes centered off of the input/output center line.

Suppose, for example, that we had an application which required a compound gear ratio (too large for a simple gear ratio) and the center distance required was such that we could not accommodate an on-line compound shaft.

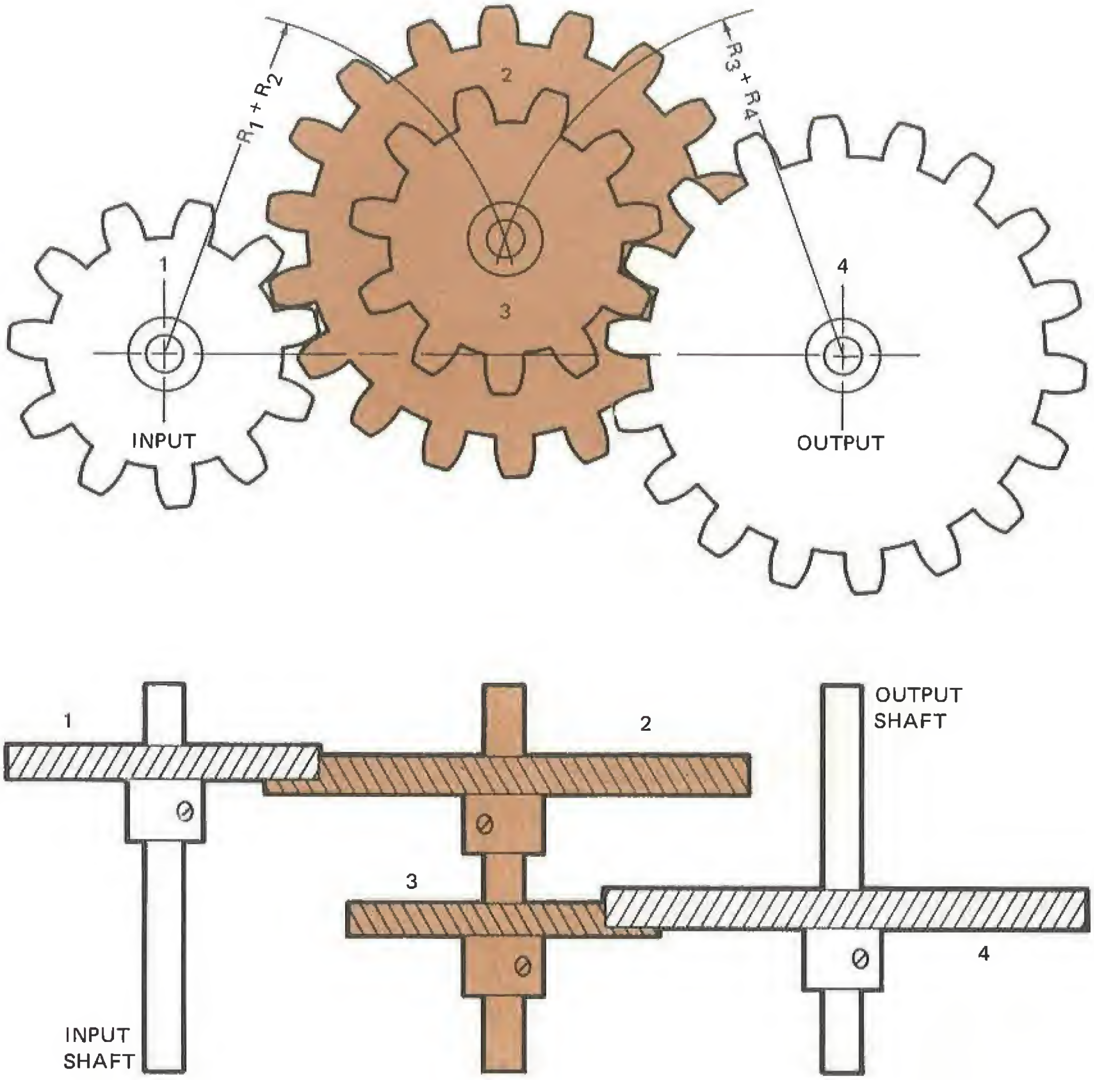


Fig. 8-3 An "Off-line" Compound Gear Train

In such a case the compound shaft can be located off of the input/output center line as shown in figure 8-3.

We can locate the center of the compound shaft by striking an arc from the center of the *input driver.* This arc must have a radius equal to the sum of the input driver radius plus the compound input gear pitch radius. The second arc is struck from the center of the output gear. Its radius is equal to the pitch radius of the output gear plus that of the compound output gear. The intersection of these two arcs locates the center of the compound shaft.

In the case of a relatively large center distance, it is possible to use more than one off-line compound shaft. Such a case is shown in figures 8-4 and 8-5. A complete discussion of the geometry involved in locating shaft centers in such a case is beyond the scope of this experiment. However, center locations in such a case can be determined experimentally with very little difficulty.

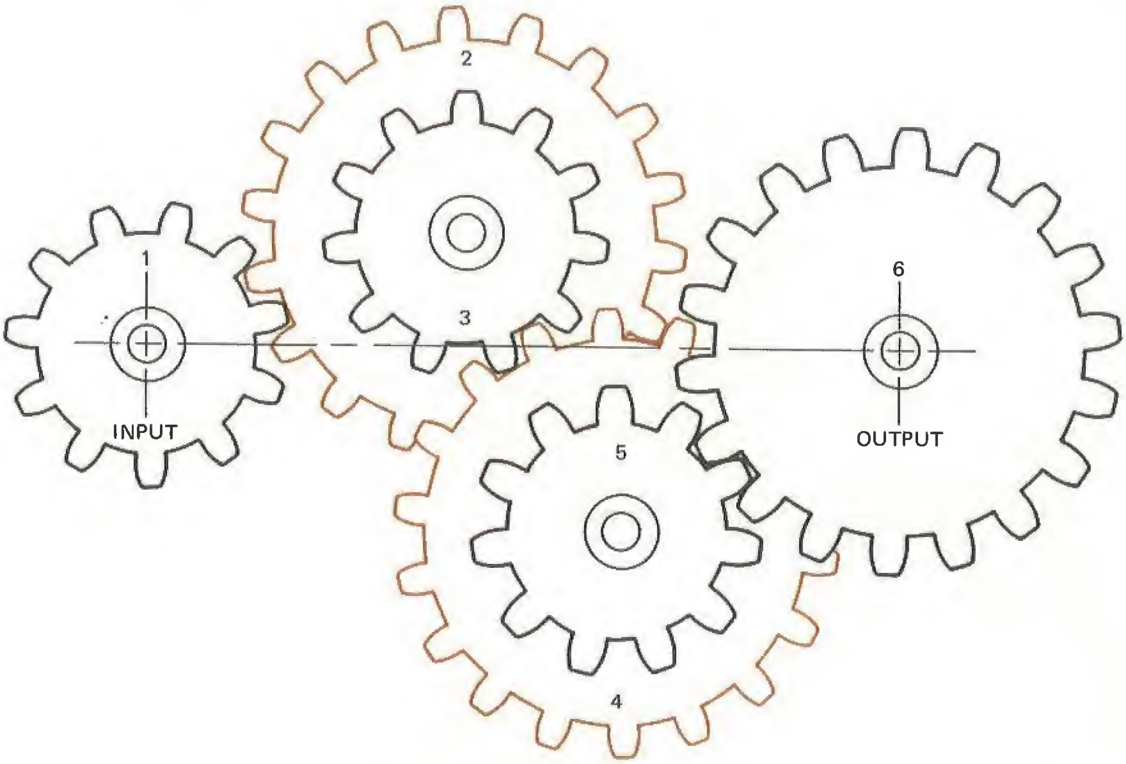


Fig. 8-4 Multiple Off-line Compound Shafts

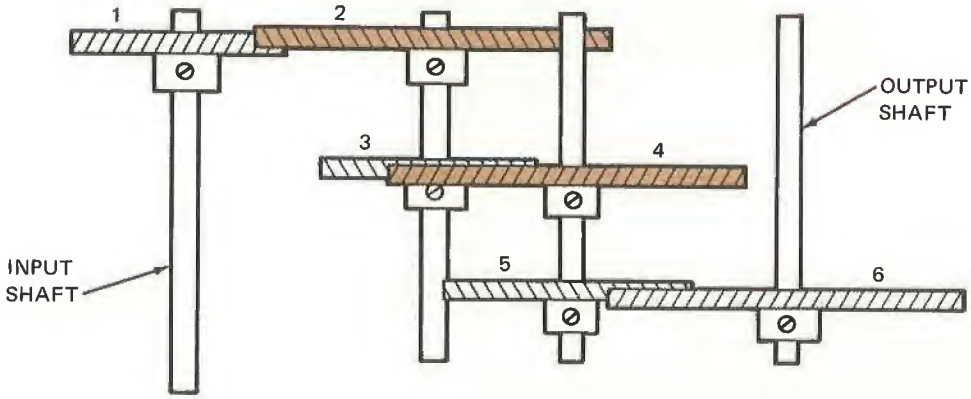


Fig. 8-5 Multiple Off-line Compound Shafts

MATERIALS

1 Spur gear, approx. 2 in. OD
1 Spur gear, approx. 1-1/2 in. OD
1 Spur gear, approx. 1 in. OD
1 Spur gear, approx. 3/4 in. OD
3 Shafts 4″ X 1/4″
6 Bearing mounts
6 Bearings
2 Bearing plates with spacers

1 Breadboard with legs and clamps
2 Collars
1 Shaft coupling
2 Dials with 1/4 in. bore hubs
1 Motor and mount
1 Power supply
1 Stroboscope
1 Dial caliper

PROCEDURE

1. Inspect each component for possible damage.
2. Count the number of teeth on each gear and record the results in the Data Table. The smallest gear is number one, the largest is number two, the smaller of the remaining gears is number three while the larger is number 4.
3. Assemble the mechanism shown in figure 8-6. *Take considerable care in aligning the bearings.*

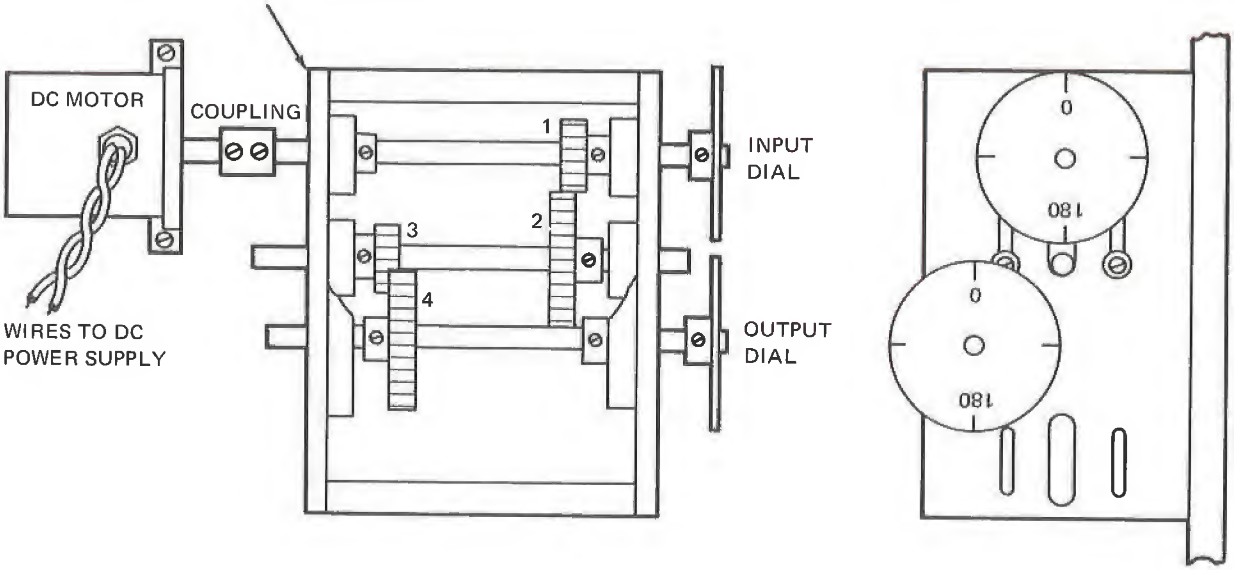


Fig. 8-6 The Experimental Mechanism

4. Turn on the mechanism and allow it to run for several minutes. During this time turn on the stroboscope and allow it to warm up.
5. Set the DC power supply to about 24 volts.
6. Using the stroboscope, measure and record the input and output shaft speeds, (ω_1 and ω_4).
7. Turn off the mechanism and measure the OD of each gear. Record the results in the Data Table, (D_{o1}, D_{o2}, D_{o3}, and D_{o4}).
8. Compute and record the pitch diameter of each gear (D_1, D_2, D_3, and D_4).
9. Using the pitch diameters and the measured input speed, *compute* the output speed, (ω_4').
10. Using the measured output speed and the number of teeth on each gear, compute the input shaft speed, (ω_1').

11. Turn the mechanism on and set the DC power supply for about 21 volts. Repeat steps 6, 9, and 10.
12. Reduce the DC power supply setting to about 18 volts and repeat steps 6, 9, and 10.

Gear Parameters

Gear No.	N	D_o	D
1			
2			
3			
4			

Shaft Speeds

Qty.	$E_{DC} \approx 24V$	$E_{DC} \approx 21V$	$E_{DC} \approx 18V$
ω_1			
ω'_1			
ω_4			
ω'_4			

Fig. 8-7 The Data Table

ANALYSIS GUIDE. The purpose of this experiment has been to examine the operating characteristics of compound gear trains. Specifically, we have measured input and output shaft speed and computed them, using pitch diameters as well as tooth counts. In analyzing your results you should consider the extent to which they agreed with the material presented in the discussion. Also you should consider the advantages of using a compound train as opposed to a simple train.

PROBLEMS

1. Four 48-pitch gears are used in a compound gear train. The input and output gears have 24 and 62 teeth respectively, while the compound follower and driver have 70 and 36 teeth. What is the overall gear ratio of the train?

2. If the gear train in Problem 1 has all three shafts on the same center drive, what is the input/output center distance?

3. If the input shaft in Problem 1 turns at 3500 RPM, what is the angular velocity of each of the gears?

4. If the gear train in Problem 1 has an output speed of 200 RPM and an output shaft torque of 460 in.-oz., what is the input power? (Give answer in HP.)

5. The gear train in figure 8-4 uses 5 pitch gears having tooth counts of:

$$N_1 = 24 \qquad N_3 = 26 \qquad N_5 = 24$$

$$N_2 = 80 \qquad N_4 = 72 \qquad N_6 = 95$$

   What is the output shaft speed if the input turns 4800 RPM?

6. If the output torque in Problem 5 is 290 ft.-lb., what is the horse-power that is being delivered to the input shaft?

experiment 9 REVERTED GEARS

INTRODUCTION. A variety of gear applications require that the input and output shafts have the same axis. A compound gear-train configuration which satisfies this requirement is called a *reverted* train. In this experiment we shall examine the construction and operation of such a train.

DISCUSSION. Compound gear trains of the type shown in figure 9-1 are called reverted trains. They offer the advantages of being compact and having a coaxial input/output. Inspection of this gear arrangement reveals that it is a compound gear train.

There are two conditions that a reverted gear train must satisfy in order to function properly. These conditions are:

1. *The overall gear ratio must be correct for the application.*
2. *The center distance between each simple mesh must be the same.*

The simplest way to satisfy both these conditions simultaneously is to use two identical simple meshes. For example, suppose that we wish to build a reverted train having an overall gear ratio of 9. The overall ratio for a compound train is the product of the simple ratios. So, we have

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = 9$$

in this case.

However, if we choose to let N_2/N_1 and

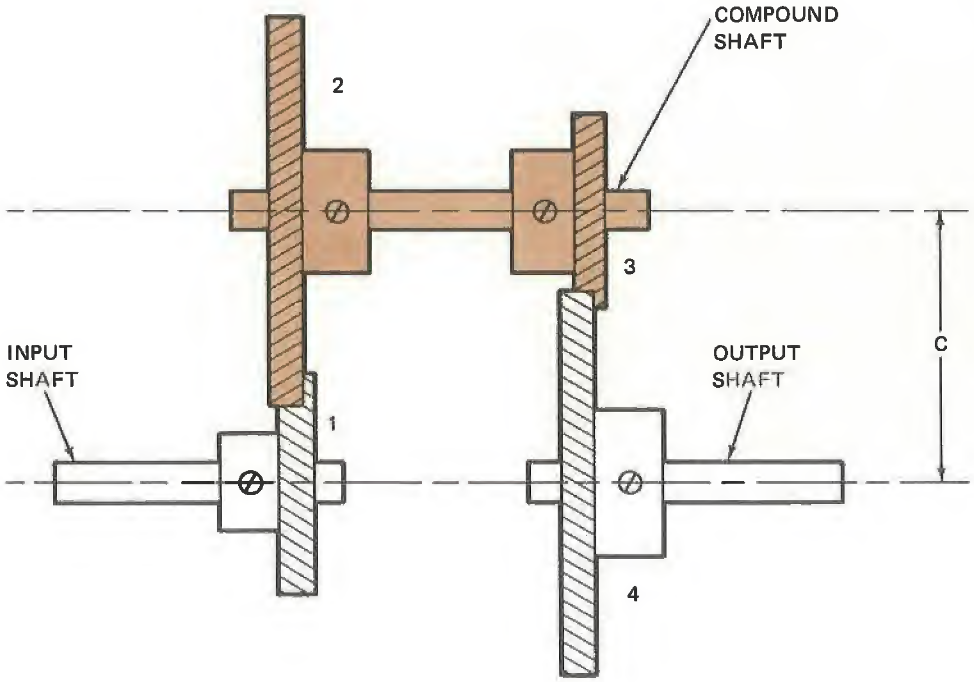


Fig. 9-1 A Reverted Gear Train

N_4/N_3 be equal, we can substitute N_2/N_1 for N_4/N_3 and have

$$\frac{N_2}{N_1} \times \frac{N_2}{N_1} = 9$$

or

$$\left(\frac{N_2}{N_1}\right)^2 = 9$$

Therefore,

$$\frac{N_2}{N_1} = \sqrt{9} = 3$$

As a result, we would choose the gears for the two simple meshes to have a ratio of 3 and to be identical to each other.

The center distance between the input/output shafts and the compound shaft will be equal to the sum of pitch radii of the simple meshes.

The above example worked out very well because the specified overall ratio was a perfect square. Unfortunately, this will not always be the case. Suppose, for instance, that the desired overall ratio was 6. The square root of 6 is not a whole number, and fractional gear ratios, while they often occur, are awkward to handle. Therefore, we would probably choose two simple ratios, $N_2/N_1 = 2$ and $N_4/N_3 = 3$, $(2 \times 3 = 6)$. Now, we must worry about the center distance. Both gear ratios *must* have the same value of C. The pitch diameter of each gear is

$$D = \frac{N}{P} \qquad \textbf{(9.1)}$$

where P is the dimetrical pitch of the gear; and if we divide by two, we have the pitch radius of each gear in the form

$$R = \frac{D}{2} = \frac{N}{2P_1}$$

The center distance at the input is the sum of the pitch radii of gears 1 and 2

$$R_1 + R_2 = \frac{N_1}{2P_1} + \frac{N_2}{2P_1} = \frac{N_1 + N_2}{2P_1} \qquad \textbf{(9.2)}$$

Similarly, the center distance at the output is

$$R_3 + R_4 = \frac{N_3}{2P_2} + \frac{N_4}{2P_2} = \frac{N_3 + N_4}{2P_2} \qquad \textbf{(9.3)}$$

Then, since these two center distances must be equal, we have

$$\frac{N_1 + N_2}{2P_1} = \frac{N_3 + N_4}{2P_2} = C \qquad \textbf{(9.4)}$$

Multiplying this relationship completely through by $2P_1P_2$ gives us

$$P_2 (N_1 + N_2) = P_1 (N_3 + N_4) = 2P_1P_2C$$

Factoring N_1 from the left-hand quantity and N_3 from the middle quantity renders

$$P_2N_1 \left(1 + \frac{N_2}{N_1}\right) = P_1N_3 \left(1 + \frac{N_4}{N_3}\right) \qquad \textbf{(9.5)}$$

$$= 2P_1P_2C$$

We can separate this relationship into two statements:

$$P_2N_1 \left(1 + \frac{N_2}{N_1}\right) = 2P_1P_2C \text{ and}$$

$$P_1N_3 \left(1 + \frac{N_4}{N_3}\right) = 2P_1P_2C$$

Solving these equations in the usual way is troublesome because each has two unknowns (N_1 and C on the left, and N_3 and C on the right). But we observe that P_2, N_1 and $(1 + N_2/N_1)$ are whole numbers, as are P_1, N_3, and $(1 + N_4/N_3)$. Therefore, $2P_1P_2C$ must also be a whole number. Moreover, if $2P_1P_2C$ is to satisfy both of the statements given above, it *must be divisible by* P_1, P_2, $(1 + N_2/N_1)$ *and* $(1 + N_4/N_3)$. There are, of course, lots of numbers which would be divisible by all these things. One of these many numbers is their *least common multiple.* That is, one of the values that $2P_1P_2C$ may have is the product of these four factors. So,

$$(K)\ 2P_1P_2C = P_1P_2\left(1 + \frac{N_2}{N_1}\right)\left(1 + \frac{N_4}{N_3}\right) \qquad \textbf{(9.6)}$$

may be substituted into equation 9.5. Making this substitution into each of the two statements of equation 9.5, we may reduce them to

$$\boxed{N_1 = P_1\left(1 + \frac{N_4}{N_3}\right) \text{ and}}$$

$$\boxed{N_3 = P_2\left(1 + \frac{N_2}{N_1}\right)} \qquad \textbf{(9.7)}$$

Using these two relationships we can solve for the number of teeth on each gear provided that we know the mesh ratios and the pitches.

For example, suppose we decide to use 24-pitch gears for the ratios $N_2/N_1 = 2$ and $N_4/N_3 = 3$ respectively. The number of teeth on each of the gears will be

$$N_1 = P_1\left(1 + \frac{N_4}{N_3}\right) = 24\ (1 + 3) = 96 \text{ teeth}$$

$$N_2 = \left(\frac{N_2}{N_1}\right)N_1 = 2\ (96) = 192 \text{ teeth}$$

$$N_3 = P_2\left(1 + \frac{N_2}{N_1}\right) = 24\ (1 + 2) = 72 \text{ teeth}$$

$$N_4 = \left(\frac{N_4}{N_3}\right)N_3 = 3\ (72) = 216 \text{ teeth}$$

Using equation 9.6, we can find the required center distance

$$\boxed{C = \frac{1}{2}\left(1 + \frac{N_2}{N_1}\right)\left(1 + \frac{N_4}{N_3}\right)} \qquad \textbf{(9.8)}$$

$$C = \frac{1}{2}(1 + 2)\ (1 + 3) = 6 \text{ in.}$$

For many applications this solution would not be satisfactory because the gears are too big. As was mentioned before, this is only one of many possible solutions. We can find other solutions simply by multiplying or dividing these values by a common number. For example, if we divide this solution by 3, we have

$$N_1 = \frac{96}{3} = 32 \text{ teeth} \qquad N_3 = \frac{72}{3} = 24 \text{ teeth}$$

$$N_2 = \frac{192}{3} = 64 \text{ teeth} \qquad N_4 = \frac{216}{3} = 72 \text{ teeth}$$

$$C = \frac{6}{3} = 2 \text{ in.}$$

which is another equally acceptable solution.

Equations 9.7 and 9.8 also allow us to use different pitch gears for the input/output pairs. For example, suppose that we wish to construct a reverted train having an overall ratio of 12 using 48-pitch gears for the input pair and 32-pitch gears for the output pair.

We would solve this problem by choosing the input and output mesh ratios:

let

$$\frac{N_2}{N_1} = 4 \quad \text{and} \quad \frac{N_4}{N_3} = 3$$

Then use equation 9.7 to find N_1 and N_3.

$$N_1 = P_1 \left(1 + \frac{N_4}{N_3}\right) \qquad N_3 = P_2 \left(1 + \frac{N_2}{N_1}\right)$$

$$N_1 = 48\ (1 + 3) \qquad N_3 = 32\ (1 + 4)$$

$$N_1 = 192 \text{ teeth} \qquad N_3 = 160 \text{ teeth}$$

Now we solve for N_2 and N_4 using the mesh ratios:

$$N_2 = N_1 \left(\frac{N_2}{N_1}\right) \qquad N_4 = N_3 \left(\frac{N_4}{N_3}\right)$$

$$N_2 = 192\ (4) \qquad N_4 = 160\ (3)$$

$$N_2 = 768 \text{ teeth} \qquad N_4 = 480 \text{ teeth}$$

Finally, we compute the center distance using equation 9.8.

$$C = \frac{1}{2}\left(1 + \frac{N_2}{N_1}\right)\left(1 + \frac{N_4}{N_3}\right) = \frac{1}{2}(5)\ (4) = 10 \text{ in.}$$

Since this center distance and the tooth counts are very large, we may choose to reduce each one proportionally. In this case, we can divide each one by 8 giving us

$$N_1 = \frac{192}{8} = 24 \text{ teeth} \qquad N_3 = \frac{160}{8} = 20 \text{ teeth}$$

$$N_2 = \frac{768}{8} = 96 \text{ teeth} \qquad N_4 = \frac{480}{8} = 60 \text{ teeth}$$

$$C = \frac{10}{8} = 1.25 \text{ in.}$$

Using these techniques, we can handle virtually all reverted gear train problems.

Torque, speed, and displacement relationships for a reverted gear train are the same as those for other compound trains.

Reverted gear trains are widely used in many gear applications, automobile transmissions, lathe gear boxes, and clocks being perhaps the most frequently encountered.

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Identical spur gears, approx. 3/4 in. OD
- 2 Identical spur gears, approx. 2 in. OD
- 2 Bearing plates with spacers
- 4 Bearing mounts
- 4 Bearings
- 2 Dials with 1/4 in. bore hubs
- 2 Dial indices with mounting hardware
- 2 Hollow shafts 2" X 1/4" with 1/8" bore
- 1 Shaft 4" X 1/8"
- 1 Shaft 4" X 1/4"
- 1 Dial caliper

PROCEDURE

1. Inspect each component to insure that it is undamaged.
2. Measure and record the OD of each gear wheel (use D_{o1} and D_{o3} for the small gears, D_{o2} and D_{o4} for the large gears).
3. Count and record the number of teeth on each gear (N_1 and N_3 are the small gears, N_2 and N_4 are the large gears).
4. Assemble the mechanism shown in figure 9-2.
5. Set the two dials so that they both read zero.
6. Rotate the input dial about 1/2 revolution and record the *angle* through which the input and output shafts turn (θ_1, θ_4).
7. Again, rotate the input dial in half turn steps, and record the angle through which the input and output shafts rotate. Continue this process until the output shaft has turned at least one complete revolution.
8. Compute and record the ratio of θ_1 to θ_4 for each pair of values.
9. Compute and record the average value of the ratio θ_1/θ_4.
10. Compute and record the tooth ratio of the input mesh.
11. Compute and record the tooth ratio of the output mesh.

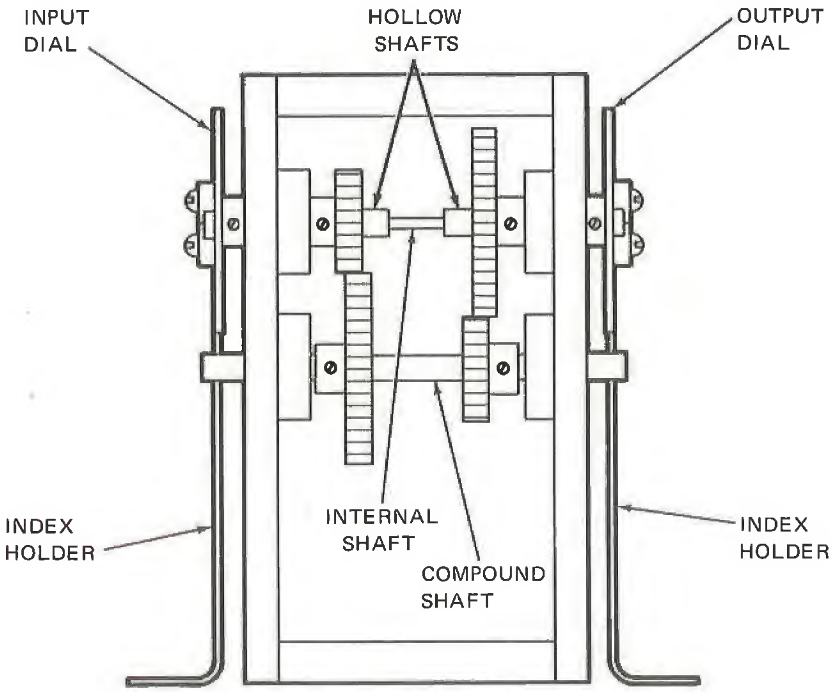


Fig. 9-2 The Experimental Mechanism

Gear No	OD	N	D	P
1				
2				
3				
4				

θ_1	θ_2	θ_1/θ_2
Ave. Value		

N_2/N_1	N_4/N_3	Overall Gear Ratio

Center Distance

Measured at Input	
Measured at Output	
Computed at Input	
Computed at Output	

Fig. 9-3 The Data Table

12. Using the results of steps 10 and 11, compute and record the overall gear ratio.
13. Compute the pitch diameter of each gear and record it in the Data Table, (D_1, D_2, D_3, D_4).
14. Compute the diametrical pitch of each gear and record the results.

15. Remove the dial from the input shaft and measure the center distance between the input shaft and the compound shaft, (C_1).

16. Repeat step 15 using the output and compound shaft, (C_2).

17. Using the pitch radii of the appropriate gears, compute the two center distances measured in steps 15 and 16, (C'_1 and C'_2).

ANALYSIS GUIDE. In analyzing the results from this experiment, there are several important points that you should discuss:

1. To what extent does the overall gear ratio agree with average displacement ratio?
2. Do the measured and computed values of C agree with each other?

In preparing your report on the results of this experiment, you should include these points as well as discussing any others which seem important to you.

PROBLEMS

1. A reverted gear train is to have an overall gear ratio of 4. All of the gears are to have the same pitch, and no gear may have less than 18 teeth. Specify the pitch, number of teeth of each gear, and center distance.

2. Repeat Problem 1 for an overall gear ratio of 3.

3. A reverted train is to have an overall ratio of 12. The pitch of the input pair is to be 6, while that of the output pair is to be 4. No gear may have fewer than 12 teeth. Specify the tooth counts and center distance.

4. Two pairs of 48-pitch gears are to be used in a reverted chain. If the gears have 30 and 72 teeth, respectively, what is the overall gear ratio and center distance?

5. A reverted train is to have a ratio of 20 using four gears of the same pitch, all having at least 12 teeth. Specify the tooth count of each gear and the ratio of each mesh.

experiment 10 INTERNAL GEARS

INTRODUCTION. The teeth of a spur gear may be formed either outside the root circle or inside it. Gears with their teeth formed inside the root circle are called internal or *annular* gears, and they have several useful characteristics. In this experiment we shall examine some of these characteristics.

DISCUSSION. An internal gear is formed by cutting the gear teeth on the inside of the root circle, as shown in figure 10-1.

Internal gears are formed to mate with standard involute external gear teeth. Consequently, they are shaped in such a way that the internal teeth are the *inverse* of the external gear tooth shape. (See figure 10-2.)

Since the tooth direction is reversed (the teeth point in toward the center), the tooth addendum and dedendum are also reversed. The tooth proportions are the same as those for an external gear except that the addendum is very slightly shorter to prevent trimming of the teeth when the gear is made. For practical purposes, the addendum (a), dedendum (b), and clearance (c) for the three common tooth configurations are taken to be those given in figure 10-3.

The diametrical pitch (P_d) of an internal gear

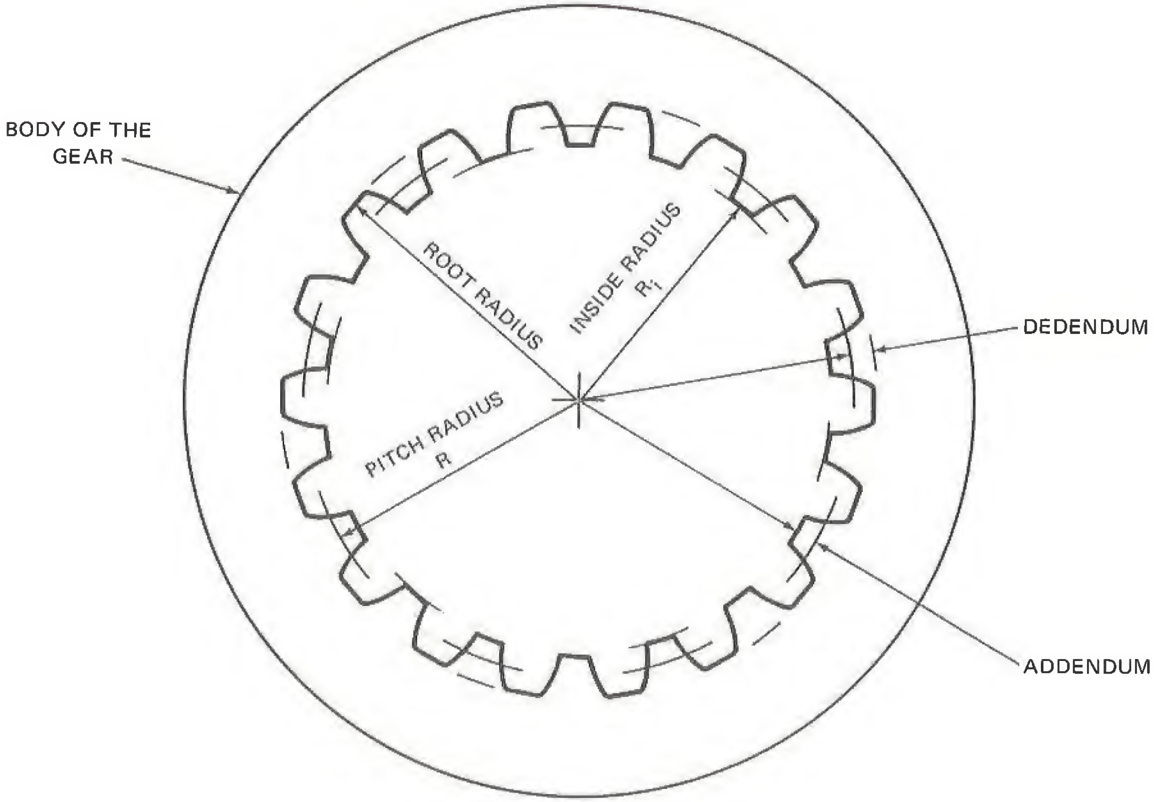


Fig. 10-1 An Internal Gear

is defined in the same way as it is for an external tooth gear

$$P_d = \frac{N}{D} \qquad (10.1)$$

where N is the number of teeth and D is the pitch diameter in inches.

In dealing with internal gears, the *outside diameter* is useful only to establish the space required to mount the gear. The *inside diameter* is used for making pitch diameter calculations. These two measurements are shown in figure 10-4.

Considerable care must be used in measuring the inside diameter to insure that the measurement is taken across the *center* of the gear.

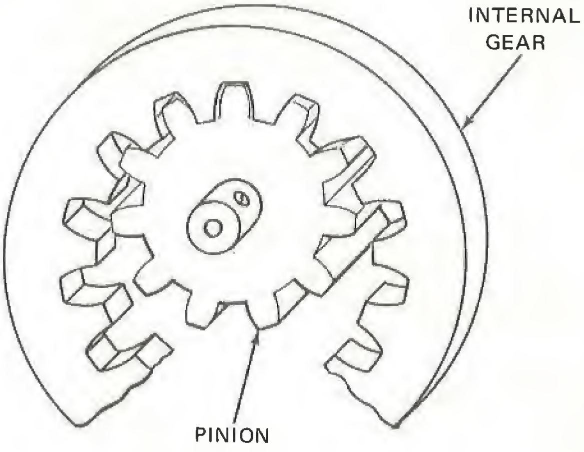


Fig. 10-2 An Internal Gear Meshed with a Pinion

Fig. 10-3 Tooth Shape

Tooth Shape	a	b	c
American standard full involute	$\frac{1}{P_d}$	$\frac{1.157}{P_d}$	$\frac{0.157}{P_d}$
American standard 20 degree stub involute	$\frac{0.8}{P_d}$	$\frac{1}{P_d}$	$\frac{0.2}{P_d}$
Fellows stub tooth	$\frac{1}{P}$	$\frac{1.25}{P}$	$\frac{0.25}{P}$

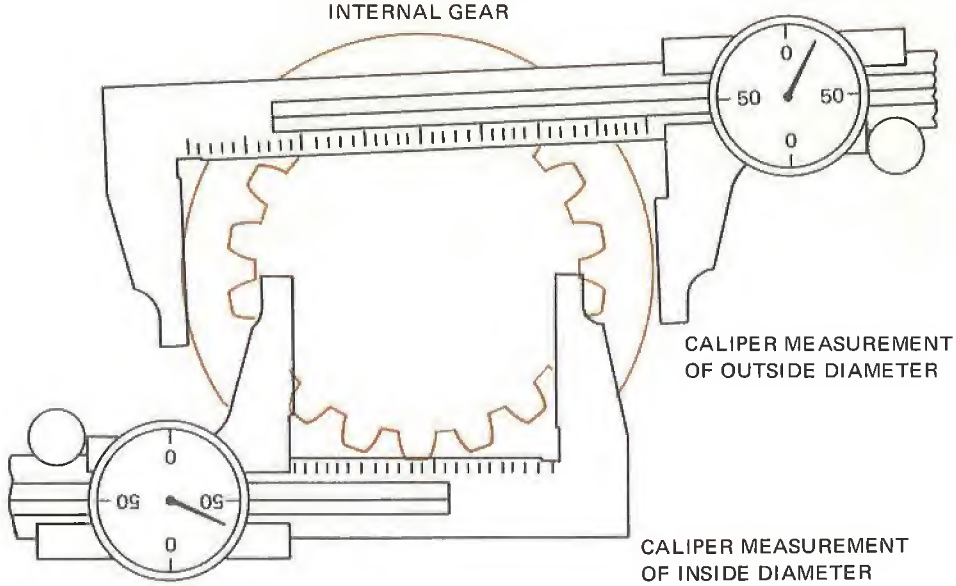


Fig. 10-4 Diameter Measurement

With the inside diameter (D_i) known, the pitch diameter (D) may be calculated by observing that

$$D_i = D - 2a$$

And, if we assume the gear to be an American standard full involute type, we may substitute $1/P_d$ for a rendering,

$$D_i = D - \frac{2}{P_d}$$

Then, recalling that P_d is equal to N/D, we have

$$D_i = D - \frac{2D}{N}$$

Multiplying both sides by N leads to

$$ND_i = ND - 2D$$

And factoring out the D on the right provides

$$ND_i = D(N-2)$$

Solving this relationship for D gives us

$$\boxed{D = D_i \frac{N}{N-2}} \qquad \textbf{(10.2)}$$

which is *very* useful in working with actual internal gears.

When an internal gear is meshed with a pinion, as shown in figure 10-5, the gear ratios are similar to those for an external gear mesh. Notice that the tooth velocity of the internal gear is the same as that of the pinion. Such a mesh constitutes a simple gear train. Also, notice that the *internal gear rotates in the same direction as the pinion.* With these observations in mind, we can write the ratios appropriate for the mesh.

$$\boxed{\frac{n}{N} = \frac{\theta_g}{\theta_p} = \frac{\omega_g}{\omega_p} = \frac{T_p}{T_g} = \frac{d}{D}} \qquad \textbf{(10.3)}$$

where n = pinion tooth count
N = internal gear tooth count
d = pinion pitch diameter
D = internal gear pitch diameter

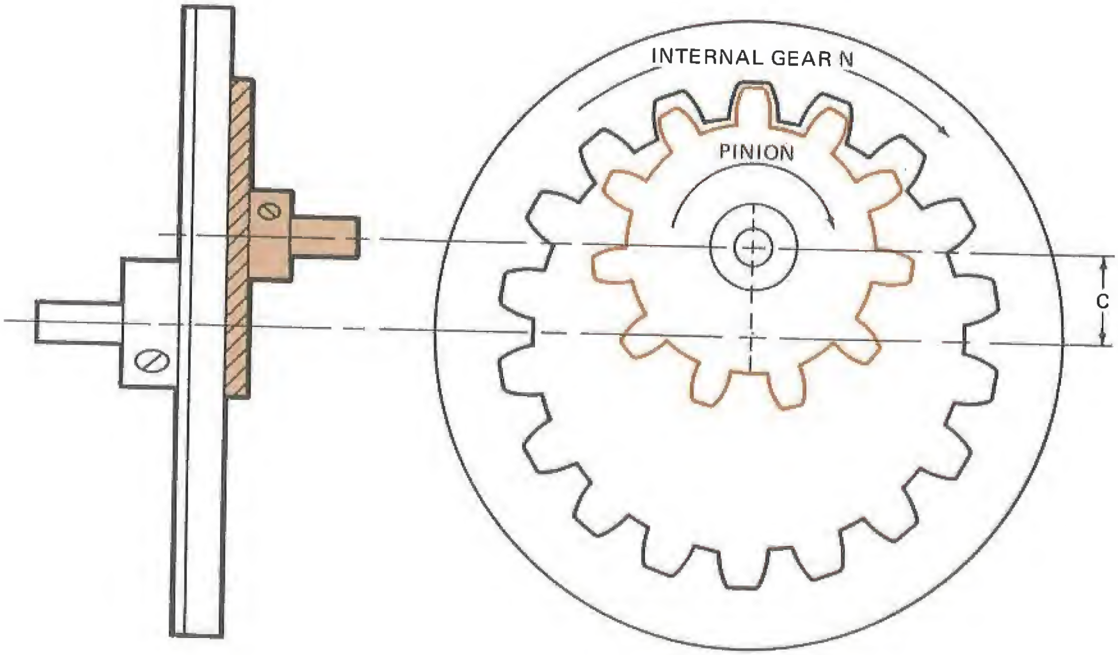


Fig. 10-5 An Internal Gear and Pinion

θ = angular displacement (subscript p for pinion and g for internal gear)
ω = angular velocity
T = torque

These ratios are, of course, the same as those for a simple external gear mesh except for the direction of rotation of the follower gear. This directional factor is one of the important differences between internal and external meshes.

Another important difference is that there are more teeth in contact with each other in an internal mesh than would be the case with an equivalent external mesh. For this reason, heavier loads can normally be handled by an internal mesh.

The center distance (C) between an internal gear shaft and the pinion shaft is not the same as for an external mesh. Referring to figure 10-4, we observe that the center distance is the *difference* between the internal gear pitch radius (R) and the pinion pitch radius (r):

$$C = R - r \tag{10.4}$$

and, since the pitch radii are half the pitch diameters, we have

$$C = \frac{D - d}{2}$$

Then, because $D = N/P_d$ and $d = n/p_d$, we may write the center distance equation in the form

$$\boxed{C = \frac{N - n}{2P_d}} \tag{10.5}$$

which is a convenient form in many practical cases.

We should notice at this point that an internal mesh will always have a shorter center distance than will an equivalent external mesh. This compactness factor is often cited as being one of the principal advantages in using internal gears.

Internal gears do have several disadvantages. Perhaps the most important ones are:

1. *They are more difficult to mount than are external gears.*
2. *They cannot be used for relatively small gear ratios because the pinion must fit inside and have considerable clearance to prevent tooth interference.*
3. *They cannot be manufactured to as high a level of precision as can external gears.*

In spite of these disadvantages, internal gears have found very wide application.

MATERIALS

1 Internal gear assembly, approx. 3 in. ID

1 Pinion, approx. 3/4 in. OD

2 Bearing plates with spacers

1 Bearing mount clearance drilled

3 Bearing mounts tapped

4 Bearings

3 Collars

1 Shaft 4" X 1/4"

1 Shaft 2" X 1/4"

2 Dials with 1/4 in. bore hubs

1 Breadboard with legs and clamps

2 Dial indices with mounting hardware

1 Dial caliper

PROCEDURE

1. Inspect the components to insure that they are not damaged.
2. Measure and record the inside diameter of the internal gear (D_i) and the outside diameter of the pinion (d_o).
3. Assemble the mechanism shown in figure 10-6. Notice that the method used to mount the pinion is normally not considered to be good construction practice. However, since we are only interested in examining basic internal gear-train operation and will not be loading the mechanism, this method will be adequate.
4. Adjust the input and output dial so that they both read zero.

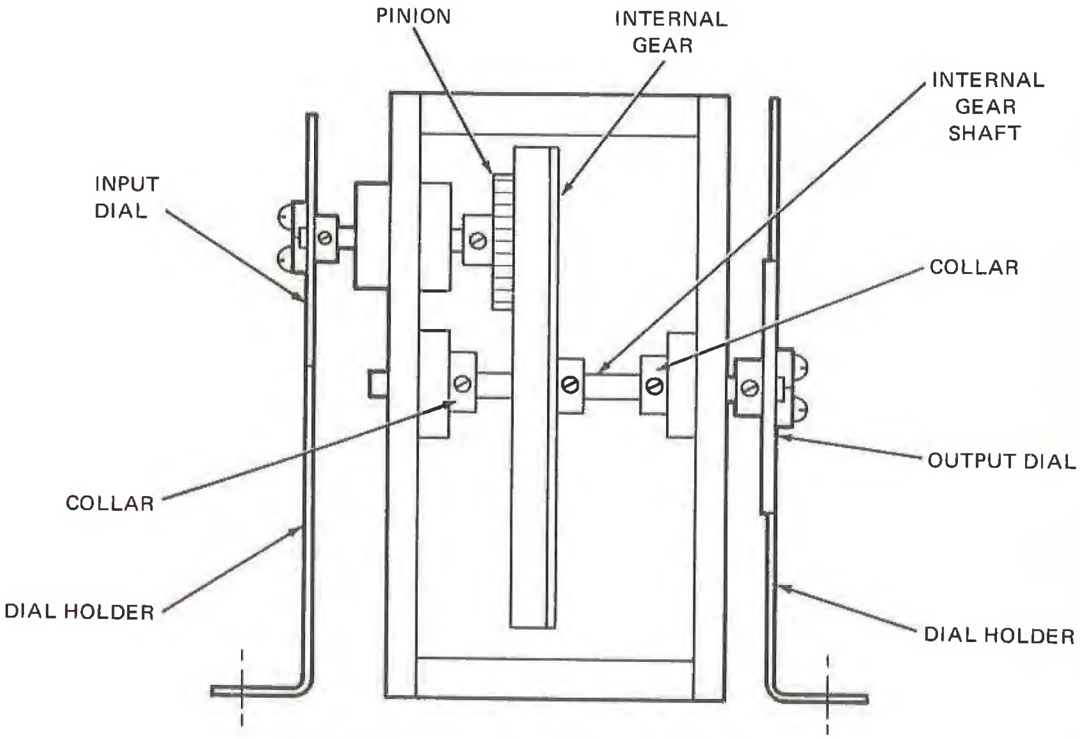


Fig. 10-6 The Experimental Mechanism

5. Rotate the pinion dial about 1/2 turn and record the angle of rotation of both input and output shafts.
6. Repeat step 5 in half-turn increments until the output shaft has made at least one complete revolution. Record the input and output rotation each time.
7. Compute the ratio of θ_g/θ_p for each pair of data.
8. Compute the average value of the θ_g/θ_p ratio.
9. Remove the input dial and index, then measure the center distance between the two shafts. (C)

10. Count the number of teeth on the pinion (n) and on the internal gear (N). Record the results.
11. Compute the tooth ratio n/N and record the results.
12. Compute the pitch diameter of each gear (d and D).
13. Compute the pitch diameter ratio (d/D).
14. Compute the diametrical pitch of each gear (p and P).
15. Compute the center distance (C′).

θ_p	θ_g	θ_g/θ_p
Ave. Value		

D_i	d_o	n	N

d	D	p	P

C	C′	n/N	d/D

Fig. 10-7 The Data Table

ANALYSIS GUIDE. In analyzing the results from this experiment, you should consider the extent to which θ_g/θ_p, n/N, and d/D agreed with each other. Also, consider the extent of agreement between the values of C.

PROBLEMS

1. A 48-pitch pinion having 24 teeth is driving a 120-tooth internal gear. What is the center distance?
2. If the pinion in problem 1 is turning 3800 RPM, how fast is the internal gear turning?
3. What is the tooth velocity in problem 2?
4. What is the ID of the internal gear in problem 1?
5. If the internal gear shaft in problem 2 is producing 40 ft-lb of torque, what is the power (in HP) applied to the pinion shaft?
6. Make a sketch showing how two internal gears and two pinions can be arranged into a reverted gear train.

experiment 11 PLANETARY TRAINS

INTRODUCTION. Planetary gear trains are used in a wide variety of machine applications. In this experiment we shall examine some of the operating characteristics of planetary type trains.

DISCUSSION. When James Watt was developing his steam engine, he was prevented from using certain mechanisms by existing patents. As a result, he developed a number of new mechanical arrangements. One such arrangement was the *planetary* or *epicyclic* gear train in figure 11-1. These gear trains are characterized by having at least one center of rotation that is not fixed.

A planetary gear may operate in any one of three different modes. These modes of operation are:

1. If the planet gear is *locked* to the planet carrier, then turning either the sun or carrier shafts causes the whole mechanism to rotate.
2. If the carrier shaft is locked so that it cannot turn, then the sun and planet gears form a simple train. In such a case, the gear ratio between the sun and planet shafts is

$$\boxed{\frac{\omega_p}{\omega_s} = \frac{N_s}{N_p}} \qquad (11.1)$$

3. If the sun shaft is locked and the carrier shaft is rotated, then the planet gear rotates about its own center *and* also about the sun gear. Turning the carrier one revolution causes the planet gear to make *one* complete revolution about the sun gear. At the same time, the planet gear is rotating around its own center a num-

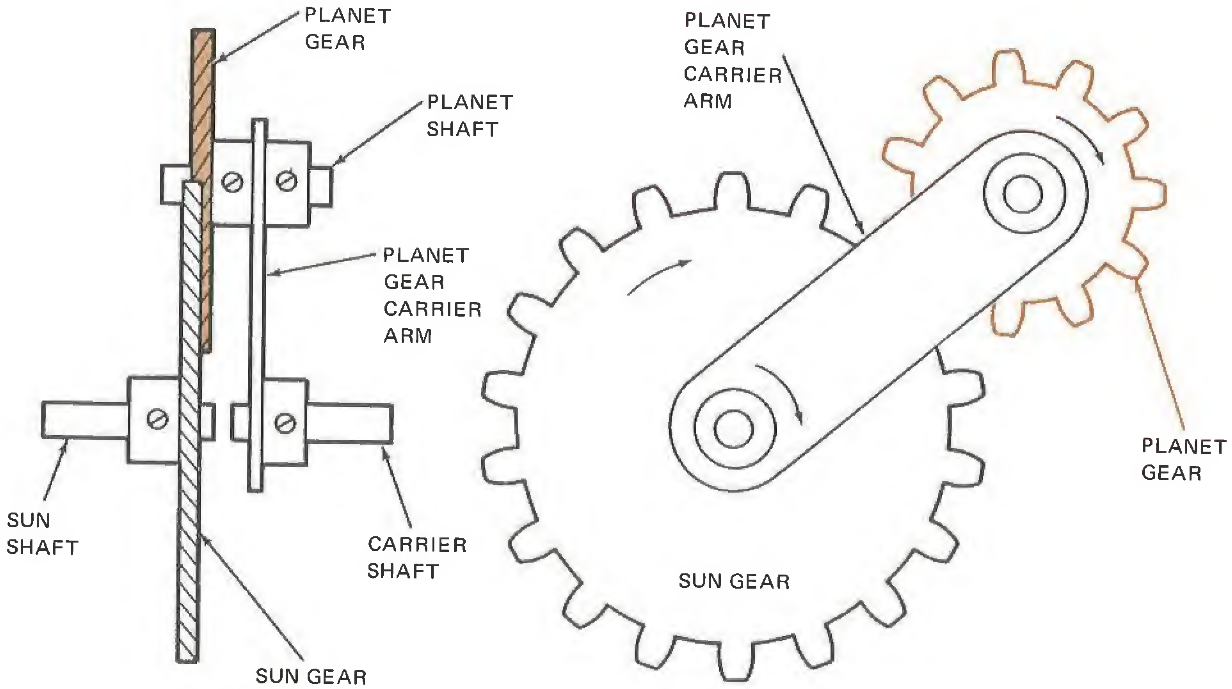


Fig. 11-1 A Basic Planetary Gear Train

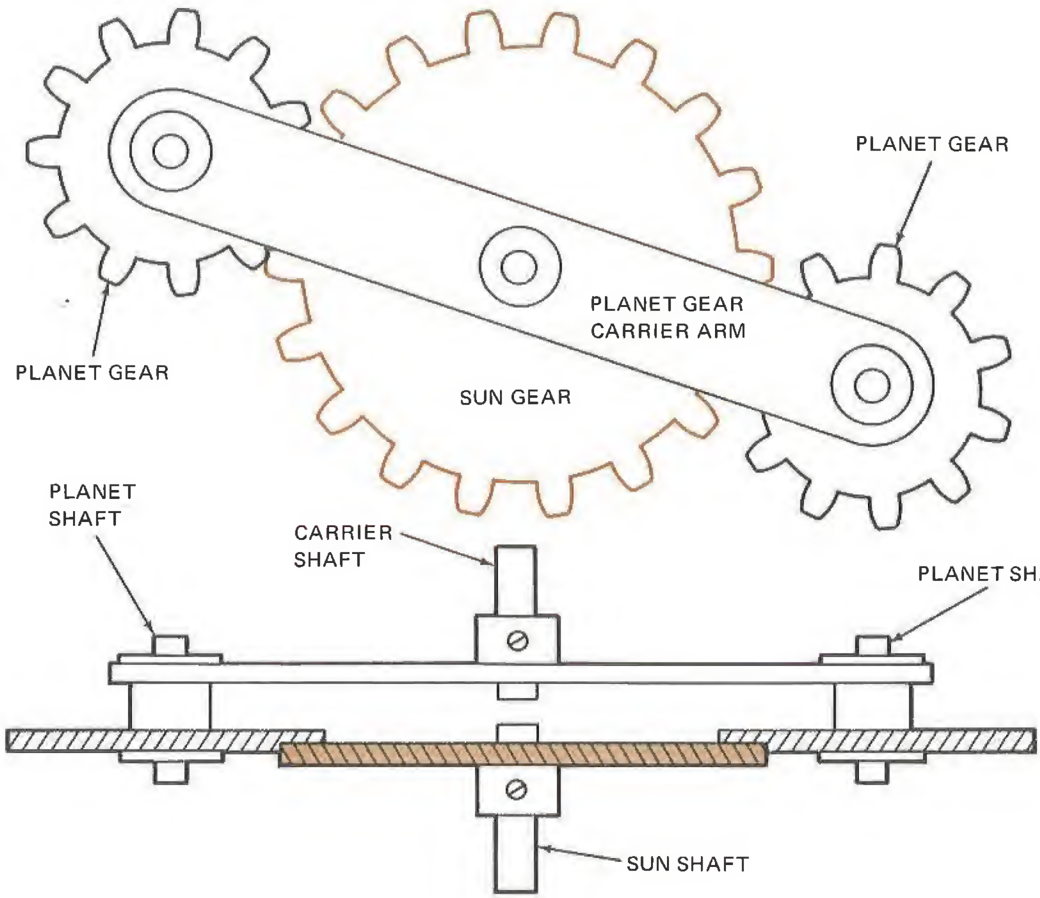


Fig. 11-2 A Planetary Train with Two Planet Gears

ber of times determined by the tooth ratio of the sun and planet gears. The total rotation of the planet gear is equal to the sum of these two rotations. Since these things occur during the time of one carrier arm revolution, the ratio of the planet gear's angular velocity to that of the carrier arm is

$$\boxed{\frac{\omega_p}{\omega_c} = \frac{N_s}{N_p} + 1} \qquad \textbf{(11.2)}$$

Notice that the carrier and planet gear rotate in the *same* direction (i.e., clockwise or counterclockwise); therefore, the ratio N_s/N_p is positive.

One of the worthwhile advantages of a planetary gear arrangement is that more than one planet gear may be used. Additional planet gears do not alter the relationships cited above, but do allow the transmitted load to be distributed over a greater number of teeth. Figure 11-2 shows a planetary system with two planet gears.

Any reasonable number of planet gears may be used to distribute the load in this manner. In many practical instances, space limitations will not allow more than four planet gears to be accommodated. In actual practice, three planet gears are very frequently used.

Planetary trains may also be built using an internal gear instead of the sun gear shown

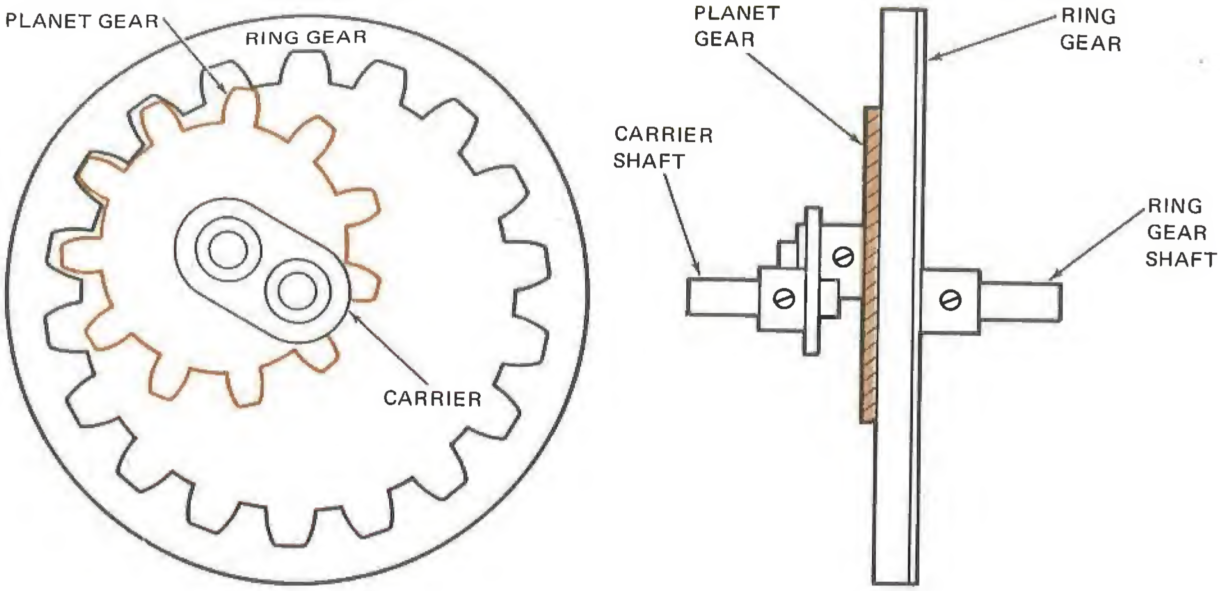


Fig. 11-3 A Ring and Planet Train

in figures 11-1 and 11-2. When this is done, the internal gear is usually referred to as a *ring gear.* Figure 11-3 shows this type of gear train.

The same three possible modes of operation apply for the ring and planet as do for the sun and planet arrangement:

1. If the planet gear is locked to the carrier, then the whole mechanism rotates as a unit.
2. If the carrier is locked, then the ring and planet act as a simple gear train. Since the ring and planet turn in the same direction, the train ratio is

$$\boxed{\frac{\omega_p}{\omega_c} = \frac{N_r}{N_p}} \qquad \textbf{(11.3)}$$

   where ω is angular velocity and N is the tooth count.
3. If the ring gear is locked, the planet gear turns one revolution for each carrier revolution *and* also turns a number of revolutions determined by the tooth ratio.

Since the planet and carrier rotate in opposite directions (clockwise and counterclockwise), the velocity ratio is

$$\boxed{\frac{\omega_p}{\omega_c} = 1 - \frac{N_r}{N_p}} \qquad \textbf{(11.4)}$$

The operation of sun/planet and ring/planet trains is very similar. However, it is very important to use the correct sign with the tooth ratio in equation 11.2 and 11.4; otherwise, serious numerical errors will result.

Up to this point we have considered the rotation of various planetary train components. We have not considered how we might couple an output from the planet gear. Since each planet gear rotates about the carrier center as well as about its own center, coupling to this gear presents a problem.

We could perhaps use a flexible shaft connected to the planet shaft, thereby effecting the desired coupling. Flexible shafts do, however, present some additional problems.

A much more common method of coupling is to use both a sun and ring gear arrangement like the one shown in figure 11-4. The addition of another gear allows the mechanism to operate in the previously discussed modes and one additional mode. The modes of operation now are:

1. If the planet gears are locked to the carrier, then rotating any one of the shafts causes the whole assembly to rotate as a unit.
2. If the carrier is locked, then the sun, planet, and ring gears operate as a simple gear train with a velocity ratio of

$$\frac{\omega_s}{\omega_r} = -\frac{N_R}{N_S} \qquad (11.5)$$

3. If the sun gear is locked, then equation 11.2 applies to the velocity ratio of the carrier and planet gears,

$$\boxed{\frac{\omega_p}{\omega_c} = \frac{N_s}{N_p} + 1} \qquad (11.2)$$

As the carrier and planet gears rotate around the sun gear, the ring gear rotates in the same direction as the carrier. The ring gear turns one revolution for each carrier revolution plus an additional amount due to the planet gear rotation. The ring gear advances one tooth thickness each time the planet gear rotates one tooth thickness against the sun gear. Consequently, the velocity ratio of the ring gear and carrier is

$$\frac{\omega_r}{\omega_c} = 1 + \frac{N_s}{N_r} \qquad (11.6)$$

when the sun gear is locked.

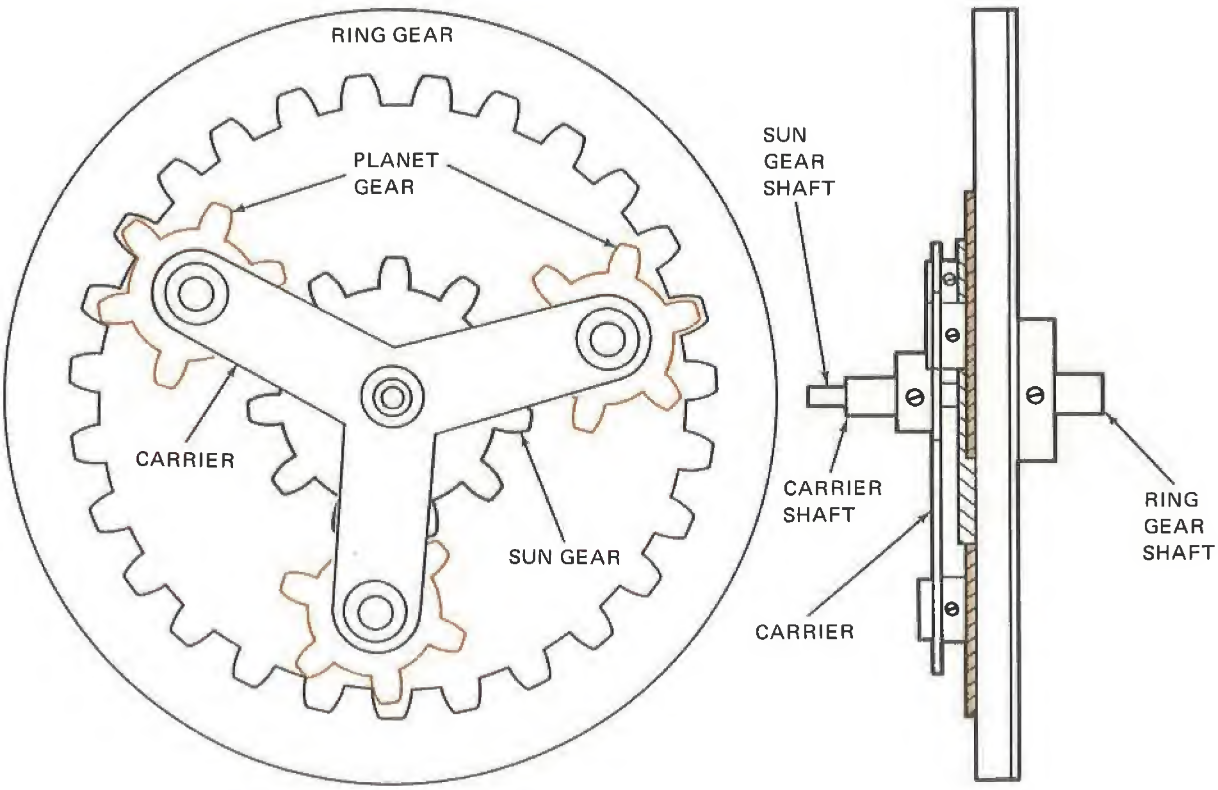


Fig. 11-4 A Typical Planetary Gear Assembly

4. If the ring gear is locked, then substantially the same thing happens as discussed above. However, since the carrier and sun gears revolve in opposite directions, the sign of the tooth ratio is negative. Therefore, the velocity ratio of the sun gear to that of the carrier is

$$\frac{\omega_s}{\omega_c} = 1 + \frac{N_r}{N_s} \qquad \textbf{(11.7)}$$

when the ring gear is locked in position.

It is also possible to put two rotary inputs into a planetary gear train and take a single rotary output from it. When this is done, the general relationship

$$\boxed{\omega_s = \omega_c\left(1 + \frac{N_r}{N_s}\right) - \omega_r \frac{N_r}{N_s}} \qquad \textbf{(11.8)}$$

may be used to solve for the various angular velocities. In using this equation, particular attention must be paid to whether N_r/N_s is positive or negative. If the sun and ring gears rotate in the same direction, N_r/N_s is positive. If the sun and ring gears rotate in opposite directions, N_r/N_s is negative.

A practical point that is worth mentioning in connection with the construction of planetary trains like figure 11-4 is that the pitch radius of the ring gear (R_r) is equal to the sum of the sun gear pitch radius (R_s) and the planet gear pitch diameter (D_p). That is

$$R_r = R_s + D_p \qquad \textbf{(11.9)}$$

In terms of pitch diameters, we may write this expression as

$$D_r = D_s + 2D_p \qquad \textbf{(11.10)}$$

And, of course, all of the gears must be of the same diametrical pitch.

MATERIALS

1 Sun gear, approx. 1-1/2 in. OD 1/8 in. bore
3 Planet gears, approx. 3/4 in. OD, 1/4 in. bore
1 Internal gear assembly, 3 in. ID
2 Bearing plates with spacers
1 Breadboard with legs and clamps
2 Bearing Mounts
2 Bearings
2 Dials with 1/4 in. bore hubs
1 Dial 1/8″ hub
1 Dial clamp
3 Dial indices with mounting hardware
2 Hollow shafts 2″ X 1/4″ with 1/8″ bore
1 Shaft 4″ X 1/8″
2 Collars
1 Planet carrier and 1/4 in. bore hub
1 Dial caliper
3 Planet spacers

PROCEDURE

1. Inspect all of your components to insure that they are undamaged.
2. Measure and record the ID of the ring gear and the ODs of the sun and planet gears.
3. Count the number of teeth on each gear wheel and record the results.
4. Compute and record the pitch diameter of each gear wheel. Compare your results to the requirements for a gear train of the type shown in figure 11-4. When you are sure that your gears are compatible with this type of assembly, proceed with step 5.

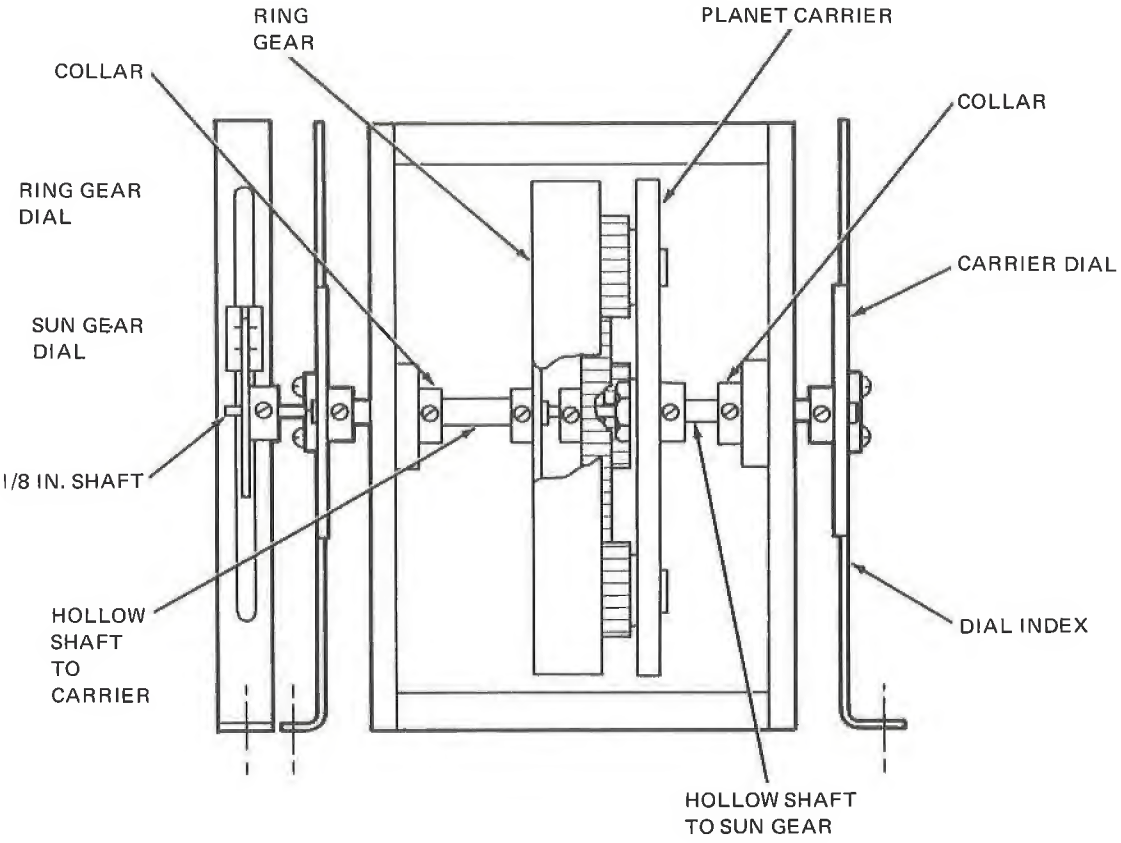


Fig. 11-5 The Experimental Mechanism

5. Assemble a planetary gear train of the type shown in figure 11-4 using the layout indicated in figure 11-5.
6. Loosen the sun gear dial clamp and position both the carrier and ring gear dials for a reading of zero. Tighten the sun gear dial clamp.
7. Slowly rotate the carrier dial one complete turn clockwise while carefully observing the amount and direction of rotation of the ring gear dial. Record the amount of rotation of each dial in degrees (θ_c and $\pm\theta_r$). Use a positive sign for θ_r if dials turn in the same direction, and a negative sign if they rotate oppositely.
8. Interchange the dial clamp and *the ring gear dial index.*
9. Repeat steps 6 and 7 using θ_c and $\pm\theta_s$.
10. Interchange the dial clamp and *the carrier dial index.*
11. Repeat steps 6 and 7 again by turning the sun gear dial (θ_s) and observing the ring gear dial ($\pm\theta_r$).
12. Remove the dial clamp completely and replace it with a dial index.
13. Adjust all three dials to read zero.

	ID or OD	N	D_p
Ring Gear			
Planet #1			
Planet #2			
Planet #3			
Sun Gear			

Sun Gear Locked				Ring Gear Locked				Carrier Locked				Unlocked			
θ_c	θ_r	θ_c/θ_r	ω_c/ω_r	θ_c	θ_s	θ_s/θ_c	ω_s/ω_c	θ_s	θ_r	θ_s/θ_r	ω_s/ω_r	θ'_c	θ'_r	θ'_s	θ''_s

Fig. 11-6 The Data Table

14. Hold the ring gear dial and turn the carrier dial about 80 degrees clockwise. Then hold the carrier dial and turn the ring gear dial about 60 degrees counterclockwise. Record the readings of the *three* dials (θ'_c, θ'_r, and θ'_s). Be sure to use the correct algebraic sign for each angle.

15. Compute and record the ratio of θ_c/θ_r using the values from step 7. Similarly, record the ratios θ_s/θ_c and θ_s/θ_r using the values from steps 9 and 11 respectively.

16. Using the *appropriate* equations from the discussion and the gear tooth counts, compute and record ω_c/ω_r, ω_s/ω_c, and ω_s/ω_r.

17. With the carrier and ring gear displacements from step 14 (θ'_c and θ'_r) as well as the tooth counts, compute the sun gear displacement (θ''_s) using equation 11.8.

ANALYSIS GUIDE. In analyzing the results from this experiment, you should discuss the extent to which your displacement ratios agreed with the velocity ratios. Also explain *why* the velocity and displacement ratios *should* agree. Also discuss the agreement between θ'_s and θ''_s. Finally, explain why the methods used to get θ'_s and θ''_s were valid.

PROBLEMS

1. A simple planet gear assembly like the one in figure 11-1 is built using a 60-tooth sun gear and a 24-tooth planet gear. How fast will the planet gear rotate if the carrier turns at 2000 RPM?

2. What angle will the carrier in problem 1 rotate through each time the planet turns one complete revolution?

3. An internal gear having 78 teeth is to be substituted for the sun gear in problem 1. When this is done, what would be the results of problem 1 and 2?

4. If the gears in problem 1 are 24 pitch, what would be the distance between the planet and carrier center shafts in problem 1?

5. Repeat problem 4 using the gears in problem 3.

6. A planetary gear assembly similar to figure 11-4 is to be constructed using 32-pitch gears. If the ring and sun gears have 96 and 48 teeth respectively, how many teeth does each planet gear have?

7. If the ring gear in problem 6 rotates at 3200 RPM and the sun gear is locked, what is the angular velocity of the carrier?

8. The sun gear in problem 6 is rotating at 400 RPM clockwise and the ring gear is rotating at 200 RPM counterclockwise. What is the carrier doing?

experiment 12 HELICAL GEARS

INTRODUCTION. In some applications the relatively low contact ratio of spur gears is a disadvantage. The use of helical gears is one way to overcome this disadvantage. In addition to having relatively large contact ratios, helical gears also typically run quieter than spur gears. In this experiment we shall examine some of the operating characteristics of helical gears.

DISCUSSION. Helical gears, like those shown in figure 12-1, have their teeth cut at an angle across the width of the gear blank.

Virtually all of the relationships which are appropriate for spur gears also apply to helical gears. Specifically the gear ratios are

$$\boxed{\frac{n}{N} = -\frac{\theta_g}{\theta_p} = -\frac{\omega_g}{\omega_p} = -\frac{T_p}{T_g}} \qquad (12.1)$$

where

n = Pinion tooth count
N = Gear tooth count
θ = Angular displacement (subscript p for pinion, g for gear)
ω = Angular velocity
T = Torque

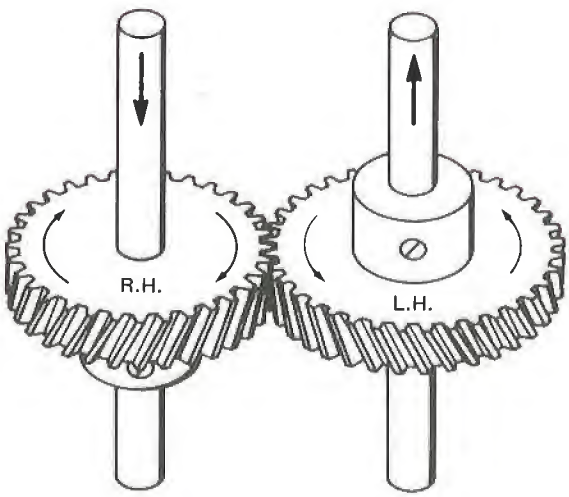


Fig. 12-1 Meshed Helical Gears

The angle between the side of the gear blank and a line perpendicular to the teeth is called the *helix angle* ψ (psi) of the gear. This is, of course, the same angle as that between the teeth and the gear shaft center line. Figure 12-2 shows these angles.

Having the teeth set at an angle makes it possible to define pitch differently from that of a spur gear. You will recall that for a spur gear, circular pitch (P_c) is defined as the ratio of the pitch circumference to the number of teeth on the gear, where D is the pitch diameter of the gear.

$$P_c = \frac{\pi D}{N} \text{ in./tooth} \qquad (12.2)$$

In other words, the circular pitch of a gear is the distance along the pitch circle between the teeth.

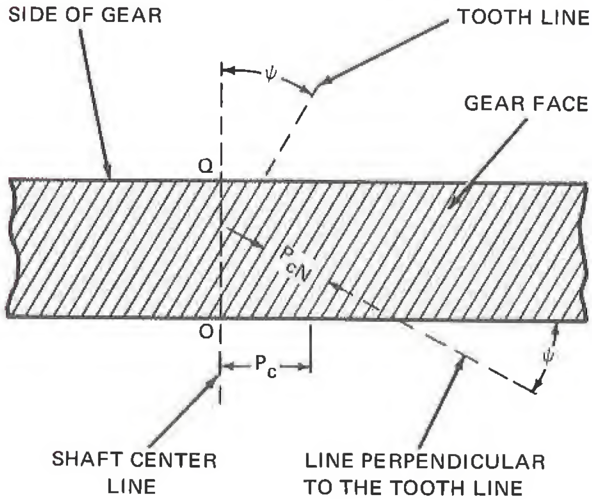


Fig. 12-2 Helix Angle and Normal Circular Pitch

In the case of a helical gear we could measure this distance parallel to the side of the gear or perpendicular to the tooth line. In practice, both systems are used. The term *circular pitch* (P_c) refers to the pitch measured parallel to the gear side. This is the same as for a spur gear. The pitch measured perpendicular to the tooth line is called the *normal circular pitch* (P_{cN}). Examining figure 12-2 closely we see that P_c and P_{cN} form an angle equal to ψ as shown in figure 12-3. Moreover, we see that P_c, P_{cN}, and the tooth line form a right triangle with P_c as the hypotenuse; P_{cN}, the adjacent side, and ψ, the enclosed angle. For such a right triangle we know that

$$\cos \psi = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Consequently, we may write

$$\cos \psi = \frac{P_{cN}}{P_c}$$

or

$$P_{cN} = P_c \cos \psi \qquad \textbf{(12.3)}$$

as the relationship between the circular and normal circular pitches.

In the case of spur gears, the circular and diametrical pitches are related by

$$P_c = \frac{\pi}{P_d}$$

This same relationship is also used for the normal circular pitch and *normal diametrical pitch* (P_N) of a helical gear. That is

$$P_{cN} = \frac{\pi}{P_N} \qquad \textbf{(12.4)}$$

Substituting this quantity into equation 12-3 for P_{cN} gives us

$$\frac{\pi}{P_N} = P_c \cos \psi$$

Then, substituting π/P_d for P_c provides

$$\frac{\pi}{P_N} = \frac{\pi}{P_d} \cos \psi$$

or

$$\boxed{P_N = \frac{P_d}{\cos \psi}} \qquad \textbf{(12.5)}$$

which is a useful relationship between the two diametrical pitches.

The helix angle selected for a particular gear application is usually such that there is some overlap of a meshing gear from one tooth to another. This can occur if point Q in figure 12-2 is in line with, or to the right of, point O. The triangle formed by line OQ, the side of the gear, and the tooth line is shown in figure 12-4. The distance OQ is the width of gear τ (tau).

From the triangle we see that

$$\frac{P_c}{\tau} = \tan \psi$$

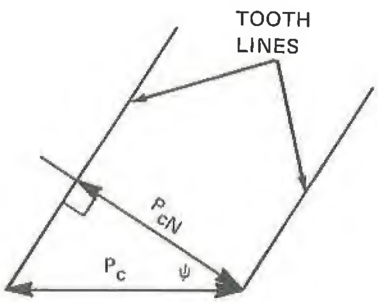


Fig. 12-3 The P_c, P_{cN} Triangle

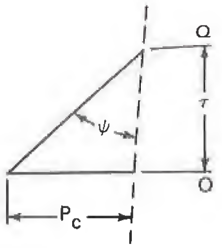


Fig. 12-4 Circular Pitch, Width, and Helix Angle

or

$$\psi = \text{arc TAN } \frac{P_c}{\tau} \qquad \textbf{(12.6)}$$

Helical gears may be made with either a righthand or lefthand helix angle. A righthand helix angle makes the teeth run in the direction of a righthand screw thread. Figure 12-1 shows both right- and lefthanded helix angles.

If helical gears are to be meshed between parallel shafts, they must have equal pitches, equal helix angles, and be of opposite "hand". That is, one must be righthanded and the other lefthanded as shown in figure 12-1.

In addition to being able to handle greater torque due to the increased contact ratio, helical gears also tend to run considerably quieter than do spur gears.

Unfortunately helical gears are more expensive to manufacture than are spur gears. An even more troublesome disadvantage of helical gears is their *side thrust.* To better understand this problem, let's observe that if a pinion is delivering a torque (T), then force on the pinion teeth is

$$F = \frac{T}{r}$$

where r is the pitch radius of the pinion. In terms of the pitch diameter this is

$$F = \frac{T}{d/2} = \frac{2T}{d}$$

This force is transmitted to the teeth of the gear. However, since the helical gear's teeth are inclined at an angle ψ, we have a situation somewhat as shown in figure 12-5a. The force tends to do two things. Part of it is transmitted to the gear along a line perpendicular to the tooth line. At the same time the helix angle incline tries to force the pinion sideways. From the force diagram we see that this side thrust force (f_y) has a value of

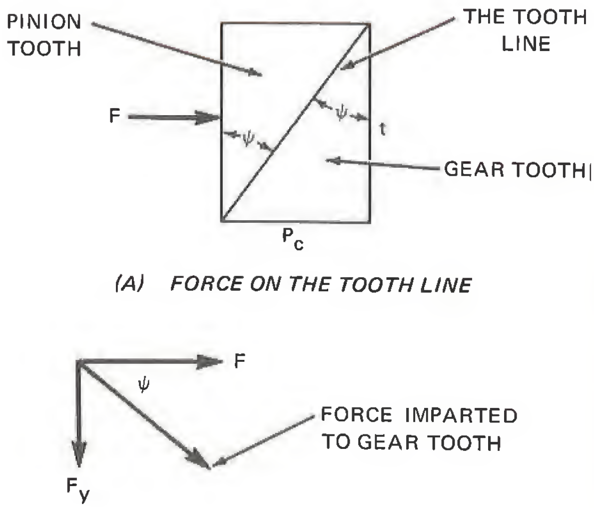
$$f_y = F \text{ TAN } \psi$$

And since we have found that

$$F = \frac{2T}{d}$$

we have

$$\boxed{f_y = \frac{2T}{d} \text{ TAN } \psi} \qquad \textbf{(12.7)}$$



(A) FORCE ON THE TOOTH LINE

(B) THE FORCE DIAGRAM

Fig. 25-5 Force on Helical Teeth

as the equation for the side thrust on a helical gear. This side thrust effect can be very large and sometimes causes troublesome problems in mounting helical gears.

To eliminate the side thrust problem, helical gears are frequently mounted in opposing pairs as shown in figure 12-6. When helical gears are mounted in this way, the side forces of the two pinions oppose and cancel each other. And, in the same manner, the side forces of the two gears cancel each other. As an overall result there is no effective side force to deal with. The only real disadvantage of this system is that it is quite expensive.

From figure 12-6 it is apparent that the same job could be accomplished with a single gear pair if each gear had two opposing sets of helical teeth. Such gears are manufactured and are called *herringbone* gears. A sketch of this type of gear wheel is shown in figure 12-7.

There are no universally accepted standards for the tooth proportions of helical gears. A number of systems have been tried successfully. Helical gears are normally purchased in pairs and are rarely meshed with others. Therefore, the lack of standardization rarely causes problems. However, it should be kept in mind when dealing with this type of gears. Also, since tooth proportions vary, pitch diameters should *not* be used in computing the gear ratio of a mesh. The ratio of the tooth counts may, however, be relied upon for gear ratio calculations.

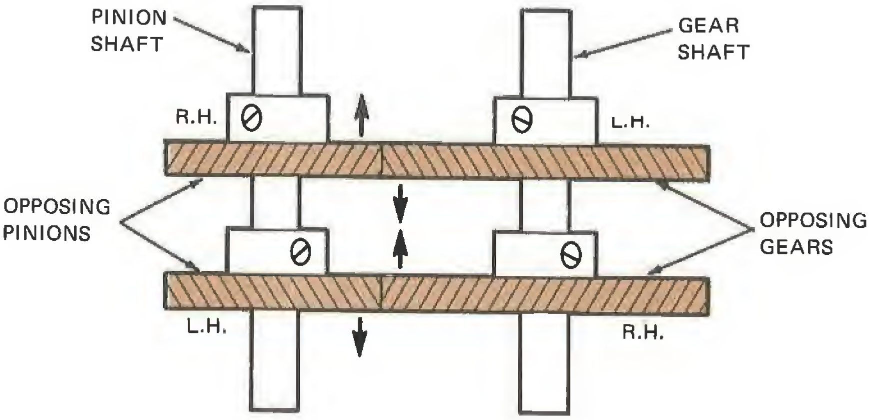


Fig. 12-6 Opposing Helical Gear Pairs

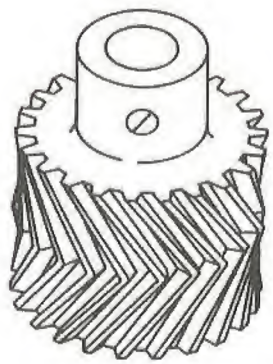


Fig. 12-7 A Herringbone Tooth Pattern

MATERIALS

1 Righthand helical pinion, approx. 3/4 in. OD.

1 Lefthand helical pinion, same N as R.H. pinion

1 Righthand helical gear, approx. 1-1/2 in. OD.

1 Lefthand helical gear, same N as R.H. gear.

2 Bearing plates with spacers

4 Bearing mounts

4 Bearings

2 Shafts 4" X 1/4"

4 Collars

1 Shaft coupling

1 Motor and mount

1 Power supply

1 Breadboard with legs and clamps

2 Dials with 1/4 in. bore hubs

1 Stroboscope

PROCEDURE

1. Examine all of the components to insure that they are undamaged.
2. Identify the right- and lefthand helical gears. Count and record the number of teeth on each gear wheel.
3. Assemble the mechanism shown in figure 12-8. Use an instrument grade oil to lubricate the shafts at the bearings.

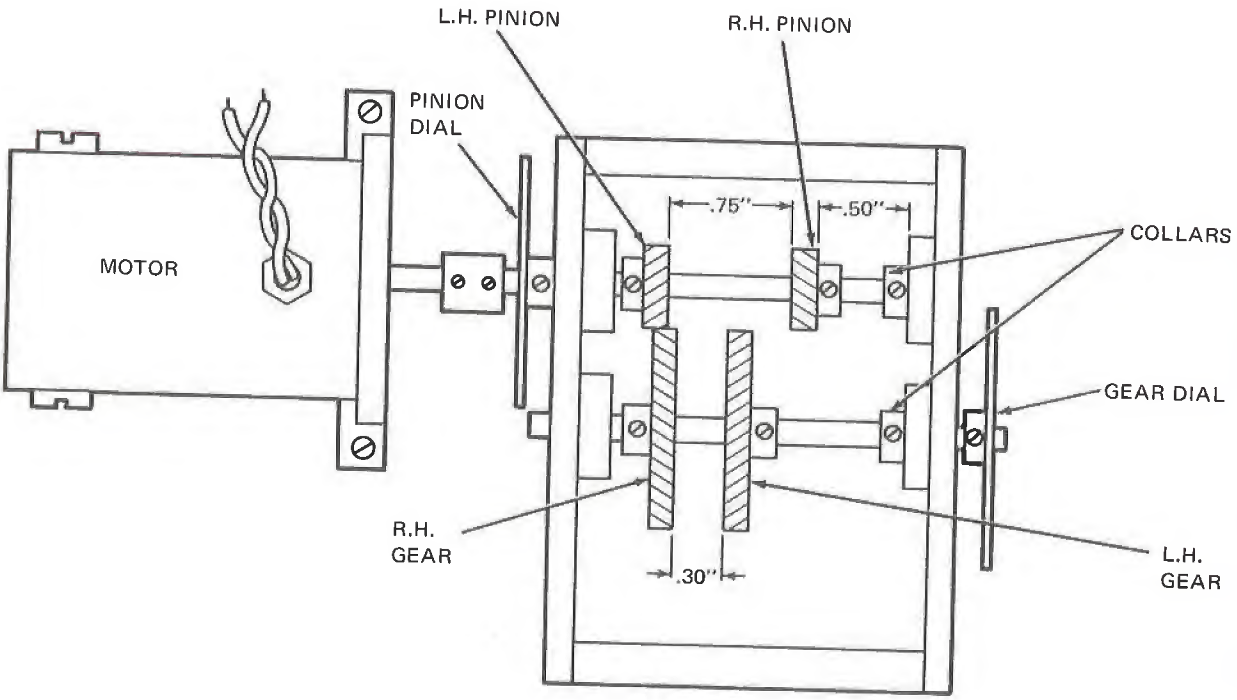


Fig. 12-8 The Experimental Mechanism

4. Rotate the mechanism several times by hand to insure that it is free.
5. Connect the motor wires to the DC power supply. Starting at zero volts, increase the voltage slowly until the pinion is turning a few hundred RPM. The pinion dial should be turning clockwise as viewed from the right side of figure 12-8.
6. With the pinion running clockwise, slowly increase the speed until the power supply is set for about 10 volts. Strobe the pinion dial to determine the angular velocity of the motor, ωp.
7. Strobe the gear dial and record the angular velocity (ωg).
8. If the presence of side thrust is not already apparent, carefully reach in and touch the gear shaft with your finger. This should increase the load and the side thrust. Make a note of the gear's reaction.
9. Repeat steps 6, 7, and 8 for voltages of 12, 14, and 16 volts.
10. Move the L.H. helical pinion into mesh with the R.H. gear, forming an opposing pair of the type shown in figure 12-6.
11. Repeat steps 6 through 9 and record your results in the Data Table as ω_g', ω_p'.
12. Compute the tooth ratio n/N and record the results.
13. Compute the velocity of the single mesh, ω_g/ω_p.
14. Compute the velocity ratio of the opposing pair mesh, ω_g'/ω_p'.

n (R.H.)	n (L.H.)	N (R.H.)	N (L.H.)	ω_g	ω_p	f_y	ω_g'	ω_p'	f_y'

n/N	ω_g/ω_p	ω_g'/ω_p'

Fig. 12-9 The Data Table

ANALYSIS GUIDE. In the analysis of these results there are three main points that you should consider. They are:

1. Did the tooth ratio and the single mesh velocity ratio agree? How closely?

2. Did using the double mesh significantly change the velocity ratio?
3. Did using the double mesh significantly change the side thrust? Why?

In addition to these points you should discuss the general accuracy of the experiment and any difficulties you encountered.

PROBLEMS

1. A 30-tooth R.H. helical pinion is meshed with a 72-tooth L.H. gear. The helix angle is 30° and the normal diametrical pitch is 4. What is the diametrical pitch?
2. What is the circular pitch of the gears in problem 1?
3. What is the normal circular pitch in problem 1?
4. What should be the minimum tooth width in problem 1?
5. If the gear in problem 1 turns 75 degrees clockwise, through what angle will the pinion rotate?
6. What is the velocity ratio in problem 1?
7. If 940 in.-lb. of torque is transmitted by the gear in problem 1, what is the side thrust?
8. In problem 7, what torque is transmitted by the pinion?

experiment 13 BEVEL GEARS

INTRODUCTION. Bevel gears are one type of gears that are used to transmit motion at an angle. They have their teeth cut on the surface of a cone instead of a cylinder. In this experiment we shall examine some of the operating characteristics of this type of gear.

DISCUSSION. The gear wheel shown in figure 13-1 is called a *bevel gear.* If we slice the gear along the center line of the shaft, the cross section will appear as shown in figure 13.2.

As you might expect, the OD of a bevel gear is measured at the large end of the teeth. The pitch and pitch diameter are defined in the same way as for a spur gear. Therefore, we have

$$\boxed{P_d = \frac{N}{D}} \qquad \textbf{(13.1)}$$

where P_d is the diametrical pitch, D is the pitch diameter, and N is the number of teeth. However, since bevel gear teeth are not normally proportioned like spur gear teeth, we *cannot* use the O.D. to determine pitch diameter as we do with a spur gear.

The velocity ratios of a pair of bevel gears are the same as for spur gears:

$$\boxed{\frac{n}{N} = -\frac{\theta_g}{\theta_p} = -\frac{\omega_g}{\omega_p} = -\frac{T_p}{T_s} = \frac{d}{D}} \qquad \textbf{(13.2)}$$

where n = Pinion tooth count
N = Gear tooth count
d = Pinion pitch diameter
D = Gear pitch diameter
θ = Angular displacement (subscript p for pinion, G for gear.)
ω = Angular velocity
T = Torque

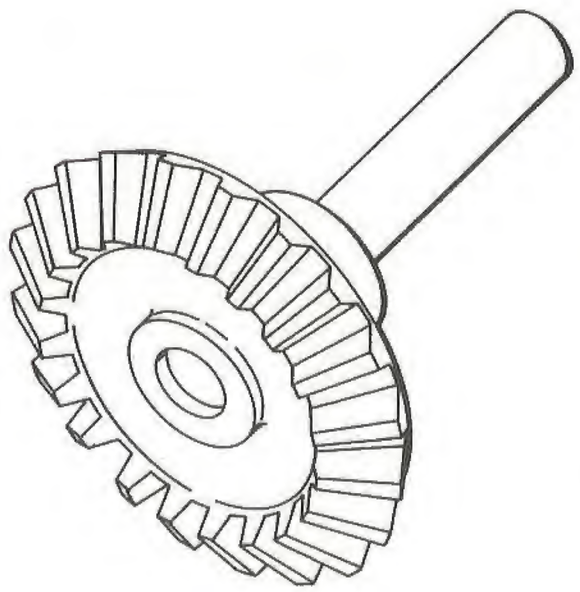


Fig. 13-1 A Bevel Gear

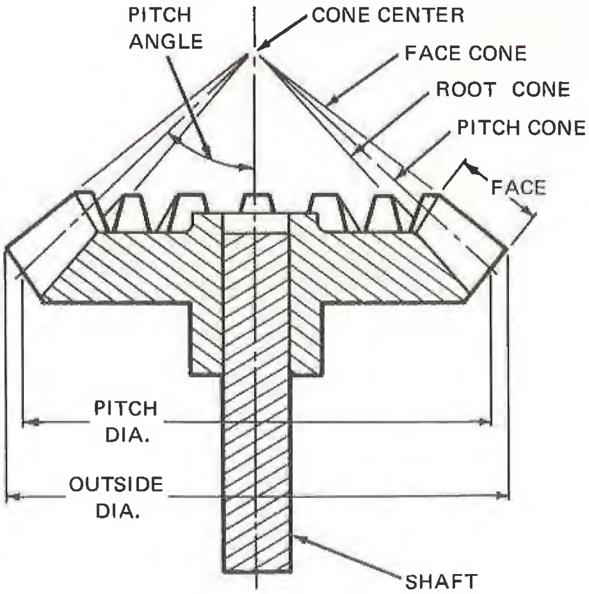


Fig. 13-2 Bevel Gear Terminology

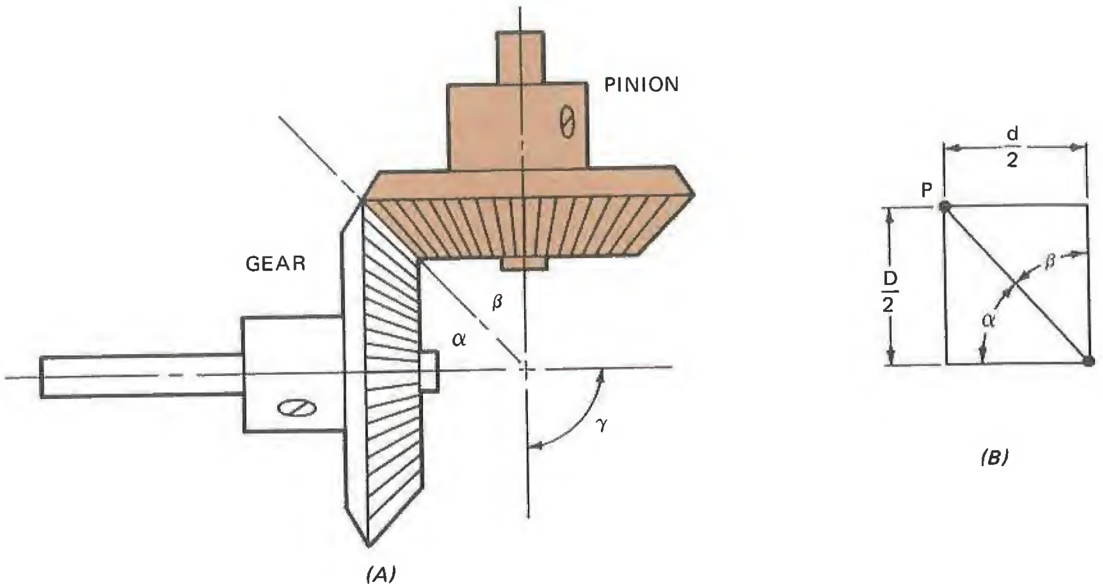


Fig. 13-3 Meshed Bevel Gears

The negative signs associated with the θ, ω, and T ratios indicate that the pinion and gear rotate in opposite directions.

If we mesh two bevel gears as shown in figure 13-3, we see that the angle (γ) between the shafts is equal to the sum of the two pitch angles (α and β). That is

$$\gamma = \alpha + \beta \qquad \textbf{(13.3a)}$$

Moreover, if we examine the pitch angle triangles more closely, we see that they are formed by the pitch radii and the line OP. Then, looking at each of the right triangles shown in figure 13-3 (b), we see that

$$\frac{d}{2} = \text{OP SIN } \beta$$

and

$$\frac{D}{2} = \text{OP SIN } \alpha$$

Dividing the first equation by the second and canceling appropriate terms, we have

$$\frac{d}{D} = \frac{\text{SIN } \beta}{\text{SIN } \alpha} \qquad \textbf{(13.4)}$$

The ratio of the pitch diameters is, of course, the same as the gear ratio. Consequently, we see that the gear ratio of a bevel gear set is related to the pitch angles.

Further, equation 13.3 shows us that the angle between the shafts also depends on the pitch angles. It is therefore very important to be able to determine the pitch angles which will produce a desired gear ratio at a given shaft angle. To determine one of the pitch angles we start with equation 13.4,

$$\frac{d}{D} = \frac{\text{SIN } \beta}{\text{SIN } \alpha} \qquad \textbf{(13.4)}$$

But, since equation 13.3a tells us that

$$\gamma = \alpha + \beta$$

we see that

$$\beta = \gamma - \alpha \qquad \textbf{(13.3b)}$$

Substituting this into 13.4 gives us

$$\frac{d}{D} = \frac{\text{SIN } (\gamma - \alpha)}{\text{SIN } \alpha}$$

or

$$\frac{d}{D} \text{SIN } \alpha = \text{SIN } (\gamma - \alpha)$$

This sine of $\gamma - \alpha$ is somewhat difficult to manipulate in its present form. However, from trigonometry we recall that the sine of the difference in two angles may be written as

$$\text{SIN } (\gamma - \alpha) = \text{SIN } \gamma \text{ COS } \alpha - \text{COS } \gamma \text{ SIN } \alpha \qquad \textbf{(13.5)}$$

Substituting this identity for SIN $(\gamma - \alpha)$ gives us

$$\frac{d}{D} \text{SIN } \alpha = \text{SIN } \gamma \text{ COS } \alpha - \text{COS } \gamma \text{ SIN } \alpha$$

Collecting SIN α terms on the left, we have

$$\frac{d}{D} \text{SIN } \alpha + \text{COS } \gamma \text{ SIN } \alpha = \text{SIN } \gamma \text{ COS } \alpha$$

or

$$\text{SIN } \alpha \left(\frac{d}{D} + \text{COS } \gamma\right) = \text{SIN } \gamma \text{ COS } \alpha$$

Then, dividing both sides by $(\frac{d}{D} + \text{COS } \gamma)$ and COS α renders

$$\frac{\text{SIN } \alpha}{\text{COS } \alpha} = \frac{\text{SIN } \gamma}{(\frac{d}{D} + \text{COS } \gamma)}$$

The ratio on the left, we recognize from trigonometry as tan α. That is

$$\frac{\text{SIN } \alpha}{\text{COS } \alpha} = \text{TAN } \alpha$$

We may, therefore, write

$$\text{TAN } \alpha = \frac{\text{SIN } \gamma}{(\frac{d}{D} + \text{COS } \gamma)}$$

or

$$\boxed{\alpha = \text{TAN}^{-1} \left(\frac{\text{SIN } \gamma}{\frac{d}{D} + \text{COS } \gamma} \right)} \qquad \textbf{(13.6)}$$

To illustrate the use of this equation, let's suppose that we wish to build a bevel gear set having a gear ratio (d/D) of 1/2 and an angle of 90° between the shafts. The pitch angle (α) of one of the gears must be

$$\alpha = \text{TAN}^{-1} \left(\frac{\text{SIN } 90^\circ}{\frac{1}{2} + \text{COS } 90^\circ} \right) = \text{TAN}^{-1} \left(\frac{1}{\frac{1}{2} + 0} \right)$$

$$\alpha = \text{TAN}^{-1} (2) \approx 63.5^\circ$$

The pitch angle (β) of the other gear can be found using equation 13.3b,

$$\beta = \gamma - \alpha \approx 90^\circ - 63.5^\circ = 26.5^\circ$$

Bevel gears can be manufactured for almost any reasonable ratio and shaft angle. However, 30°, 45°, and 90° shaft angles are the most common. Gear ratios of 1/3, 1/2, and 1/1 are very frequently used.

A shaft angle of 90° is probably the most frequently encountered, and when this angle is coupled with a 1/1 gear ratio, the gears are called *miter* gears.

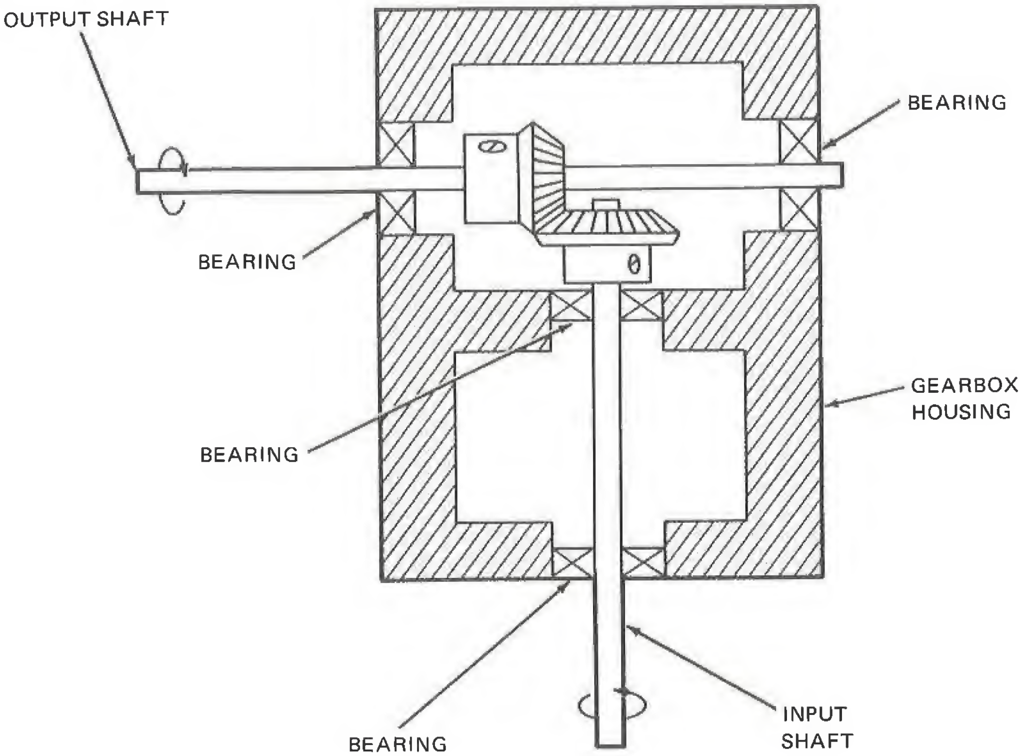


Fig. 13-4 Mounting Bevel Gears

Mounting bevel or miter gears can cause a few problems since each shaft must be supported by two bearings. Figure 13-4 shows one popular type of bevel gear box.

Retaining rings and shims to position and hold the bearings and shafts are not shown but would, of course, be required in a practical application.

Up to this point we have considered bevel gear operation without regard to the tooth pattern. There are, in fact, several different tooth patterns which are commonly used. Figure 13-5 shows the most often encountered patterns.

The *straight tooth* bevel gear (figure 13-5a) has its teeth cut in straight lines along the sides of the face cone. If the pitch angle is less than 90°, the bevel gear is an *external* toothed gear. One with a pitch angle of 90° has its teeth on the flat end of a cylinder and is called a *face gear* or a *crown* gear. For a pitch angle greater than 90°, the teeth are on the inside of a cone. Such a gear is called an *internal* toothed bevel gear.

A *zerol gear* (figure 13-5b) has its teeth cut on the surface of the face cone with a *circular cutter* so that the teeth are in the form of arcs. Such gears tend to run more quietly than straight tooth bevel gears and can sustain somewhat larger loads.

Spiral tooth bevel gears (figure 13-5c) offer many of the advantages of zerol gears in larger amounts but are more expensive to

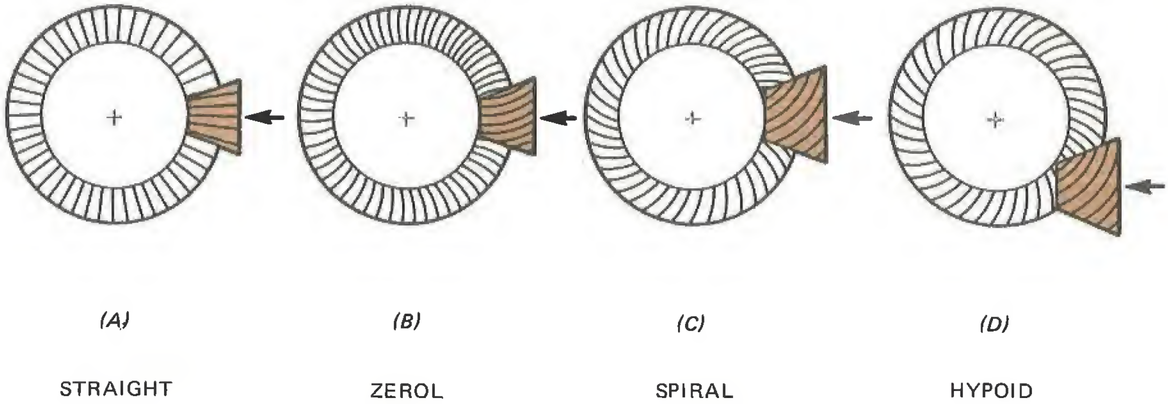


Fig. 13-5 Bevel Gear Tooth Patterns

manufacture. They are most often found in high-speed, relatively high-load applications.

Hypoid gears (figure 13-5d) are much like spiral gears but have their shafts offset. Hypoid gears have found wide acceptance in automobile "rear ends" since they allow the drive shaft to enter the gear box below the center line of the rear wheels. This feature allows lower and "more stable" automobile designs.

Bevel gears are occasionally constructed integrally with one or more spur or helical gears as shown in figure 13-6.

Such an arrangement can be quite economical and is used widely in mechanical design. These multiple-gear constructions are called *cluster gears.*

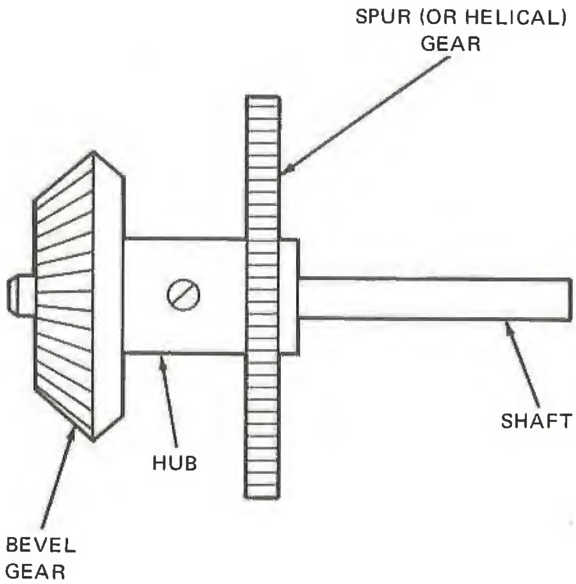


Fig. 13-6 Cluster Gear

MATERIALS

- 1 Breadboard with legs and clamps
- 2 Bearing plates with spacers
- 1 Motor and mount
- 1 Shaft coupling
- 2 Bearing mounts
- 2 Shaft hangers with bearings
- 2 Bearings
- 2 Dials with 1/4 in. bore hubs
- 3 Collars
- 1 Stroboscope
- 1 Power supply
- 1 Bevel pinion
- 1 Bevel gear
- 2 Shafts 4" X 1/4"

PROCEDURE

1. Inspect each of your components to insure that they are all undamaged.
2. Count the number of teeth on each bevel gear and record the results.
3. Assemble the mechanism shown in figure 13-7.
4. Turn the mechanism several times by hand without the belt in place to insure free rotation.

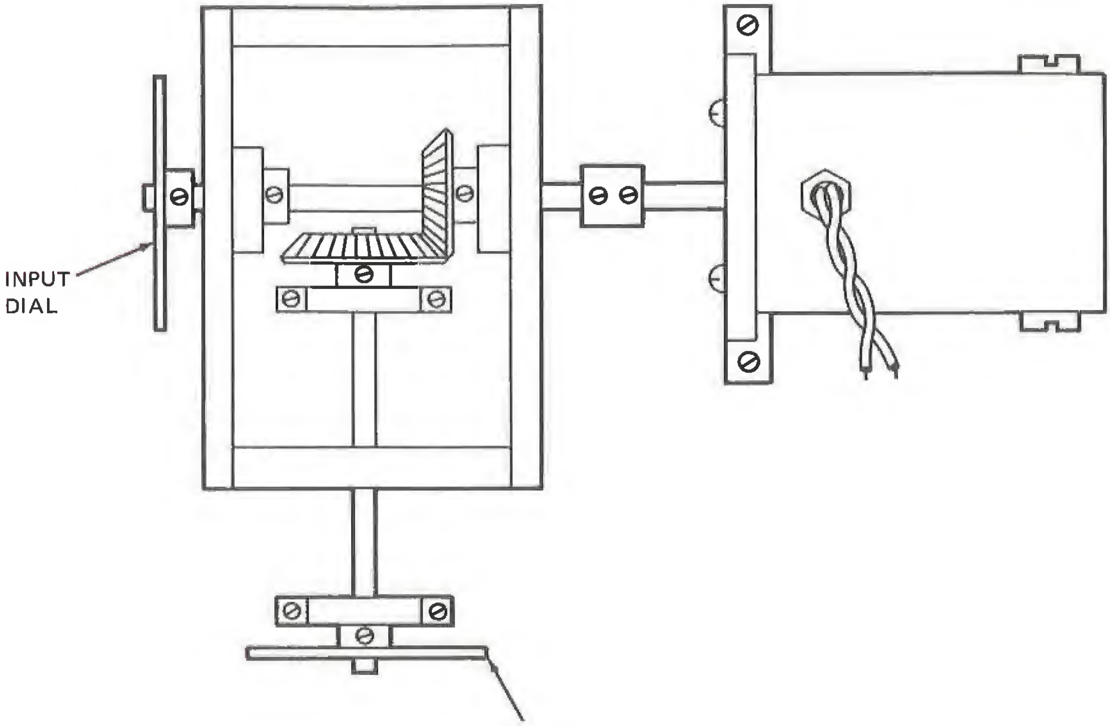


Fig. 13-7 The Experimental Mechanism

5. Install the belt and start the motor. Set the motor voltage to about 24 volts.
6. Using the stroboscope, measure and record the angular velocity of the input shaft (ω_p) and the output shaft (ω_g).
7. Compute and record the ratio of the number of teeth on the pinion to the number of teeth on the gear (n/N).
8. Compute and record the velocity ratio (ω_g/ω_p).
9. Compute the percent difference between the two ratios.

$$\%\ \text{Diff} = 100\,\frac{(n/N) - (\omega_g/\omega_p)}{(n/N)}$$

10. Repeat steps 6 through 9 for motor voltages of 18 and 14 volts.

Voltage	n	N	n/N	ω_p	ω_g	ω_g/ω_p	% Diff.
24 V							
18 V							
14 V							

Fig. 13-8 The Data Table

ANALYSIS GUIDE. In analyzing these data you should consider the extent to which your results agreed with those predicted by the discussion. Also consider the cause of any difference between the two ratios. Were the differences within reason, considering the methods and components used? How could the differences be reduced?

PROBLEMS

1. A 48-pitch bevel gear has 45 teeth. What is the pitch diameter?
2. The gear in problem 1 is to be meshed with a 30-tooth pinion. What is the gear ratio?
3. What is the pitch diameter of the pinion in problem 2?
4. The gears in problem 2 are to be used with shafts that are 90° apart. What is the pitch angle of each gear?
5. Two meshing bevel gears have pitch angles of 31.75° and 13.25°, respectively. What is the angle between the two shafts?
6. A 72-pitch bevel gear set is to be used between shafts that are 30° apart. The velocity ratio is to be 1:3. Specify pitch angles and tooth counts for the gears.

experiment 14 RACK AND PINION

INTRODUCTION. Many practical applications of mechanics require that rotary motion be converted to linear motion. One of the mechanisms used to make this conversion is the rack-and-pinion. In this experiment we will examine several characteristics of the rack-and-pinion mechanism.

DISCUSSION. A *rack* is a straight bar with involute gear teeth cut into one surface. It is normally meshed with a pinion as shown in figure 14-1.

There are several approaches that can be taken to analyze the action of a rack-and-pinion assembly. One popular approach is to consider the rack to be a gear wheel of infinite diameter. Such a gear would have a straight line for its pitch circle. Analysis of pitch circle velocities can then be carried out.

The infinite gear approach might be somewhat confusing since most racks are only a few inches long. We shall therefore consider a rack to be simply a toothed bar free to move in its slide.

Referring to figure 14-1, if we turn the input shaft by an amount θ_p, the pitch circle of the pinion will move through an arc length (S) of

$$S = r\theta_p$$

where r is the pitch radius of the pinion. In terms of the pitch diameter, this distance is

$$S_r = \frac{1}{2} d\,\theta_p \qquad \textbf{(14.1)}$$

And since the rack is free to move, this equation 14.1 gives the movement of the rack. If the pinion turns one complete revolution, the rack moves

$$S_r = \pi d \text{ in. per rev.}$$

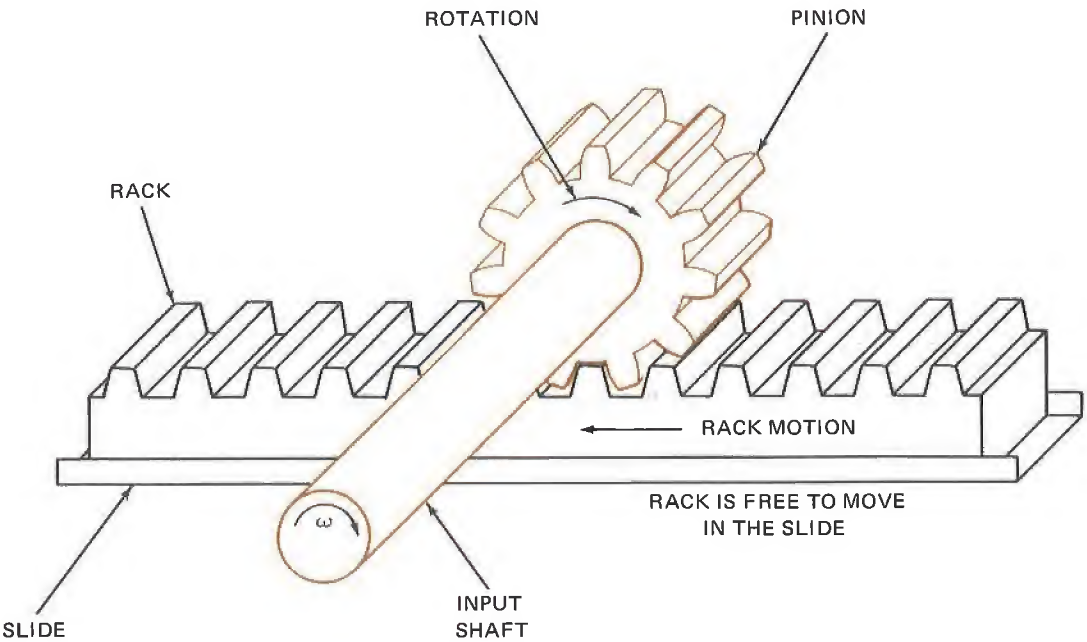


Fig. 14-1 A Rack-and-Pinion

And if the pinion turns at an angular velocity of ω revolutions-per-minute, then the rack velocity is

$$V_r = \pi\omega d \text{ in. per min.}$$

But since we normally measure such velocities in inches per second, it is more convenient to use

$$V_r = \frac{1}{60}\pi\omega d \text{ in. per sec.}$$

Finally, since π is equal to 3.1416 we have

$$V_r = 0.0525\ \omega d \text{ in. per sec.} \qquad \textbf{(14.2)}$$

for the velocity of the rack in terms of the pinion angular velocity (in RPM) and pitch diameter.

A rack-and-pinion may, of course, be used to convert linear motion to angular motion by applying the linear input to the rack and taking the rotary output from the pinion. When this is done, equation 14.2 may be solved for the RPM in terms of rack velocity and pinion pitch diameter.

$$\boxed{\omega = \frac{V_r}{0.0525d} = 19.1\frac{V_r}{d} \text{ RPM}} \qquad \textbf{(14.3)}$$

We can find the relationship between rack force (f_r) and shaft torque (T_p) by assuming that the input and output work are equal. See figure 14-2.

The work done by the rack is

$$W_r = f_r S_r$$

and that of the pinion is

$$W_p = T_p \theta_p$$

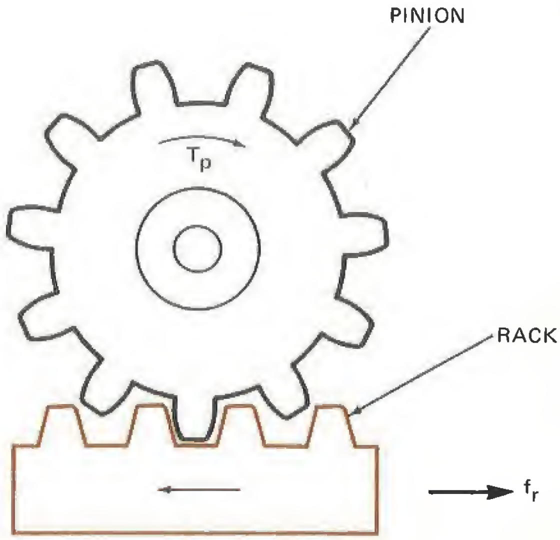


Fig. 14-2 Load Force and Pinion Torque

Equating these quantities gives us

$$T_p \theta_p = f_r S_r$$

Substituting the value of S_r from equation 14.1 renders

$$T_p \theta = \frac{1}{2} f_r d\theta$$

or

$$\boxed{T_p = \frac{1}{2} f_r d} \qquad \textbf{(14.4)}$$

which is the torque produced at the pinion shaft by a rack force of f_r.

Similarly, the rack force produced by a given pinion torque is

$$\boxed{f_r = 2\frac{T_p}{d}} \qquad \textbf{(14.5)}$$

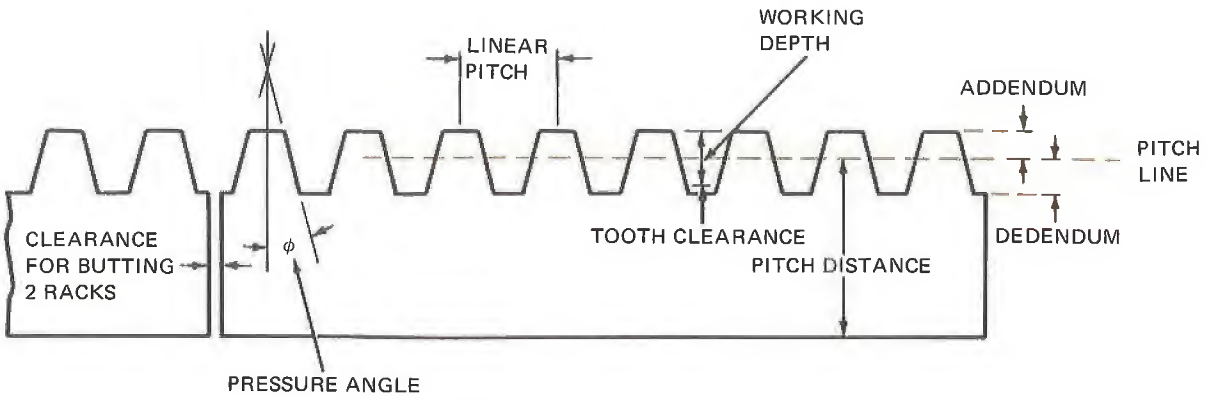


Fig. 14-3 Rack Terminology

In mounting a rack some sort of slide must be provided. Frictional losses between the rack and slide can be a very serious problem. Consequently, rack slides require careful lubrication. Roller bearing slides are frequently used in precision applications.

Rack teeth are straight-sided and inclined from the vertical at the pressure angle of the mating gear. Figure 14-3 shows the rack tooth geometry and terminology.

Racks can be manufactured to mesh with any pinion pitch. One special pitch is manufactured for use in decimal control applications. It has a linear pitch of 0.100 in.

Racks come in a great variety of lengths from fractions of an inch to several feet. Precision quality racks are rarely over a foot long because it is extremely difficult to maintain high precision straightness in longer lengths. Racks can, however, be butted together end-to-end to form longer lengths.

MATERIALS

- 2 Bearing plates with spacers
- 6 Bearing mounts
- 6 Bearings
- 3 Shafts 4″ X 1/4″
- 1 Pinion approx. 1 in. OD
- 1 Rack approx. 9 in. long
- 6 Collars
- 2 Spring balances with posts and clamps
- 1 Breadboard with legs and clamps
- 1 Lever arm approx. 2 in. long with 1/4 in. bore hub
- 1 Dial with 1/4 in. bore hub
- 1 Dial index with mounting hardware
- 1 Dial caliper

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the bearing plates and components shown in figure 14-4.
3. Mount the bearing plate assembly on the spring balance stand as shown in figure 14-5a.

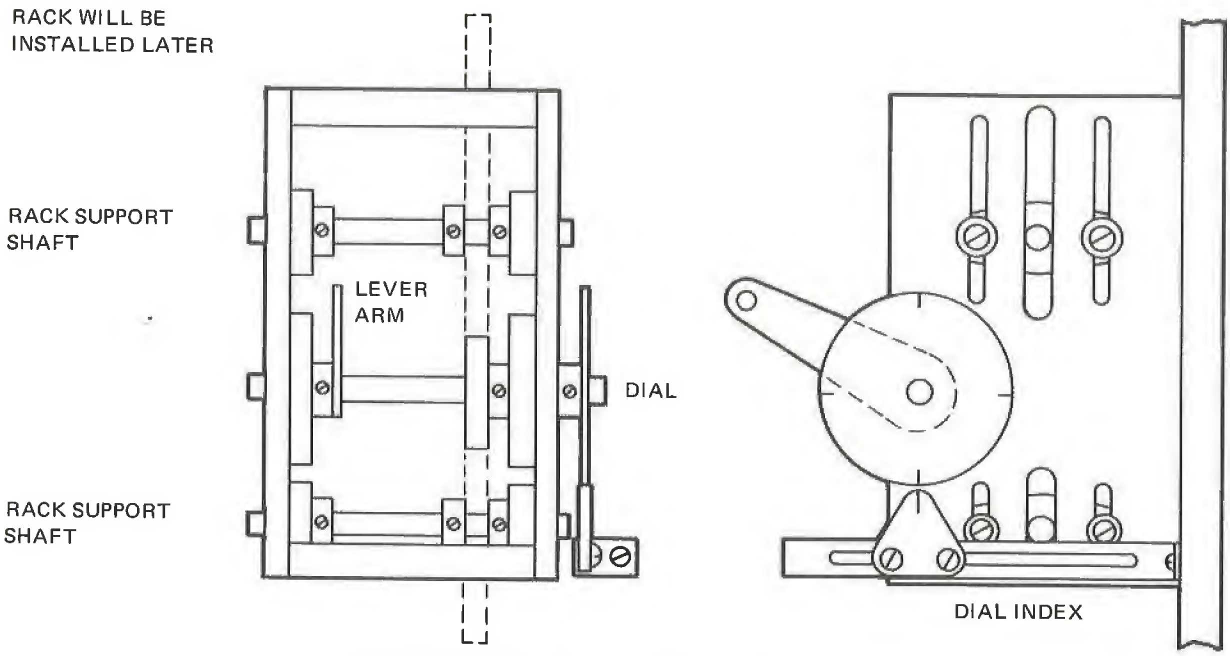
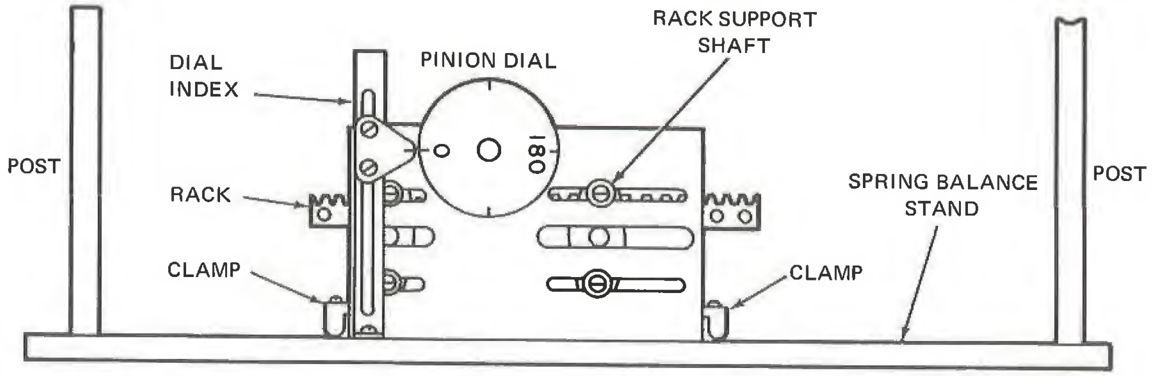


Fig. 14-4 The Bearing Plate Assembly

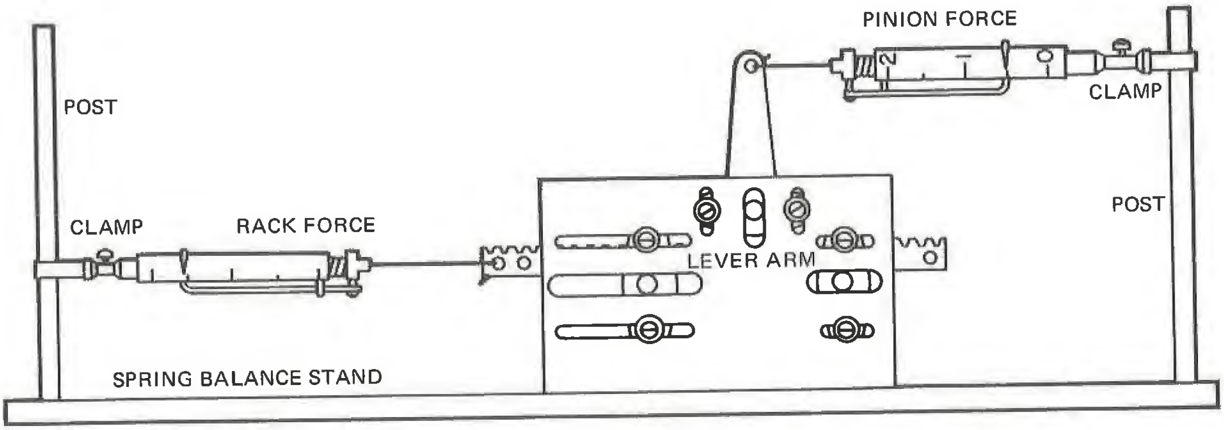
4. Carefully insert the rack on top of rack support shafts between the pairs of collars as indicated in figure 14-4. Adjust the collars so that the rack moves freely. Lubricate the bearings and rack.
5. Adjust the pinion position so that it meshes with the rack and check for freedom of movement.
6. Position the rack at some convenient reference point from which you can measure its displacement.
7. Set the pinion dial to zero with the rack in the reference position.



(A) THE FIRST EXPERIMENTAL SETUP

Fig. 14-5 The Experimental Mechanism

8. Starting at zero, rotate the dial in 10-degree steps. At each 10-degree step, measure and record the rack displacement (S_r) from the reference point.
9. Measure and record the OD of the pinion.
10. Count the number of teeth on the pinion and record the results.
11. Compute the pinion pitch diameter and record the results.
12. Using the appropriate equation from the discussion, compute and record the rack displacement for each angle used in step 8 (θ should be in radians).
13. Compute the percent difference between each pair of measured and computed rack displacement values.
14. Turn the spring balance stand around and install the spring balances as shown in figure 14-5b.



(B) THE SECOND EXPERIMENTAL SETUP

Fig. 14-5 The Experimental Mechanism

15. Adjust the posts until the lever arm is vertical and the rack force is approximately 4 oz. Record the readings of both spring balances.
16. Repeat step 4 for rack forces of 12, 16, 20, 30 oz.
17. Measure and record the distance from the center of the pinion shaft to the point of attachment of the pinion spring balance (r_ℓ).
18. For each pinion force value, compute the pinion torque and record the results. (T_p meas.)
19. Using the appropriate equation from the discussion, compute and record the pinion torque for each measured value of rack force. (T_p comp.)
20. Compute the percent difference between each pair of measured and computed torque values.

θ (Degrees)	S_r (Meas.)	S_r (Comp.)	% Diff.

d_o	n	d	r_ℓ

f_r	f_p	T_p (Meas.)	T_p (Comp.)	% Diff.

Fig. 14-6 The Data Tables

ANALYSIS GUIDE. In this experiment we have examined rack-and-pinion displacements and forces. In your analysis you should discuss the significance of the percent differences observed for the displacements and torques. What were probably the major causes of these differences? How could the differences be reduced?

PROBLEMS

1. An automobile has a rack-and-pinion steering mechanism. The rack is 9 in. long and the pinion has 28 teeth. The pinion has a pitch of 14. How far does the rack move per pinion revolution?
2. In problem 1, how many pinion revolutions are required to move the rack from end to end?
3. The steering wheel in the car above must be turned approximately 2.8 turns to move the rack from end to end. What is the gear ratio between the steering wheel and the pinion?
4. How many teeth are there on the rack in problem 1?

5. The steering wheel in problem 3 has a 12 in. diameter. When the car is standing still, the driver must exert 14 lbs. of force on the steering wheel to move the rack. What is the rack force?

6. A drill press uses a rack-and-pinion between the feed wheel and the quill (See figure 14-7). The pinion is 24 pitch and has 36 teeth. How long must the rack be if the feed wheel turns 1-1/2 times to move quill from its highest to lowest position?

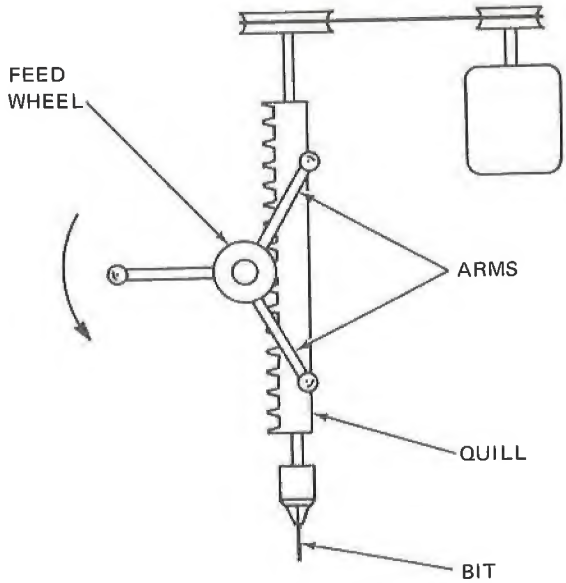


Fig. 14-7 System for Problem 6

7. How fast must the feed wheel turn to move the bit 2 in. per min.?

8. If the feed wheel arms are 8 in. long and the operator applies 30 lbs. of force to them, what is the force on the bit?

9. The drill press in figure 14-7 has a return spring which retracts the quill to its highest position when the feed wheel is released. This spring exerts 4 lbs. of force upward on the quill. How much torque must the operator apply to the pinion to overcome the spring?

10. In problem 9, what force does the operator apply to the 8-in. feed wheel arms?

experiment 15 WORM AND WHEEL

INTRODUCTION. The worm-and-wheel assembly is a very economical way to achieve large gear ratios in a relatively small size. In this experiment we shall examine some of the operating characteristics of a worm-and-wheel mechanism.

DISCUSSION. A *worm* pinion is a cylinder with a helical thread cut on its outer surface, as shown in figure 15-1. The worm thread may be either right- or lefthand.

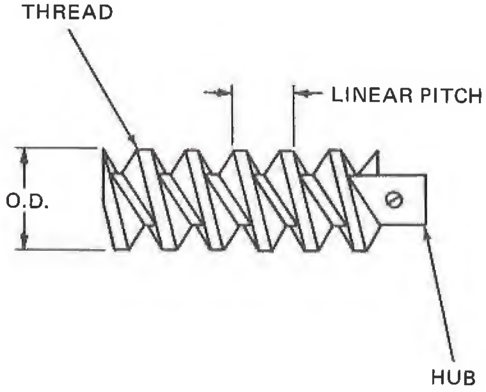


Fig. 15-1 A Righthand Worm

The worm is meshed with a specially designed gear wheel as shown in figure 15-2. The worm wheel is manufactured with curved tooth tips so that it will fit down into the worm better.

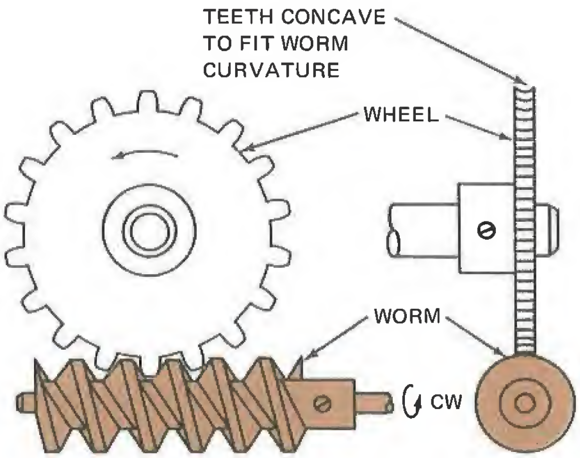


Fig. 15-2 A Worm-and-wheel

Worm wheel teeth are often shaped much like spur gear teeth. There is sliding contact between the teeth and the worm thread. Consequently, there is considerable friction loss in a worm-and-wheel mechanism. The efficiency of worm-and-wheel assemblies typically runs from about 20 percent up to about 90 percent. Gear drives, on the other hand, are usually well over 75 percent efficient.

However, because worm-and-wheel assemblies have the full face of their teeth in contact, they are able to transmit relatively large loads. To further improve this large load capability, the worm thread may be concaved as shown in figure 15-3 to bring even more tooth surface into contact. Along with the increased load carrying ability comes increased friction and reduced efficiency.

Worm-and-wheel mechanisms are also built using helical gears. These assemblies tend to be highly inefficient and wear more quickly than spur tooth types.

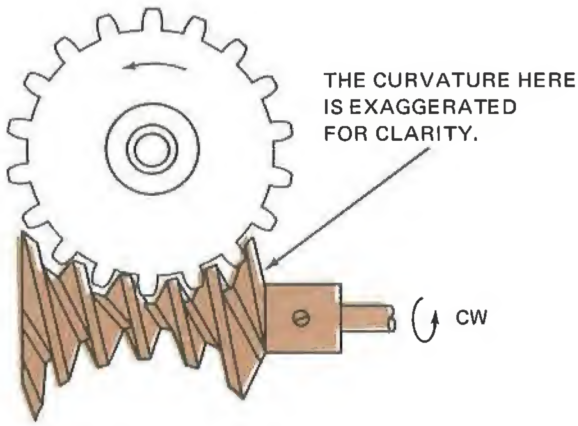


Fig. 15-3 A Contoured Worm

When a worm with a single helical thread is meshed with a gear wheel, the wheel advances one tooth for each revolution of the worm. Therefore, the gear ratio of a single thread worm-and-wheel is equal to the number of teeth on the gear.

$$N_g = \frac{\omega_w}{\omega_g} = \frac{\theta_w}{\theta_g}$$

Since the losses in a worm-and-wheel are higher than for many other mechanisms, it is wise to include the efficiency in any torque calculations. Efficiency is defined as the ratio of the output to input work. That is

$$\eta_o = 100 \frac{W_o}{W_i} \text{ Percent}$$

And since the output work is

$$W_o = T_g \theta_g$$

while the input work is

$$W_i = T_w \theta_w$$

we have

$$\eta_o = 100 \frac{T_g \theta_g}{T_w \theta_w} \text{ Percent}$$

But as we have already seen,

$$\frac{\theta_w}{\theta_g} = N_g$$

So we may write

$$\eta_o = 100 \frac{T_g}{N_g T_w} \text{ Percent}$$

In many cases it is convenient to express the efficiency as a fraction rather than as a percentage. This being the case, we have

$$\eta = \frac{T_g}{N_g T_w}$$

or

$$N_g = \frac{1}{\eta} \frac{T_g}{T_w}$$

Consequently, the worm-and-wheel gear ratio relationships may be summarized by

$$N_g = \frac{\omega_w}{\omega_g} = \frac{\theta_w}{\theta_g} = \frac{1}{\eta} \frac{T_g}{T_w} \qquad \textbf{(15.1)}$$

Worm screws can be, and are, frequently made with two, three, or four separate threads, side by side. When a multiple thread worm is meshed with a gear wheel, the wheel advances one tooth for each thread every time the worm turns a complete revolution. Consequently, the ratios for a multiple thread worm-and-wheel are

$$\boxed{\frac{N_g}{K} = \frac{\omega_w}{\omega_g} = \frac{\theta_w}{\theta_g} = \frac{1}{\eta} \frac{T_g}{T_w}} \qquad \textbf{(15.2)}$$

where K is the number of threads on the worm and is often referred to as the "lead" of the screw (i.e. a triple lead screw has a K of 3).

Gear ratios of worm-and-wheel assemblies range from about 10 to over 120 in a single mesh. Because of this wide range, they are used in a very great variety of applications.

It should be observed that ratios below about 10 are rarely used because it is impractical to make gear wheels with a small number of teeth.

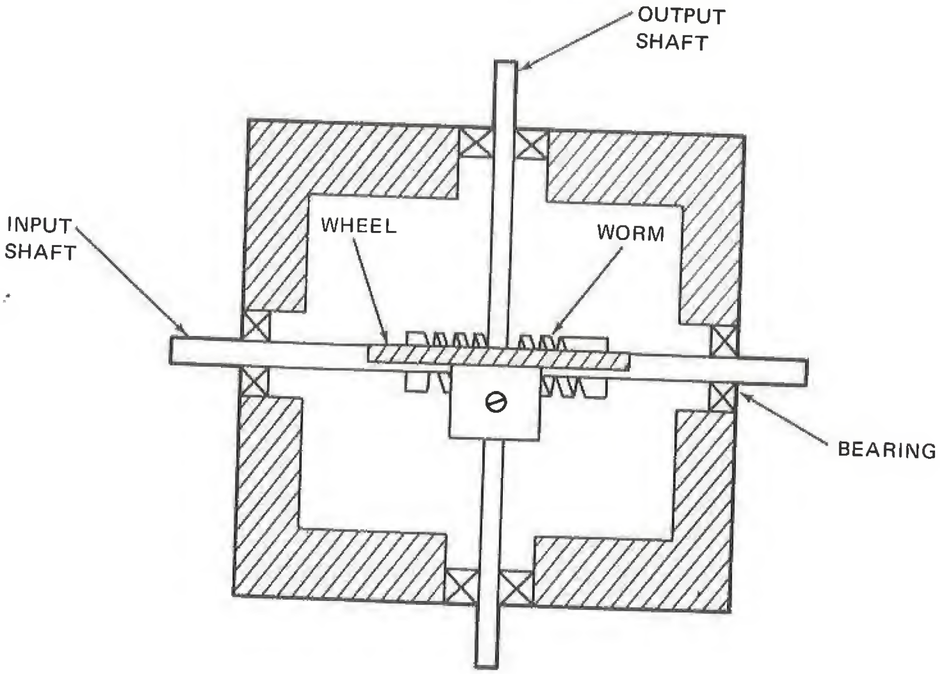


Fig. 15-4 A Worm-and-Wheel Gear Box

Worm-and-wheel assemblies are usually mounted in gear boxes more or less like the one shown in figure 15-4. Retaining rings and mounting hardware are not shown.

One of the interesting features of the worm-and-wheel is the fact that they only work well with the input applied to the worm. If the input is applied to the wheel, the teeth tend to lock against the worm. Assemblies with very low gear ratios can be driven backward but are extremely inefficient.

In precision gearing applications mounting can be a severe problem. The shafts must be exactly 90 degrees from each other to prevent binding of the teeth against the thread. Moreover, the wheel must be accurately centered on the worm because of the curved tooth tips.

Since the worm and wheel rely on sliding contact, end-thrust is created in both the worm and in the wheel. Consequently, thrust bearings are required in worm-and-wheel assemblies.

MATERIALS

- 2 Bearing plates with spacers
- 1 Breadboard with legs and clamps
- 2 Bearing mounts
- 2 Bearings
- 2 Shafts 4" X 1/4"
- 1 Worm screw
- 1 Worm wheel
- 2 Shaft hangers with bearings
- 2 Dials with 1/4 in. bore hubs
- 1 Motor and mount
- 1 Power supply
- 2 Dial indices with mounting hardware
- 1 Stroboscope
- 4 Collars

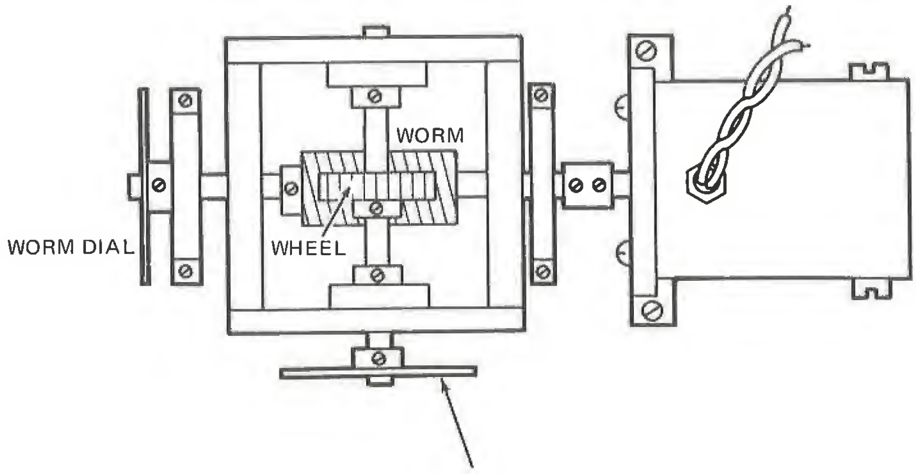


Fig. 15-5 The Experimental Mechanism

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Carefully examine the worm to determine the number of threads. Record the results in the Data Table (K).
3. Count the teeth on the wheel and record the results (N).
4. Construct the mechanism shown in figure 15-5, but do not install the pulley belt.
5. Lubricate the worm, collars, and bearings lightly.
6. Set both dials to zero.
7. Rotate the worm dial and count the number of revolutions required to turn the wheel one complete revolution *clockwise*. Record the results (θ_w/θ_g) using a + sign if the worm was turned clockwise, and a - sign if the worm was turned counterclockwise.
8. Carefully try rotating the wheel dial and watch the action of the teeth. **DO NOT FORCE THE DIAL!**
9. Install the pulley belt.
10. Connect the DC motor to the power supply and turn it on. Set the voltage so that the worm dial rotates slowly.
11. If the dial turns counterclockwise, reverse the motor connections.
12. With the worm dial running clockwise, set the motor voltage to about 24 volts.
13. Using the stroboscope, measure and record the angular velocity of the worm and of the wheel. (ω_w and ω_g)
14. Repeat steps 12 and 13 for motor voltages of about 20, 16, and 12 volts.
15. For each pair of angular velocities, compute and record the ratio (ω_w/ω_g).
16. Compute the average of the four angular velocity ratios. (ω_w/ω_g ave.)
17. Compute the percentage difference between the average angular velocity ratio and the displacement ratio. (% Diff., 17)
18. Using the results of steps 2 and 3, compute and record the ratio N/K.
19. Compute and record the percentage difference between the results of step 18 and step 16. (% Diff., 19)

K	N	θ_w/θ_g	ω_w/ω_g (Ave.)	% Diff. (Step 17)	N/K	% Diff. (Step 19)

ω_w	ω_g	ω_w/ω_g

Fig. 15-6 The Data Tables

ANALYSIS GUIDE. In analyzing the results of this experiment, you should consider the extent to which your values of θ_w/θ_g and ω_w/ω_g agreed with the worm-and-wheel operation described in the discussion.

Explain what you observed in step 8 and tell when this feature would be an advantage and when it would be a disadvantage.

PROBLEMS

1. Worm screws may be either right- or lefthanded. Make a sketch showing the direction of wheel rotation if a righthanded worm is turned clockwise and counterclockwise.
2. Repeat problem 1 using a lefthanded worm.
3. Make a sketch showing the direction of end-thrust for the worms in problems 1 and 2.
4. Repeat problem 3 for the worm wheel.
5. A 100-tooth worm wheel is to be used in a speed reducer. What is the gear ratio if the worm has:
   (a) A single thread (c) A triple thread
   (b) A double thread (d) A quadruple thread
6. A double-threaded worm drives a 120-tooth wheel, and the mechanism has an efficiency of 70 percent. If the output torque is 250 in.-oz., what is the input torque?
7. The worm in problem 6 is turning 1750 RPM. What is the output power?
8. What is the input power in problem 7?
9. List at least three practical applications of a worm-and-wheel mechanism.
10. How could a worm-and-wheel be made to operate between shafts that were 75 degrees apart?

experiment 16 BLOCK AND SCREW

INTRODUCTION. The block-and-screw is widely used for converting rotary motion into linear motion. In this experiment we shall examine some of the important operating characteristics of this type of device.

DISCUSSION. If the screw in figure 16-1 is rotated, the block will move laterally. The screw is normally mounted in bearings at each end. The block is prevented from turning by a fixed guide shaft. As a result the block may *only* move laterally.

If the screw has only a single thread, then the block moves a distance equal to the pitch of the thread each time the screw turns one complete revolution: that is,

$$S_b = P_s \text{ in. per rev.} \tag{16.1a}$$

where S_b is the distance moved by the block and P_s is the pitch of the thread. Consequently, if the screw turns through an angle θ_s, then the block will move a distance

$$S_b = \frac{\theta_s P_s}{2\pi} \text{ in.} \tag{16.2a}$$

where θ_s is in radians, or

$$S_b = \frac{\theta_s P_s}{360} \text{ in.} \tag{16.3a}$$

if θ_s is in degrees.

Similarly if the input shaft rotates at ω_s revolutions-per-minute, the block will have a linear velocity of

$$v_b = \omega_s P_s \text{ in. per minute}$$

or

$$v_b = \frac{1}{60}\,\omega_s P_s \text{ in. per second} \tag{16.4a}$$

Screws of this type are called *single thread* or *single lead.* Screws with double, triple, and quadruple threads are also used. The effect of multiple threads on the equations given above is simply to multiply each

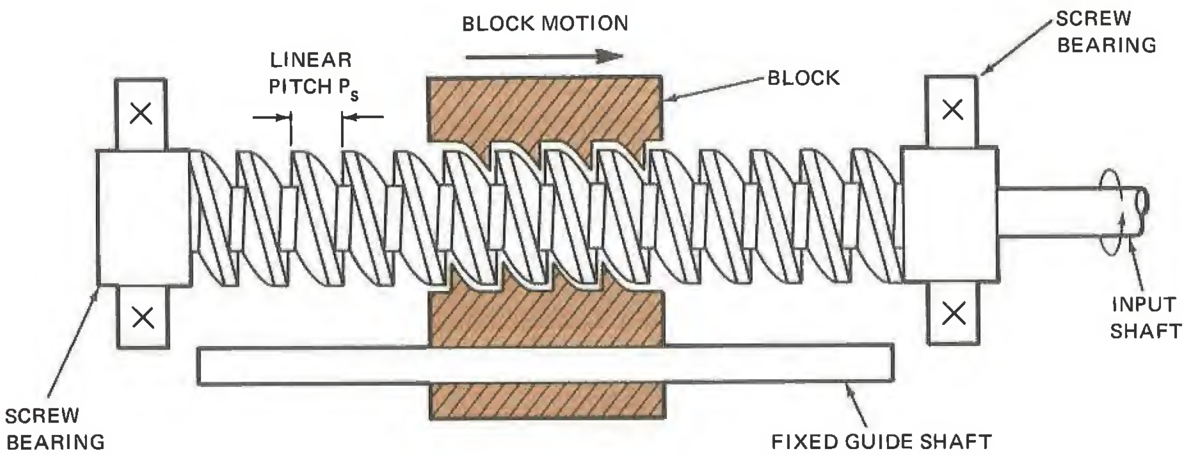


Fig. 16-1 A Block-and-Screw

by the number of threads. That is

$$S_b = kP_s \text{ in. per rev.} \qquad \textbf{(16.1b)}$$

$$S_b = k\frac{\theta_s P_s}{2\pi} \text{ in. } (\theta_s \text{ in radians}) \qquad \textbf{(16.2b)}$$

$$S_b = k\frac{\theta_s P_s}{360} \text{ in. } (\theta_s \text{ in degrees}) \qquad \textbf{(16.3b)}$$

$$v_b = k\omega_s P_s \text{ in. per min.}$$

$$v_b = \frac{1}{60} k\omega_s P_s \text{ in. per sec.} \qquad \textbf{(16.4b)}$$

where k is the number of threads on the screw and is often referred to as the "lead" of the screw. For instance, a screw with four separate threads is called a quadruple lead screw.

Because of its very large contact area the block-and-screw is quite useful for handling large loads. In many cases the block is fixed and the screw has both rotary motion and translation or linear motion. Automobile jacks, C-clamps and vises are examples of this type of application.

In any case, the threads inside the block and those on the screw are in sliding contact just like a block on an inclined plane. Because of this sliding contact, the friction between the surfaces can be very great. Moreover, with a moving block assembly there is additional friction between the block and its guides. In the case of a fixed block assembly, there is additional friction at the screw pivot point and against any end collar. The overall result is that the block-and-screw tends to be very inefficient. Typical efficiency values range all the way from around 1% for a large load assembly like an automobile jack to over 95% for a precision, light load instrument.

Inefficiency is not always undesirable. If an automobile jack were too efficient, it would unwind itself as soon as you released the handle.

In any case the efficiency of a machine can be defined in terms of the input and output work as

$$\boxed{\eta_o = 100\frac{W_o}{W_i} \text{ percent}} \qquad \textbf{(16.5)}$$

Referring to the block-and-screw in figure 16-2, the work done by the block in moving the load f_L a distance S_b is

$$W_o = f_L S_b$$

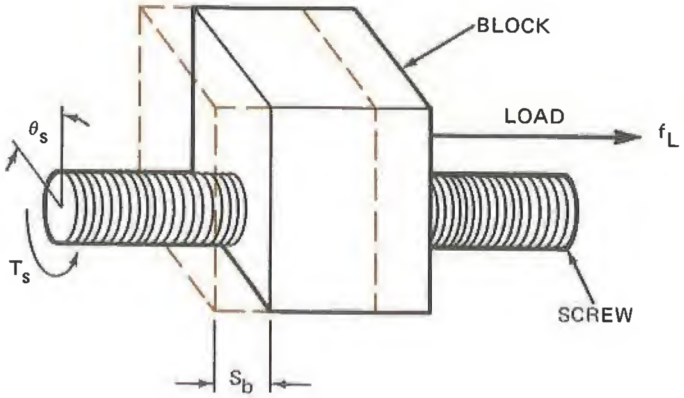


Fig. 16-2 A Loaded Block-and-Screw

and the work done at the input to rotate the screw is

$$W_i = T_s\theta_s$$

Substituting these quantities into the relationship for efficiency gives us

$$\eta_o = 100\frac{f_L S_b}{T_s\theta_s}\text{ percent}$$

It is often convenient to express efficiency as a fraction rather than as a percentage. When this is the case, we have simply

$$\eta = \frac{f_L S_b}{T_s\theta_s}$$

Also, since the block distance is related to screw pitch and the number of threads (equation 16.2b), we can write the efficiency fraction as

$$\eta = \frac{f_L k \dfrac{\theta_s P_s}{2\pi}}{T_s\theta_s}$$

or

$$\boxed{\eta = \frac{f_L k P_s}{2\pi T_s}} \qquad (16.6)$$

Therefore, the torque required by the screw to move a load f_L is

$$T_s = \frac{f_L k P_s}{2\pi\eta}$$

Generally speaking, block-and-screw assemblies are not intended to operate in the reverse direction. Indeed, most light duty precision assemblies can be seriously damaged by attempting to force the block.

MATERIALS

- 1 Breadboard with legs
- 2 Shaft hangers with bearings
- 1 Lead screw with block and guide
- 1 Dial with 1/4 in. bore hub
- 1 Dial index with mounting hardware
- 1 Dial caliper

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Carefully examine the screw to determine how many threads it has. Record the number in the Data Table. (k)
3. Count the total number of thread tips from one end of the screw to the other. Record the number in the Data Table. (N_s)
4. Measure the total length of the threaded portion of the screw. (ℓ)
5. Using the results of steps 3 and 4, compute the linear pitch (P_s) of the screw and record it in the Data Table.

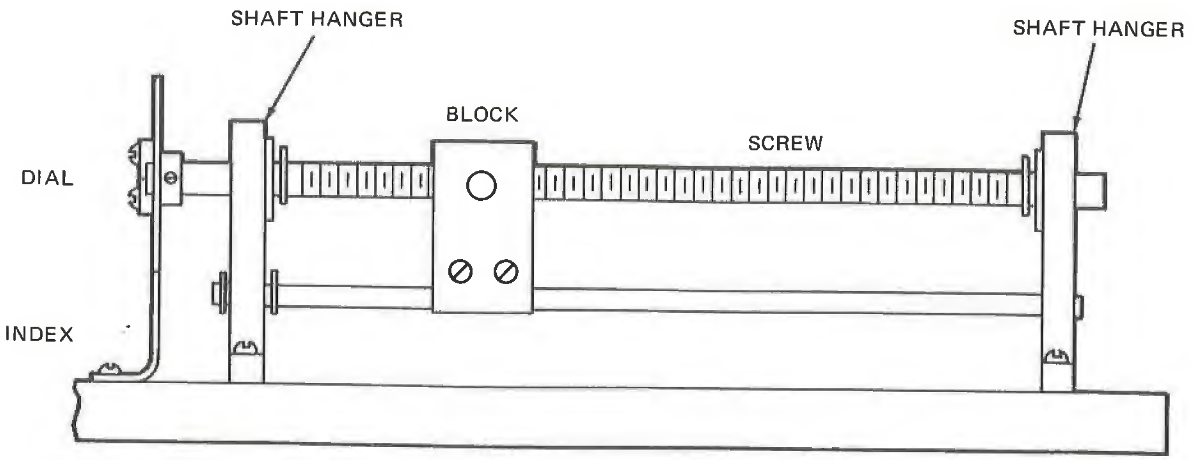


Fig. 16-3 The Experimental Mechanism

6. Construct the mechanism shown in figure 16-3.
7. Rotate the screw until the block rests at one end of its travel. Mark this position as a reference point.
8. Adjust the screw dial to read zero.
9. Rotate the screw five complete turns, then measure and record the distance that the block has moved from the reference point established in step 7 (S_b). Be very sure that the block does not rotate but moves only laterally.
10. Rotate the screw another five complete turns and again measure the distance that the block has moved from the reference point.
11. Continue in this manner five revolutions at a time until the block reaches the other end of the screw.
12. With *each* set of data, compute the block travel per revolution.
13. With the results of step 12, compute the *average* block travel per revolution.
14. Using equation 16.1b and the results of steps 2 and 5, compute the block travel per revolution.
15. Compute the percent difference between the results of steps 13 and 14.

ANALYSIS GUIDE. In analyzing your data from this experiment you should consider how well your measurements agreed with the calculated value. Discuss what causes errors in this type of experiment and how they could be reduced. What applications can you think of for the block-and-screw?

k	N_s	ℓ	P_s	S_b (Ave.)	S_b (Comp.)	% Diff.

Screw Turns	S_b	S_b Per Rev.

Fig. 16-4 The Data Tables

PROBLEMS

1. A double thread screw has a pitch of 0.1 inches. How far will the block move if the screw turns 21 times?
2. If the screw in problem 1 turns 150 RPM, how fast does the block move?
3. If the angular velocity in problem 2 were 150 radians per second, what would the result be?
4. The block-and-screw in problem 2 has an efficiency of 37% and the load on the block is 230 lbs. What is the input torque?
5. What horse power must be supplied in problem 4?

6. An automobile jack uses a single thread block-and-screw that is 21% efficient. What input torque must be supplied to lift a 1400-lb. load to a height of 8 in. if the jack handle rotates 41 times in doing the job?

7. The jack handle in problem 6 has an effective moment arm of 10 inches. What force must the operator exert on the handle to raise the load?

8. A small car jack is borrowed to change a tire on a large sedan. The jack uses a block-and-screw and has no locking device. When the job is attempted, it is noticed that the jack unwinds very slowly by itself when the handle is released. What is the problem?

9. How would you prevent unwinding in problem 8 without damaging the borrowed jack? Give several alternatives.

10. The jack in problem 8 is a single thread device and has a linear pitch of 1/8 inch. If the load in problem 8 is 1700 lbs. and a torque of 4 lb.-ft. is required to prevent unwinding, what is the efficiency of the jack in the unwinding direction?

experiment 17 COMBINED MECHANISMS

INTRODUCTION. Many practical mechanisms contain more than one type of component. In this experiment we shall examine some of the analysis techniques that are appropriate for use with such assemblies.

DISCUSSION. Spur gears, helical gears, bevel gears, worms, racks, blocks, and screws may be combined in many ways to form single mechanisms. Let's consider a few of the possible combinations.

Consider the mechanism shown in figure 17-1. In this figure we have three basic mechanisms: a simple gear train (1 and 2), a compound gear train (2, 3, and 4), and a rack-and-pinion (4 and 5).

In analyzing a combined mechanism we are usually interested in the output displacement or velocity that results from a given input. We are also normally interested in the input torque required to drive a given load.

To find the output displacement or velocity, we start at the input and assume a value of displacement θ. Then trace the resulting displacement through the mechanism to the output.

Let's try this process on the mechanism in figure 17-1.

Assuming an input displacement of θ we see that it is transmitted to gear number 2 through a simple mesh. Therefore,

$$\theta_2 = -\theta_1 \frac{N_1}{N_2}$$

Since gears number 2 and 3 are compound they have the same displacement.

$$\theta_3 = \theta_2 = -\theta \frac{N_1}{N_2}$$

Gears 3 and 4 are a simple mesh. So

$$\theta_4 = -\theta_3 \frac{N_3}{N_4}$$

Substituting the value of θ_3 from before, we have

$$\theta_4 = \theta_1 \frac{N_1 N_3}{N_2 N_4}$$

Then gear 4 drives the rack, giving it a displacement of

$$S_r = 1/2\ D_4 \theta_4$$

where D_4 is the pitch diameter of gear number 4. Substituting the relationship for θ_4 into this equation gives us

$$S_r = 1/2\ D_4 \theta_1 \frac{N_1 N_3}{N_2 N_4}$$

While this process seems rather involved, it is actually quite simple when the gear parameters are known. For example, if the mechanism in figure 17-1 is composed of 32-pitch

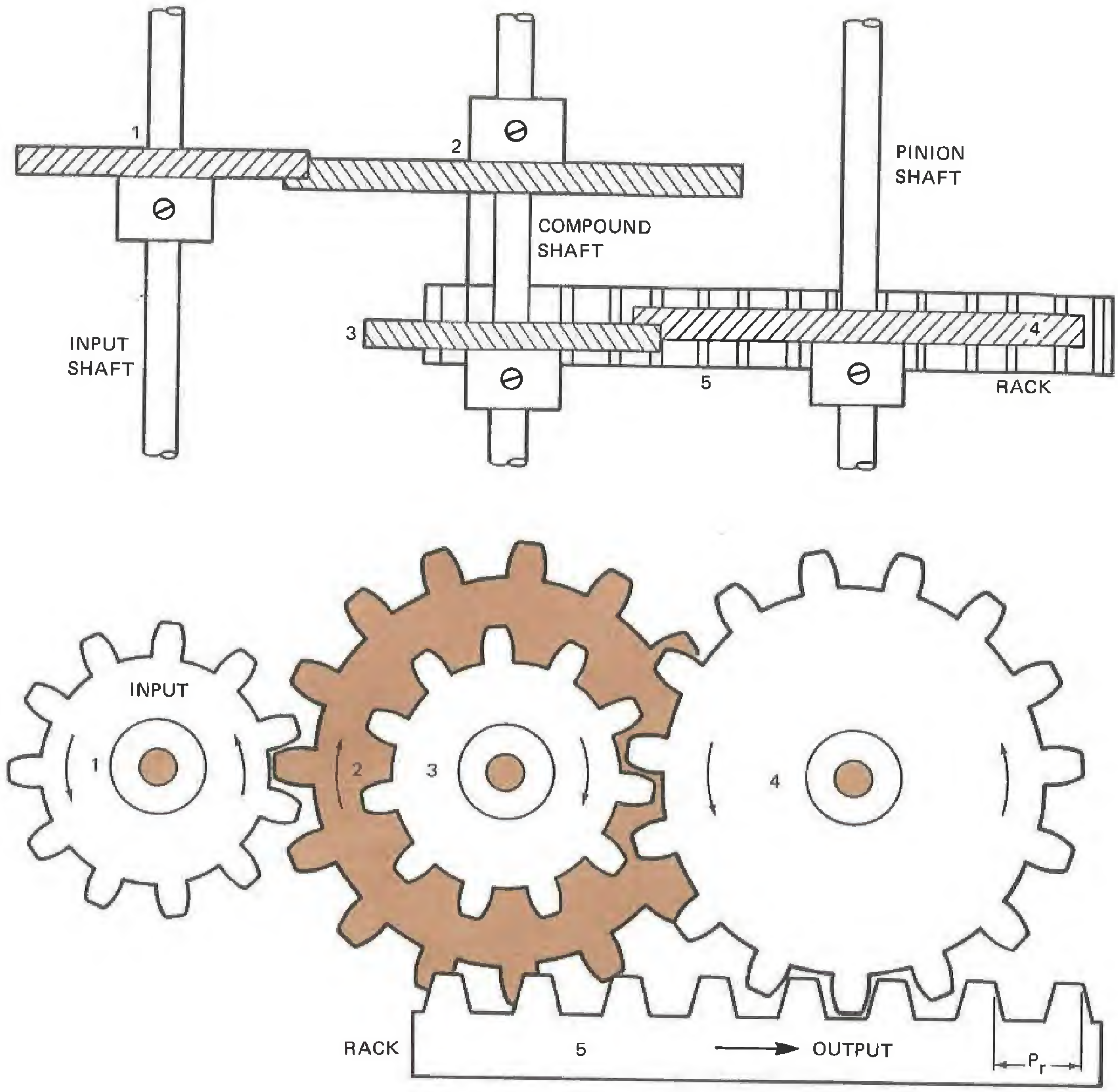


Fig. 17-1 Combined Mechanism

gears and the tooth counts are:

N_1 = 24 Teeth

N_2 = 48 Teeth

N_3 = 30 Teeth

N_4 = 60 Teeth

we proceed as follows: Assume $\theta_1 = 1.0$ radians. θ_2 is then

$$\theta_2 = -1 \frac{24}{48} = -0.5 \text{ radians}$$

and

$$\theta_3 = \theta_2 = -0.5 \text{ radians}$$

while

$$\theta_4 = -\theta_3 \frac{N_3}{N_4} = 0.5 \frac{30}{60} = 0.25 \text{ radians}$$

At this point we must calculate the pitch diameter of gear number 4.

$$D_4 = \frac{N_4}{P_d} = \frac{60}{32} = 1.875 \text{ in.}$$

We can now calculate the rack displacement.

$$S_r = 1/2\, D_4\theta_4 = 0.5 \times 1.875$$

$$\times\, 0.25 = 0.234 \text{ in.}$$

which means, of course, that the rack moves 0.234 inches per radian of input rotation.

If we know the input angular velocity in radians per second, then the rack velocity in this case is

$$V_r = 0.234 \text{ in. per sec.}$$

If we know the input velocity in revolutions per minute, the rack velocity is

$$V_r = 2\pi \frac{\text{rad.}}{\text{rev}} \times \omega \frac{\text{rev}}{\text{min}} \times \frac{1}{60}\frac{\text{min}}{\text{sec}} \times 0.234 \frac{\text{in.}}{\text{rad}}$$

$$V_r = \frac{2\pi \times \omega \times 0.234}{60} \text{ in.} \approx 0.0245 \text{ in. per sec.}$$

This *method* of displacement and velocity analysis will work for any mechanism, however complicated.

As mentioned before, we are normally also interested in the torque required at the input by a given output force.

Let's suppose that the rack in figure 17-1 is delivering a force of f_r to a load. This being the case, then the force supplied by gear number 4 at its pitch circle is also f_r and the torque delivered by gear number 4 is

$$T_4 = 1/2\, D_4 f_r$$

Gear number 4 is meshed with gear number 3; therefore, the torque supplied by gear number 3 is

$$T_3 = -T_4 \frac{N_3}{N_4} = -1/2\, D_4 f_r \frac{N_3}{N_4}$$

Gear number 2 is coupled rigidly to gear number 3 so their torques are equal.

$$T_2 = T_3 = -1/2\, D_4 f_r \frac{N_3}{N_4}$$

Then since gear number 2 is meshed with gear number 1, the torque supplied by number 1 is

$$T_1 = -T_2 \frac{N_1}{N_2} = 1/2\, D_4 f_r \frac{N_1 N_3}{N_2 N_4} \text{ in.-oz.}$$

This is, of course, the torque that must be supplied to gear number 1 by the input shaft.

Again this process seems involved until we consider a practical case. Suppose the force delivered by the rack in figure 17-1 is 100 ounces.

In such a case the torque delivered by gear number 4 is

$$T_4 = 1/2\, D_4 f_r$$

Using the gear parameters specified before, this is

$$T = 0.5 \times 1.875 \times 100 = 93.8 \text{ in.}$$

and

$$T_3 = -T_4 \frac{N_3}{N_4} = -93.8 \times \frac{30}{60} = -46.9 \text{ in.-oz.}$$

Then since $T_2 = T_3$, we have

$$T_2 = -46.9 \text{ in.-oz.}$$

And

$$T_1 = -T_2 \frac{N_1}{N_2} = 46.9 \frac{24}{48} = 23.4 \text{ in.-oz.}$$

which is the torque supplied by the input to gear number 1.

It is worth noting that the output displacement was

$$S_r = 0.234 \text{ in. } \textit{per input radian}$$

while the input torque was

$$T_1 = 0.234 \text{ in.-oz. } \textit{per input oz.}$$

The factor, 0.234, which in this instance is given by

$$0.234 = 1/2\, D_4 \frac{N_1 N_3}{N_2 N_4}$$

is called the **Transfer Function** of the mechanism. For lossless mechanisms of this type the relationships between displacements and torques are:

Output displacement

= (Transfer function) (input displacement)

and

Input Torque

= (Transfer function) (Output force)

When a mechanism involves sliding contacts, the efficiency of the mechanism must be included in the calculations.

The general patterns of analysis used in this discussion are usually appropriate. That is, displacement and velocity are normally traced from input to output while the torque is normally traced from output to input.

In testing mechanical performance there are four basic quantities that are frequently measured. They are:

1. Distances
2. Displacement
3. Velocities
4. Forces

Distances are usually measured with a rule or caliper.

Displacements are measured with a rule or caliper if they are linear, and with a radial dial and index or with a protractor if they are angular.

Velocities are most often angular and are measured with a tachometer. The electronic stroboscope is one of the most popular types of tachometers.

Forces may usually be measured with a spring balance or other "scale". Torque is defined as a force acting at the end of a lever arm. *Static torque* is measured when the mechanism is at rest. *Running torque is* measured with the mechanism in motion.

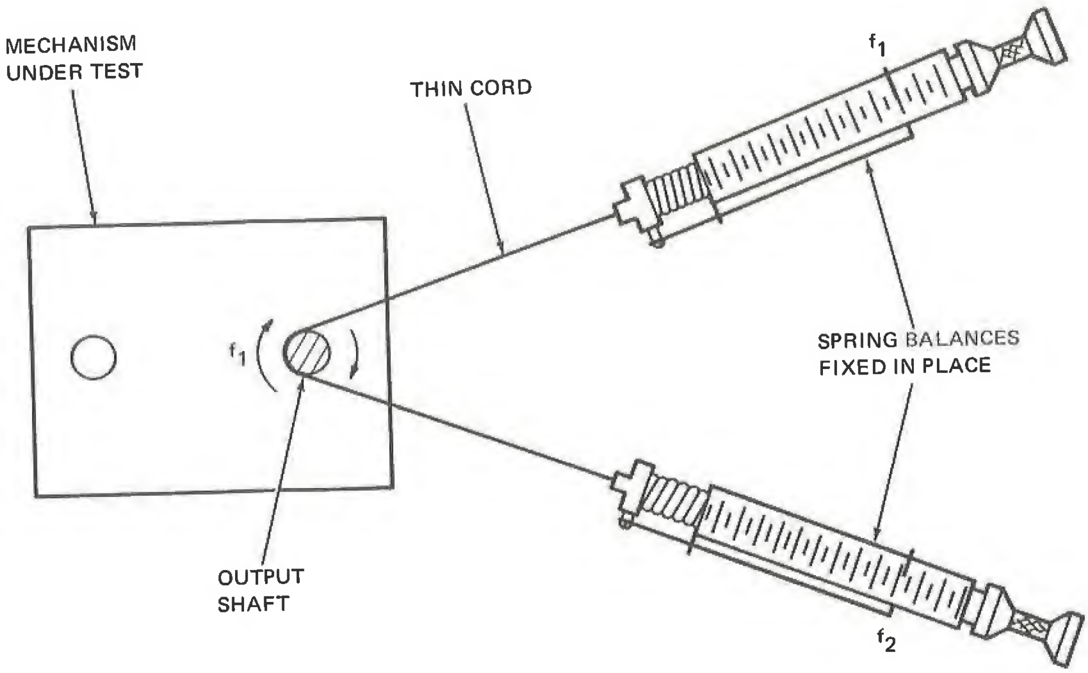


Fig. 17-2 A Prony Brake Load

We frequently wish to apply a known load to a mechanism that is in motion. One of the ways that this can be done is illustrated in figure 17-2.

Such an arrangement is called a *prony brake load,* and the force being applied to the output shaft is equal to the difference between the two spring balance readings *when the output shaft is turning.* That is

$$f_L = f_2 - f_1$$

and the torque delivered by the output shaft is

$$T = f_L\, r$$

where r is shaft radius. This relationship is only valid if the cord used on the prony brake is quite small compared to the output shaft diameter.

MATERIALS

1 Dial caliper

2 Bearing plates with spacers

1 Breadboard with legs and clamps

3 Shafts 4 X 1/4

6 Bearings

6 Collars

2 Shaft hangers with bearings

4 Spur gears, diameters approx. 3/4, 1, 1 1/2, 2 in.

1 Worm

1 Worm wheel

1 Lead screw assembly

1 Dial with 1/4 in. bore hub

1 Lead screw guide

1 Dial index with mounting hardware

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Examine *each* component and record pertinent parameters such as OD, tooth count, length, etc.
3. Construct the bearing plate assembly shown in figure 17-3.
4. Install the bearing plate assembly on the spring balance stand as shown in figure 17-4.

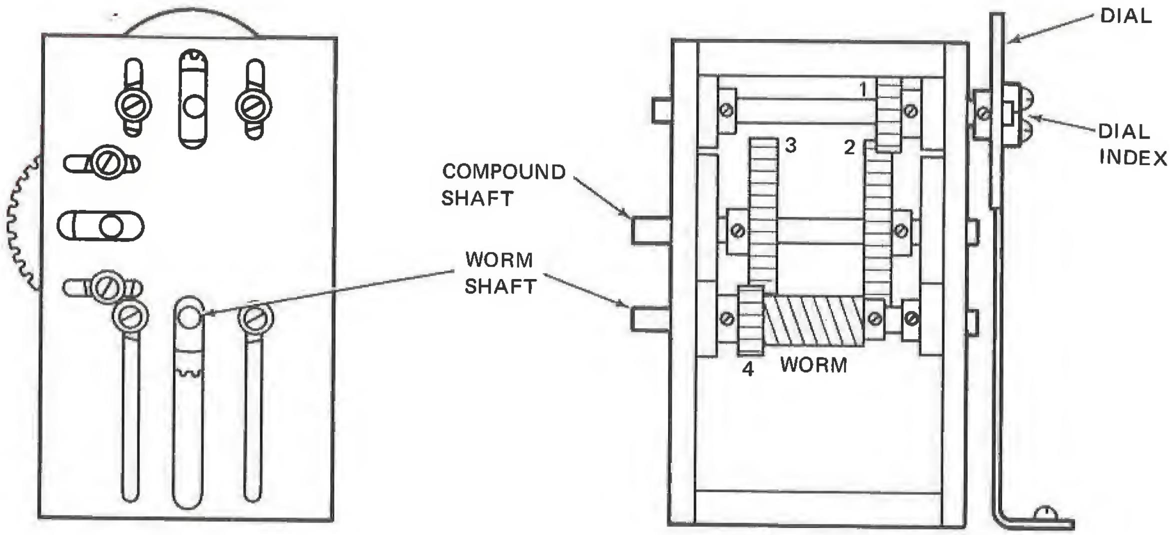


Fig. 17-3 The Bearing Plate Assembly

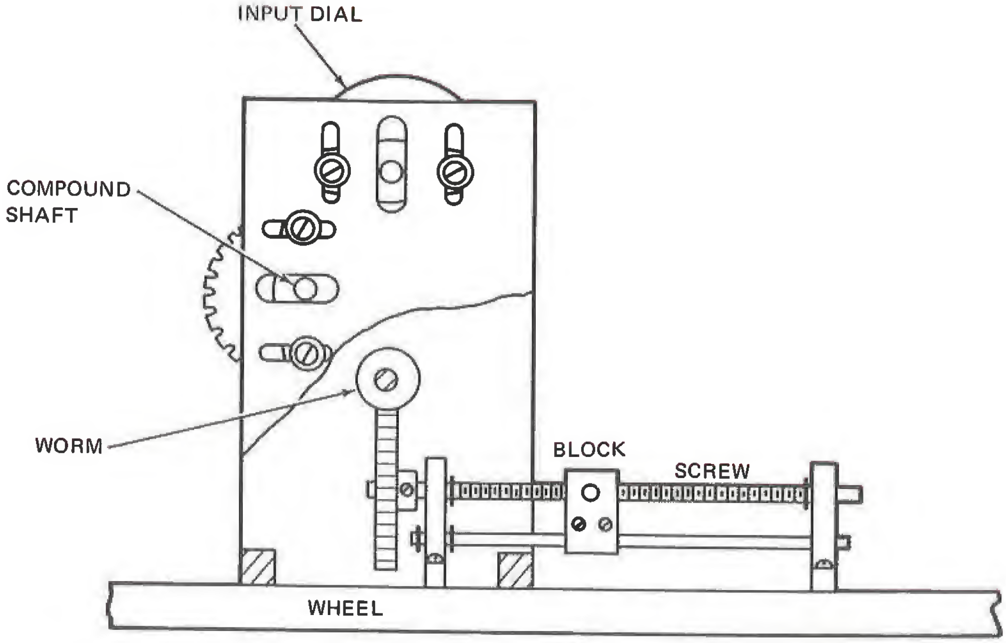


Fig. 17-4 The Experimental Mechanism

5. Make any necessary adjustments to insure that the components are properly aligned and meshed.
6. Position the block at some convenient reference point and adjust the input dial to read zero.
7. Rotate the input dial 360 degrees clockwise, being very sure that the block moves laterally only.

Gear No. 1	Gear No. 2	Gear No. 3	Gear No. 4	Worm	Wheel	Screw

θ_1	S_b	(Comp.) S_b/rev	(Ave.) S_b/rev	% Diff.

Fig. 17-5 The Data Tables

8. Measure the distance that the block has moved from the reference point. Record the results.

9. Repeat steps 7 and 8 alternately, recording the total dial displacement and the total block displacement.

10. Using the method described in the discussion, calculate the block displacement per revolution.

11. Using your measured values, compute the *average* block displacement per revolution.

12. Compute the percent difference between the results of steps 10 and 11.

ANALYSIS GUIDE. The purpose of this experiment has been to examine an analysis technique and to verify the results experimentally. Therefore, in your analysis you should concentrate on the effectiveness with which the objective was achieved.

You should also discuss any problems you had in constructing the mechanism and suggest ways that they could have been avoided.

PROBLEMS

1. If the angular velocity of the input gear in the experimental mechanism was 850 radians per second, what would be the velocity of the block?
2. If the input gear in the experimental mechanism turns 1130 RPM, what would be the linear velocity of the block?
3. How long would it take the block in problem 1 to travel the length of the screw?
4. In problem 2, what was the velocity of the worm?
5. If the block in the experimental mechanism were driving a 300-ounce load, what would be the input torque?
6. Using the velocity from problem 2 and the load from problem 5, determine the input horsepower.
7. If the torque input to the experimental mechanism is 70 in.-oz. what would be the block force?

COUNTER ROTATORS

INTRODUCTION. In many practical mechanical applications it is necessary to have two *in-line* output shafts which rotate in opposite directions. In this experiment we will examine the operating characteristics of a mechanism which provides this feature.

DISCUSSION. When we mesh a pair of bevel gears, they rotate in opposite directions as shown in figure 18-1. That is, if the input shaft **appears to turn clockwise** when **viewed from the hub side** then the output gear **appears to turn counterclockwise when viewed from the hub side.**

But if we look at the output shaft from the left in figure 18-1, it *appears* to turn clockwise. We can use this bevel gear arrangement to provide two outputs which rotate in the *same* direction.

Because the two output *appear* to rotate in opposite directions when viewed individually from the shaft ends, it is easy to become confused.' This is particularly true of situations in which both shaft ends cannot be viewed simultaneously.

The gear ratios for such a bevel gear mesh are

$$\boxed{\frac{n}{N} = -\frac{\theta_g}{\theta_p} = -\frac{\omega_g}{\omega_p} = -\frac{T_p}{T_g} = \frac{d}{D}} \qquad (18.1)$$

We can use this same type of gearing to produce counter-rotating output shafts as shown in figure 18-2.

If the input shaft rotates in the clockwise direction as viewed from the shaft end, then the two output shafts turn in opposite directions. However, if we view each output shaft individually from their shaft ends, *they both appear to rotate counterclockwise.*

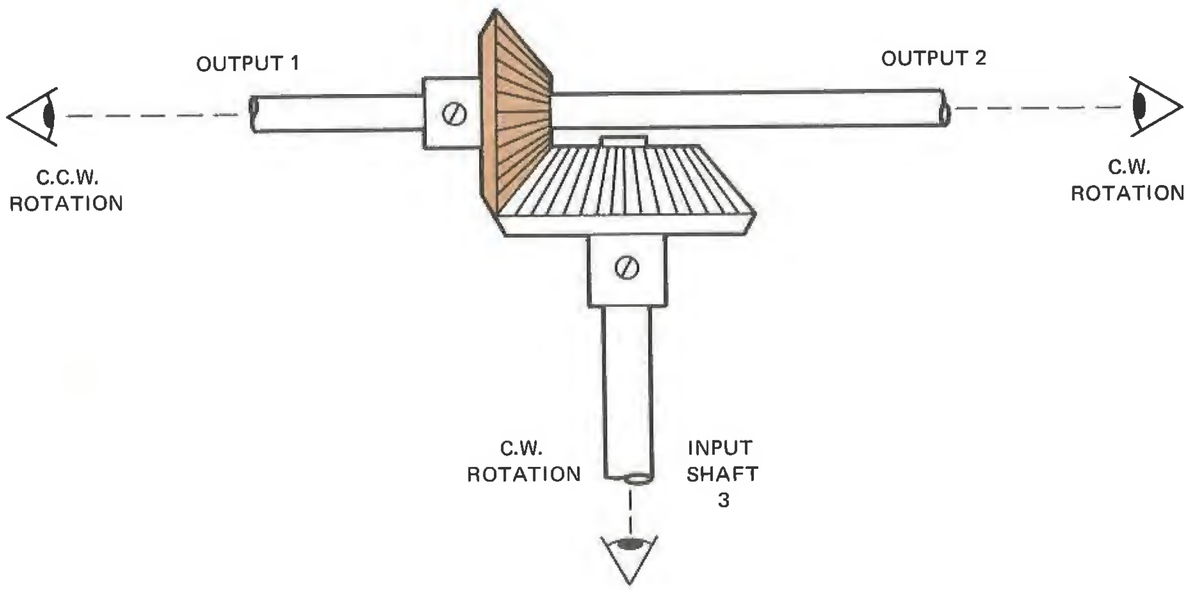


Fig. 18-1 Meshed Bevel Gears

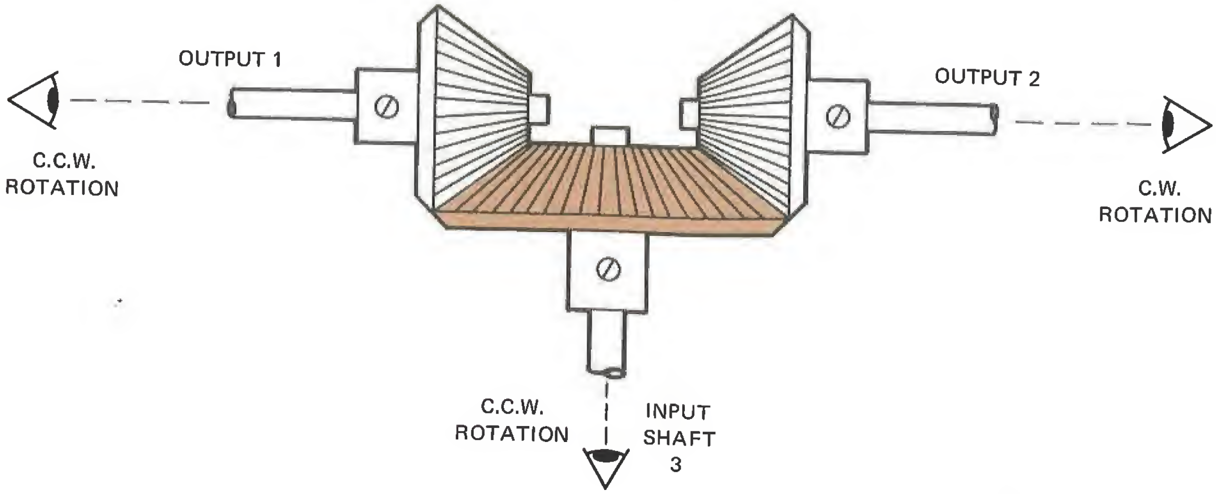


Fig. 18-2 A Counter Rotation Mechanism

The gear ratios for each mesh are similar to equation 18.1; that is

$$\boxed{\frac{N_1}{N_3} = -\frac{\theta_3}{\theta_1} = -\frac{\omega_3}{\omega_1} = \frac{D_1}{D_3}} \qquad \textbf{(18.2a)}$$

and

$$\frac{N_2}{N_3} = -\frac{\theta_3}{\theta_2} = -\frac{\omega_3}{\omega_2} = \frac{D_2}{D_3} \qquad \textbf{(18.2b)}$$

where N_1 and N_2 are output gear tooth counts, N_3 is the input gear tooth count. Subscripts 1, 2, and 3 refer to output 1, output 2, and the input, respectively.

We can also calculate the gear ratios from one of the outputs to the other by observing that

$$\boxed{\frac{N_1}{N_3} = -\frac{\theta_3}{\theta_1} \text{ and } \frac{N_2}{N_3} = -\frac{\theta_3}{\theta_2}}$$

Solving both equations for N_3 gives us

$$N_3 = -N_1\frac{\theta_1}{\theta_3} \text{ and } N_3 = -N_2\frac{\theta_2}{\theta_3}$$

Then, equating the two quantities and canceling common terms renders

$$N_1\theta_1 = N_2\theta_2$$

or

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}$$

Similarly, the velocity and diameter ratios may be determined and the overall result is

$$\boxed{\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{D_1}{D_2}} \qquad \textbf{(18.3)}$$

Notice that the ratios *do not* indicate counter-rotation. These ratios are the ones that would be observed from the shaft ends.

Equation 18.2a and 18.2b apply to each individual mesh between the input gear and an output gear. On the other hand, equation 18.3 applies from one output to the other. Notice that these equations *do not include torque relationships.*

Let's suppose that we are applying an input torque T_3 to the input shaft and supplying two output torques T_1 and T_2 respectively.

The force (f_1) applied to the pitch circle of output gear produces an output torque of

$$T_1 = \frac{1}{2} f_1 D_1$$

Consequently, f_1 must be

$$f_1 = 2\frac{T_1}{D_1}$$

Similarly the force applied to output gear 2 is

$$f_2 = 2\frac{T_2}{D_2}$$

The input gear must supply *both* of these forces so the input gear pitch circle force (f_3) is

$$f_3 = f_1 + f_2 = 2\left(\frac{T_1}{D_1} + \frac{T_2}{D_2}\right)$$

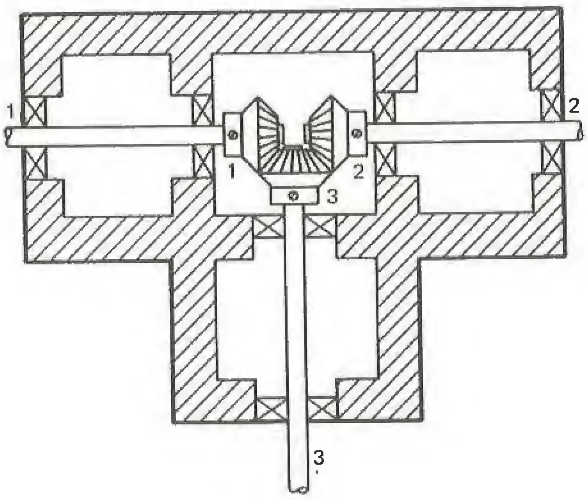


Fig. 18-3 A Counter Rotator and Housing

At the same time, the relationship between input gear pitch circle force and input torque is

$$f_3 = 2\frac{T_3}{D_3}$$

Equating these two expressions for f_1 and canceling the 2, we have

$$\boxed{\frac{T_3}{D_3} = \frac{T_1}{D_1} + \frac{T_2}{D_2}} \qquad \textbf{(18.4)}$$

which is the equation which relates the various torques transmitted by the gears.

We may, of course, apply the input torque to shafts 1 or 2 (marked "outputs" in figure 18.2). In that case we solve for that torque in terms of the other parameters. For example, if we apply the input to shaft number 1, the torque relationship is

$$\frac{T_1}{D_1} = \frac{T_3}{D_3} + \frac{T_2}{D_2}$$

When this is done we have a counter rotator with output at right angles. But notice that when this is done, gear 3 must handle its own torque *and* transmit gear 2's torque.

Mountings for counter rotators tend to be more complicated than some other gear boxes since each shaft must be supported by two bearings. Figure 18-3 shows a typical arrangement. The counter rotators that we have considered have employed only bevel gears. It is possible to make them using other gear types. Gears 1 and 2 could be spur gears and gear 3 would be a crown gear.

MATERIALS

1 Breadboard with leg and clamps
2 Bearing plates with spacers
4 Shaft hangers with bearings
2 Bearing mounts
2 Bearings
3 Bevel gears
3 Dials with 1/4 in. bore hubs
3 Dial indexes with mounting hardware
1 Motor and mount
1 Power supply
1 Stroboscope
2 Collars
1 Shaft 4" X 1/4"
2 Shafts 2" X 1/4"

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Count the number of teeth on each of your gears and record the results.
3. Assemble the mechanism shown in figure 18-4.

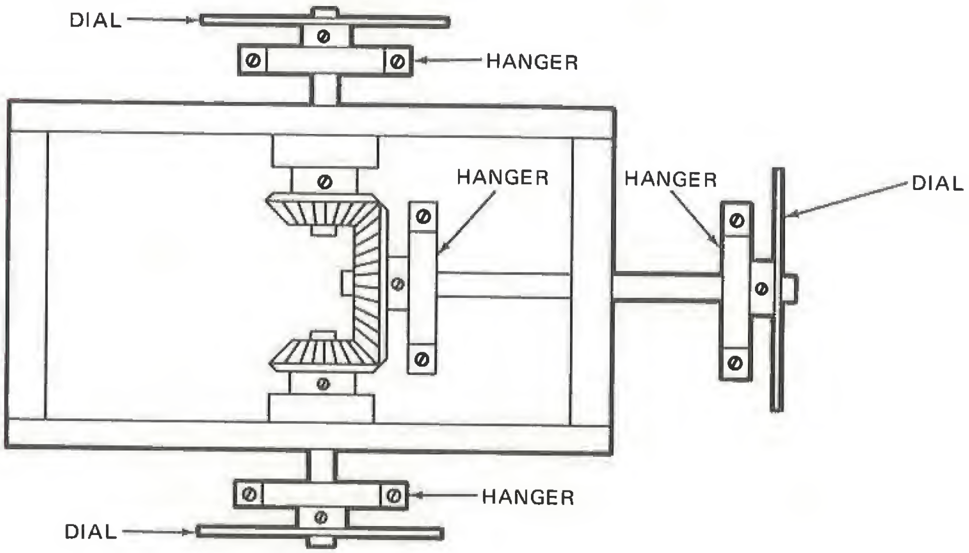


Fig. 18-4 The Experimental Mechanism

4. Adjust each shaft dial to read zero.
5. Rotate the input dial about 30 degrees clockwise. Record the amount and direction (+ for clockwise, - counterclockwise) of rotation of each dial.
6. Continue rotating the input dial in steps of about 30 degrees and recording the rotation direction and amount of all three dials. Continue in this way until all of the dials have turned at least one complete revolution.
7. Couple the DC motor to the input shaft and measure the angular velocity of each shaft for motor voltages of 24, 20, 16, and 12 volts. Record the results each time.
8. For each set of data taken in steps 6 and 7, compute the ratios θ_1/θ_3, θ_2/θ_3, θ_1/θ_3, ω_1/ω_3, ω_2/ω_3, and θ_1/ω_2.
9. Compute and record the *average* value of each ratio specified in step 8.

N_1	N_2	N_3	N_1/N_3	N_2/N_3	N_1/N_2

θ_1	θ_2	θ_3	θ_1/θ_3	θ_2/θ_3	θ_1/θ_2

ω_1	ω_2	ω_3	ω_1/ω_3	ω_2/ω_3	ω_1/ω_2

Fig. 18-5 The Data Tables

RATIO	θ_1/θ_3	θ_2/θ_3	θ_1/θ_2	ω_1/ω_3	ω_2/ω_3	ω_1/ω_2
Ave. Value						
% Diff. (N)						

10. Using the tooth counts, compute the ratios N_1/N_3, N_2/N_3, and N_1/N_2.
11. Compute the percent difference between each tooth ratio and the appropriate average ratios from step 9 (i.e., between N_1/N_3 and θ_1/θ_3, etc.)

ANALYSIS GUIDE. In analyzing the results of this experiment, you should consider two main points. First, did the experimental mechanism actually produce counter-rotating outputs? Second, did the relationships given in the discussion agree with your measured results?

PROBLEMS

1. Discuss the operation of the mechanism shown in figure 18-6.

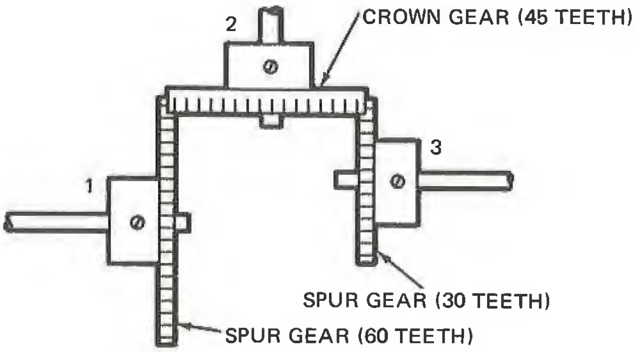


Fig. 18-6 Mechanism for Problem 1

2. If the crown gear in figure 18-6 runs clockwise (viewed from shaft end) at 1825 RPM, how fast will the two spur gears turn?
3. In problem 2, which way will each spur gear turn?
4. The counter rotator in figure 18-2 has loads at output 1 and 2 of 210 in-oz. and 145 in-oz., respectively. The tooth counts are $N_1 = N_2 = 24$ teeth. $N_3 = 60$ teeth and the gears are 32 pitch. What is the torque delivered to gear 3?
5. The counter rotator in figure 18-2 is used in such a way that shaft 2 is the input. The load on shaft 3 is 85 in-oz. while shaft 1 is loaded with 104 in-oz. What is the torque supplied to shaft 2? (Use the pitch and tooth counts from problem 4.)
6. If shaft 1 in problem 5 turns at 3600 RPM, what is the horsepower supplied by the source?
7. Repeat problem 6 for a shaft 3 velocity of 3600 RPM.
8. Discuss problems such as alignment and end thrust that you think might be encountered in mounting bevel gears.

experiment 19 MECHANICAL DIFFERENTIALS

INTRODUCTION. Basically, a mechanical differential is a mechanism which produces an output proportional to the difference between two inputs. In this experiment we shall examine some of the operating characteristics of this type of assembly.

DISCUSSION. Mechanical differentials are usually constructed with bevel or miter gears. Figure 19-1 shows a simplified exploded sketch of differential construction. Notice that a differential is basically a planetary train since it has centers of rotation that are not fixed.

Assembled and mounted, the differential is as shown in figure 19-2.

In operation the two inputs are applied to the input spur gears. The spur gears are *clustered* (permanently compounded) with the input bevel gears. These two clusters are not fixed to the *spider shaft.* They are both free to rotate on the spider shaft. Both input bevel gears are meshed with the *spider bevel gear.* The spider gear is not fixed to the *cross shaft,* but is free to rotate on it. The *junction block* is fixed to both the cross shaft and the spider

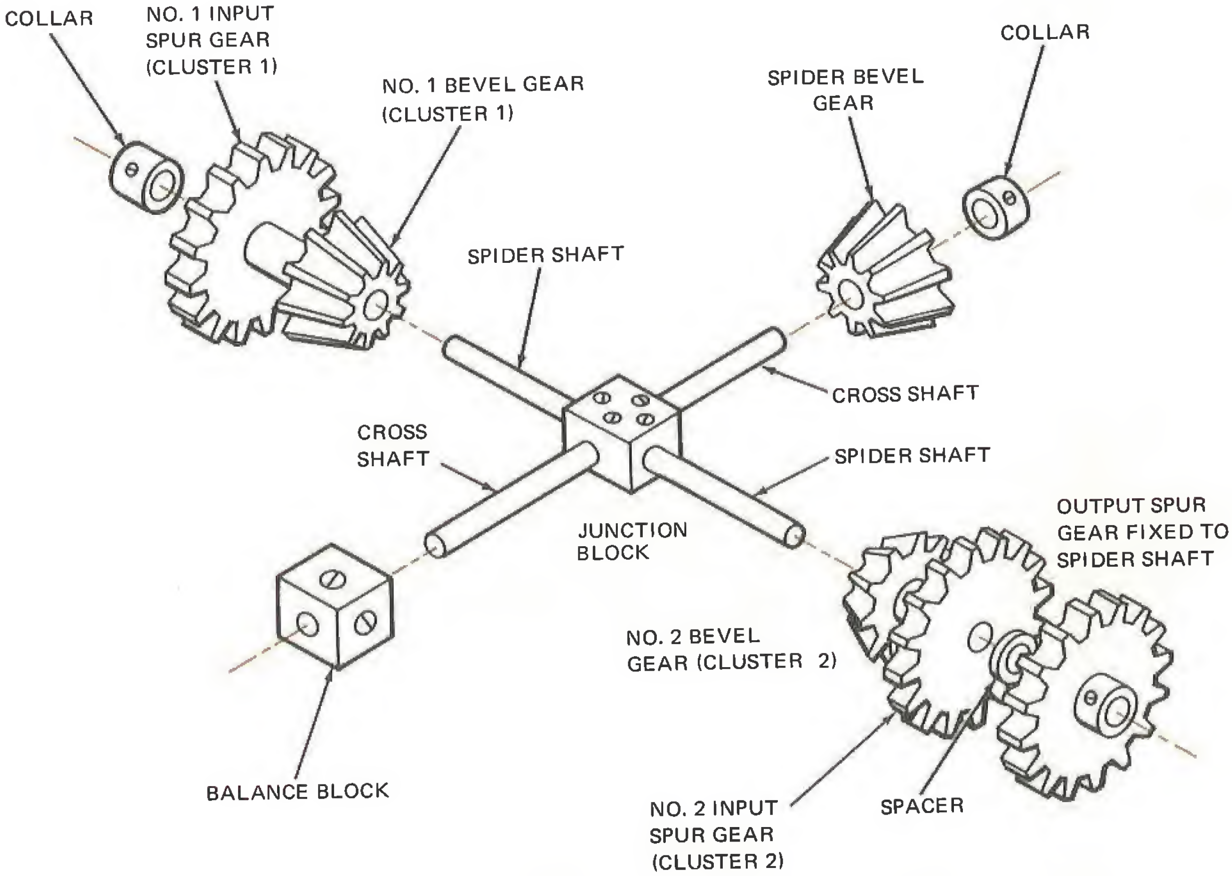


Fig. 19-1 Differential Assembly and Terminology

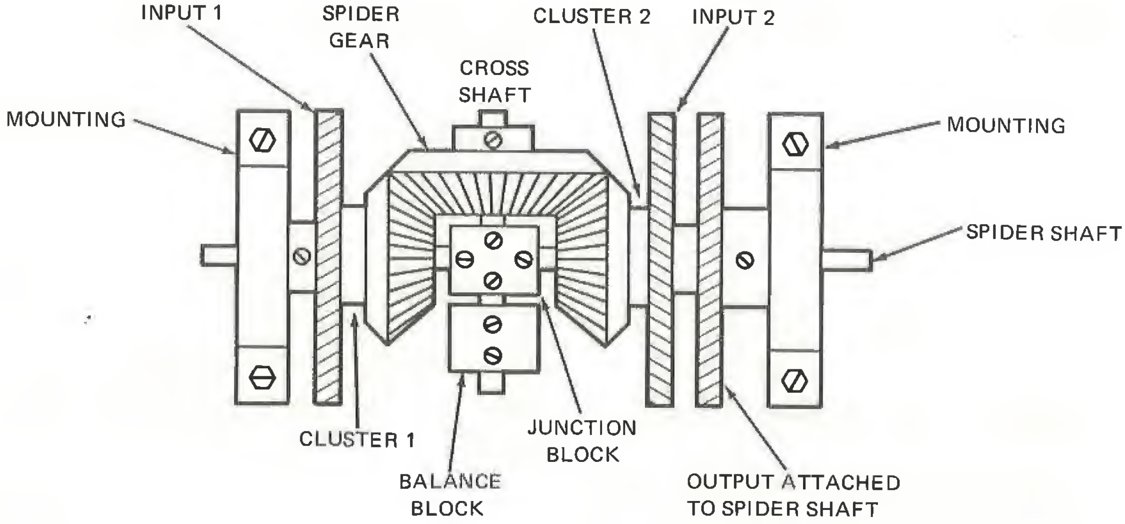


Fig. 19-2 Differential Assembly

shaft. The *balance block* serves as a counterweight to dynamically balance the cross shaft. In some cases, a fourth bevel gear is used to balance the cross shaft. When this is done, the fourth gear is mounted just like the spider gear.

When the differential is built with three gears of the same size, it is called a miter gear differential. If the spider gear is a different size than the input bevel gears, the assembly is called a bevel gear differential.

To better understand the operation of a differential assembly, let us suppose that input 2 in figure 19-2 is held fixed and input 1 is rotated. The bevel gear clustered with input 1 must turn with the input spur gear. Since the spider gear is meshed with the input bevel gear, it must also rotate. However, since the spider gear is also meshed with the input 2 bevel gear (which we are holding in a fixed position), its rotation makes the cross shaft rotate in the same direction as input 1.

Suppose the spider shaft moves through an angle which corresponds to one tooth on the fixed input bevel gear. If the spider gear did not rotate, this spider shaft movement would cause the free input bevel gear to rotate by an angle equivalent to one tooth space.

However, the spider gear must rotate if input 2 is fixed. The amount of its rotation will be an angle equivalent to one tooth space. This spider gear rotates the free input gear one additional tooth spacing. As a result of these two equal actions, the free input gear must rotate through an angle twice as big as the spider shaft rotation. That is,

$$\theta_1 = 2\theta_s \text{ if } \theta_2 = 0$$

where θ_1 = rotation of input 1

θ_s = spider shaft (output) rotation

θ_2 = rotation of input 2

We may, of course, restate this expression as

$$\theta_s = 1/2\,\theta_1 \text{ if } \theta_2 = 0$$

Similarly, if we hold input 1 fixed and rotate input 2, the result will be

$$\theta_s = 1/2\,\theta_2 \text{ if } \theta_1 = 0$$

Moreover, if we rotate both inputs at the same time, we will have simply the sum of these two actions. That is,

$$\boxed{\theta_s = 1/2\,\theta_1 + 1/2\,\theta_2} \quad \textbf{(19.1)}$$

or

$$\boxed{\theta_s = 1/2\,(\theta_1 + \theta_2)} \quad \textbf{(19.2)}$$

This equation provides several possibilities:

1. If the inputs turn in the same direction but different amounts, the output (θ_s) will turn in the same direction by an amount midway between the two inputs. For example, if $\theta_1 = 30°$ and $\theta_2 = 90°$, then $\theta_s = 1/2\,(30° + 90°) = 1/2\,(120°) = 60°$.

2. If the two inputs turn equal amounts in the same direction, then the output will do the same. For example, if $\theta_1 = \theta_2 = 60°$, then $\theta_s = 1/2\,(60° + 60°) = 60°$.

3. If the inputs turn in opposite directions by different amounts, then the output will turn in the direction of the larger input by an amount equal to half the difference between the inputs. If $\theta_1 = 90°$, $\theta_2 = -30°$, then $\theta_s = 1/2\,(90° - 30°) = 30°$.

4. If the inputs turn equal amounts in opposite directions, the output will not turn. $\theta_1 = 60°$, $\theta_2 = -60°$, then $\theta_s = 1/2\,(60° - 60°) = 0°$.

All three of the angular displacements (θ_1, θ_2, θ_s) occur in the same time interval, so we can divide each term in equation 19.1 by t (time) and have

$$\frac{\theta_s}{t} = 1/2\,\frac{\theta_1}{t} + 1/2\,\frac{\theta_2}{t}$$

And since θ/t is equal to angular velocity ω, we have

$$\boxed{\omega_s = 1/2\,\omega_1 + 1/2\,\omega_2} \quad \textbf{(19.3)}$$

or

$$\boxed{\omega_s = 1/2\,(\omega_1 + \omega_2)} \quad \textbf{(19.4)}$$

Consequently, the four possibilities named before apply to angular velocity as well as to displacement. Specifically,

1. If the inputs run in the same direction at different RPMs, then the output will run in that direction at a speed midway between the input speeds. If $\omega_1 = 900$ RPM, $\omega_2 = 300$ RPM, then $\omega_s = 600$ RPM.

2. If the inputs are equal and in the same direction, then the output will be the same. If $\omega_1 = 600$ RPM, $\omega_2 = 600$ RPM, then $\omega_s = 600$ RPM.

3. If the inputs are different speeds in opposite directions, then the output will be in the direction of the larger input and equal to half their difference. If $\omega_1 = 900$ RPM, $\omega_2 = -300$ RPM, then $\omega_s = 300$ RPM.

4. If the inputs are equal and opposite, then the output is zero. If $\omega_1 = 600$ RPM, $\omega_2 = -600$ RPM, then $\omega_s = 0$ RPM.

If the torques T_1 and T_2 are applied to the inputs 1 and 2, respectively, then the forces at the pitch circles of input bevel gears are

$$f_1 = \frac{T_1}{r_2} \text{ and } f_2 = \frac{T_2}{r_2}$$

where r_1 and r_2 are the respective pitch radii. Since pitch radius and pitch diameter are re-

lated by r = 1/2 D, and both input gears are identical,

$$f_1 = 2\frac{T_1}{D} \text{ and } f_2 = 2\frac{T_2}{D}$$

Since both of these forces are effective at the pitch circle of the spider gear, we have

$$f_s = f_1 + f_2 = \frac{2}{D}(T_1 + T_2)$$

Now if we examine figure 19-2, we see the radius arm from the center of the spider shaft to the plane of the pitch circle of the spider gear is equal to the pitch radii of input bevel gears. That is,

$$r_s = r_1 = r_2 = 1/2D$$

And the torque on the spider shaft is

$$T_s = f_s r_s = f_s \frac{D}{2}$$

Substituting the expression for f_s developed previously and cancelling D's and 2's, we have

$$\boxed{T_s = T_1 + T_2} \qquad \textbf{(19.5)}$$

Using this expression and the four conditions considered for displacement and velocity, we see that:

1. If the input torques are in the same direction and unequal, then the output torque is their sum.
2. If the input torques are in the same direction and equal, then the output torque is twice either input.
3. If the input torques are opposite and unequal, then the output torque is their difference.
4. If the input torques are opposite and equal, then the output is zero.

Mechanical differentials may be constructed using two spur gears and a crown gear as shown in figure 19-3. However, since the crown gear teeth are cut along radial lines while spur gear teeth are parallel, accurate meshing is impossible. Consequently, bevel gear differentials are much preferred for precision applications.

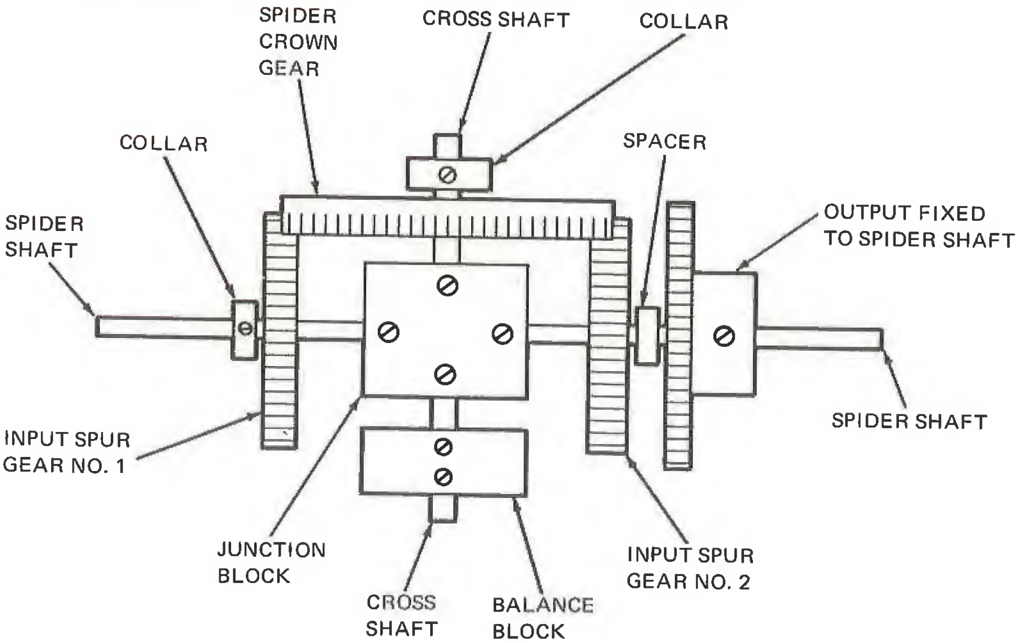


Fig. 19-3 A Spur and Crown Gear Differential

MATERIALS

- 2 Bearing plates with spacers
- 1 Junction block with shaft
- 1 Shaft coupling
- 2 Hollow shafts, 2" X 1/4" with 1/8 in. bore
- 1 Shaft, 4" X 1/8" (to fit inside the hollow shafts)
- 3 Bevel gears
- 3 Collars
- 2 Dials with 1/4 in. bore hubs
- 1 Dial with 1/8 in. bore hubs
- 3 Dial indices with mounting hardware
- 2 Bearing Mounts
- 2 Bearings
- 1 Breadboard with legs and clamps

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 19-4.
3. Before proceeding, check your assembly to insure that the following points are correct:

   (a) One of the cross shafts is fixed to the balance block and to the junction block.

   (b) The other cross shaft is fixed to the junction block. The spider gear is free to turn on this shaft. The spider gear is held on the shaft by a fixed collar.

   (c) The small internal shaft runs completely through the assembly. The internal shaft is fixed to the junction block and output dial only. The internal shaft is free to rotate inside the hollow shafts.

   (d) Each hollow shaft is fixed to an input gear and an input dial. The input gears are held in mesh with the spider gear by collars on the hollow shafts.

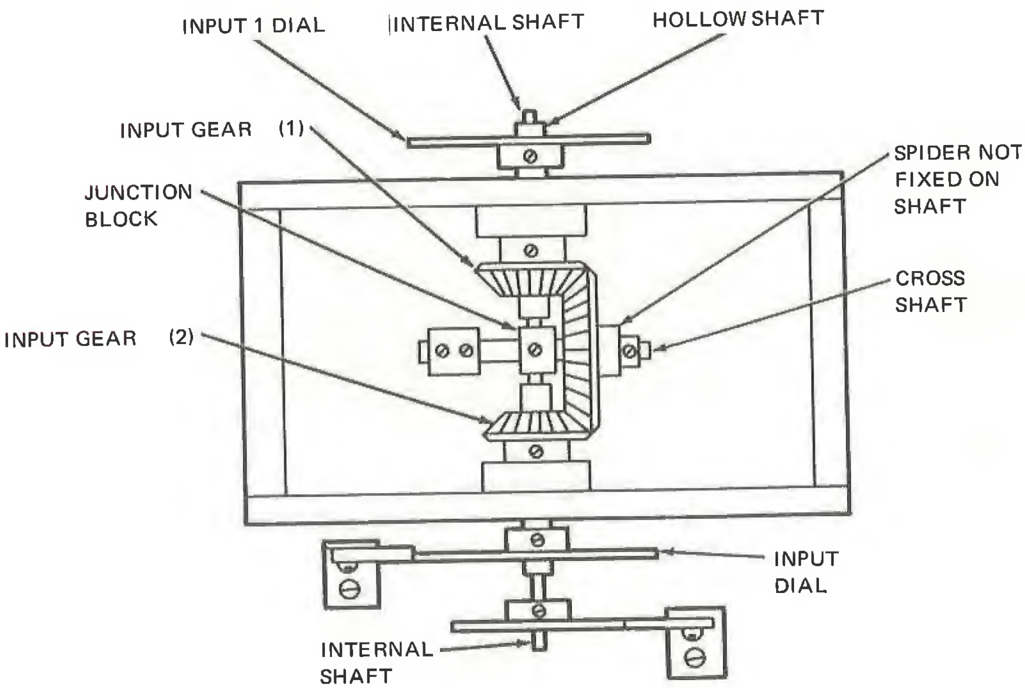


Fig. 19-4 The Experimental Mechanism

4. Carefully rotate the output dial. The differential should turn smoothly. Lubricate the hollow shafts and bearings if necessary.
5. Adjust the balance block position on the cross shaft so that the output dial will stay in any position that you set it to. If the cross shaft is unbalanced, the heavy side will swing downward changing the output dial setting.
6. After the differential is balanced, adjust all three dials to zero.
7. Hold input dial 2 fixed and rotate input dial 1 to 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, and 360 degrees. In each case, record the reading of all 3 dials.
8. Repeat steps 6 and 7 holding input 1 fixed and rotating input 2 in 30-degree steps.
9. Repeat steps 6 and 7 holding the output dial fixed at zero.
10. Using equation 19.2 and the values of input 1 and 2, compute θ_s for each set of data taken in steps 7, 8, and 9.
11. Return all three dials to zero.
12. Set input 1 to 30 degrees, then hold it while you set input 2 to 20 degrees. Record the readings of all three dials.
13. Repeat step 12 for each of the following pairs of input values:

θ_1 (degrees)	θ_2 (degrees)
45	20
45	30
45	45
45	60
60	90
60	135
90	180
135	180
180	210
200	250
270	360
360	360

14. Repeat step 10 for each of the sets of values from steps 12 and 13.
15. Repeat steps 12, 13, and 14 using the angles given, but rotate input 1 counterclockwise and input 2 clockwise.

ANALYSIS GUIDE. In analyzing the results from this experiment, there are two main things that you should do. They are:

1. Discuss the extent to which your observations agreed with the discussion.
2. Use specific examples from your data to illustrate the four possible interrelationships between the inputs and the output.

You may also wish to discuss any problems you had in constructing the differential.

Input 2 Fixed

θ_1	θ_2	θ_s	θ_s (Comp.)

Input 1 Fixed

θ_1	θ_2	θ_s	θ_s (Comp.)

Output 1 Fixed

θ_1	θ_2	θ_s	θ_s (Comp.)

Fig. 19-5 The Data Tables

Inputs Rotate Opposite Directions

θ_1	θ_2	θ_s	θ_s (Comp.)

Inputs Rotate Same Direction

θ_1	θ_2	θ_s	θ_s (Comp.)

Fig. 19-5 The Data Tables

PROBLEMS

1. List several practical applications for mechanical differentials.
2. A certain differential has an input 1 of 1750 RPM and an input 2 in the same direction of 1720 RPM. What is the spider shaft velocity and which way does it turn?
3. What would be the results in problem 2 if the inputs turned in opposite directions?
4. Make a sketch showing how a differential can be used to give two outputs from a single input. The outputs should be in line with each other and at right angles to input.
5. Where might you find a differential of the type described in problem 4?
6. A certain differential has input torques of 130 lb-ft and 90 lb-ft. What is the spider shaft torque?
7. A differential is set up with the spider shaft fixed. An input velocity of 3000 RPM is applied to input 1. A mechanical load of 175 in-oz is connected to the remaining shaft. What is the torque on the spider shaft?
8. What is the angular velocity of the load in problem 7?
9. What horsepower is being delivered to the load in problem 7?
10. What horsepower is being delivered by the source in problem 7?
11. What horsepower is being delivered to the spider shaft in problem 7?
12. What practical application can you think of for the arrangement in problem 7?
13. What do you think is meant by the statement, "Torque in a differential follows the path of least opposition?"

experiment 20 SPRING MECHANICS

INTRODUCTION. Springs are used in a great variety of mechanical applications to store potential energy. In this experiment we shall examine some of the common spring configurations and their operating characteristics.

DISCUSSION. Perhaps the two most common kinds of springs are the *coil* and *leaf* types. Of these two basic types, the coil spring is probably the most often encountered. We shall, therefore, consider only coil springs in this experiment.

Coil springs are formed by wrapping a round or square wire into a helical coil. Steel is the most commonly used material, but alloys of brass, bronze, copper and others are also used.

The most popular coil spring types are *extension, compression,* and *torsion* springs. An example of each of these is shown in figure 20-1. The extension type spring is perhaps the most familiar one since it is found on so many screen doors. In a typical application, this type extends or stretches when a load is applied to it.

Figure 20-2 shows a spring which is loaded and the same spring without a load.

When the load force F is applied to the spring, it elongates by an amount ΔL determined by the spring material and its dimen-

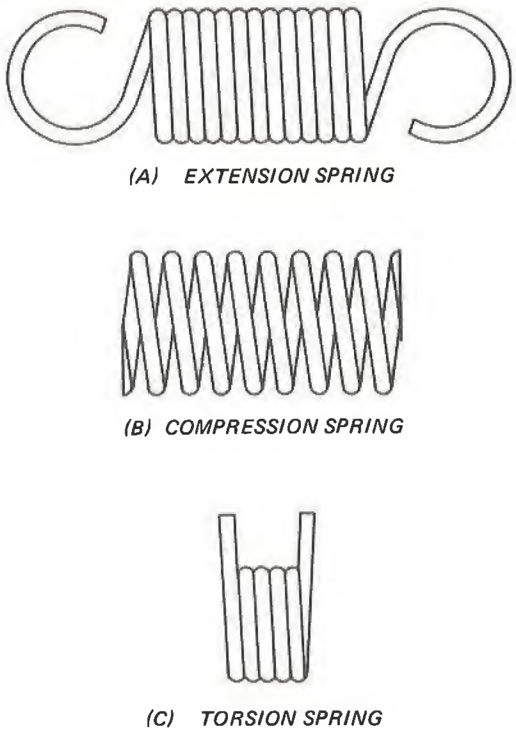


Fig. 20-1 Coil Spring Configurations

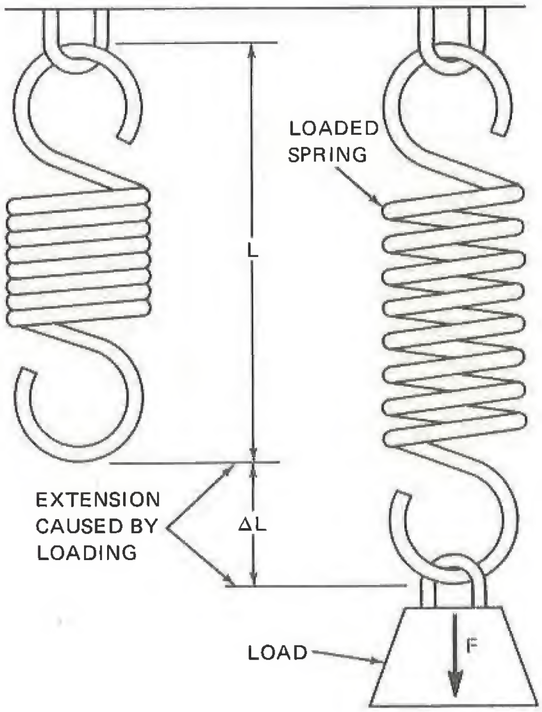


Fig. 20-2 Loading an Extension Type Spring

sions. The amount of work done in stretching the spring is, of course,

$$W = \frac{1}{2} F \Delta L$$

And if there are no losses in the spring (while this is never precisely true, spring losses tend to be very small and will be ignored in this discussion), then the potential energy stored in the spring is equal to the work performed in elongating it. That is

$$PE = W = \frac{1}{2} F \Delta L$$

If we apply a load to any kind of material, the material is said to be under *stress.* Stress is defined as the ratio of the load force to the cross-sectional area of the material. Or, in other words, the stress (S) is

$$S = \frac{F}{A} \text{ Lb. per sq. in.} \qquad (20.1)$$

Any material which is placed under stress tends to change its shape. It stretches in the direction of the applied force. This deformation is called *strain* (δ) and is defined algebraically as:

$$\delta = \frac{(\Delta L)}{L} \qquad (20.2)$$

The ability of a material to resist deformation is expressed in terms of the amount of stress required to produce one unit of strain. This ratio is called the *Modulus of Elasticity* (E) and is equal to

$$E = \frac{S}{\delta} \qquad (20.3)$$

Substituting equations 20-1 and 20-2 into this relationship renders

$$E = \frac{FL}{A(\Delta L)} \qquad (20.4)$$

This relationship is appropriate for use with most materials under compression or tension.

In the case of an extension type coil spring, *loading does not cause tension or compression within the spring material.* Stretching the spring tends to twist the spring wire. Therefore, *the stress within the wire is torsional* tending to *sheer* the wire. In the case of sheer stress, the *sheer modulus of elasticity (G)* is given by

$$G = \frac{E}{2(1 + u)} \qquad (20.5)$$

where the quantity u is called Poisson's ratio, and is the ratio of the lateral to axial strain. The value of u varies for different materials but is about 0.25 for steel and about 0.33 for brass and bronze. Consequently, the value of G is

$$G \approx 0.4 \text{ E for most springs}$$

The *spring index* of a coil spring is a parameter that is useful in describing the physical size of a spring. The spring index (C) is defined as the ratio of the mean diameter (D) of the spring coil to the diameter (d) of the wire. That is,

$$C = \frac{D}{d} \qquad (20.6)$$

In dealing with spring applications it is frequently convenient to compare springs on the basis of their performance. The most common method of comparing spring action is to use a factor called the *spring constant (K).*

The spring constant of a given spring is simply the ratio of the load force (F) to the resulting extension (or compression) (ΔL). In other words

$$\boxed{K = \frac{F}{\Delta L}} \quad \textbf{(20.7)}$$

The spring constant of helically-wound, round wire springs is related to the sheer modulus of elasticity and the spring's dimensions by

$$K = \frac{Gd}{8NC^3} \quad \textbf{(20.8)}$$

where N is the number of turns of wire in the spring that are supporting the load.

If we connect two coil springs having different spring constants in series, as shown in figure 20-3, the spring constants of the two springs are

$$K_1 = \frac{F}{\Delta L_1} \text{ and } K_2 = \frac{F}{\Delta L_2}$$

respectively. Then we observe that the total extension (ΔL_T) must be equal to the sum of the two spring extensions:

$$\Delta L_T = \Delta L_1 + \Delta L_2$$

However, from the spring constant relationships we have

$$\Delta L_1 = \frac{F}{K_1} \text{ and } \Delta L_2 = \frac{F}{K_2}$$

Substituting these values into the equation for total extension gives us

$$\Delta L_T = \frac{F}{K_1} + \frac{F}{K_2}$$

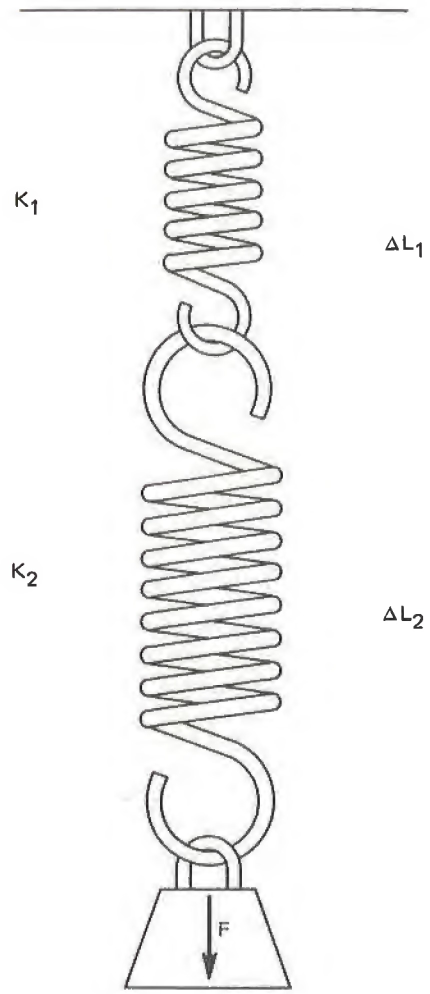


Fig. 20-3 A Series Spring System

Then dividing through by F renders

$$\frac{\Delta L_T}{F} = \frac{1}{K_1} + \frac{1}{K_2}$$

The quantity on the left is the reciprocal of the total spring constant of the system, so we can write

$$\boxed{\frac{1}{K_T} = \frac{1}{K_1} + \frac{1}{K_2}} \quad \textbf{(20.9)}$$

We can extend this relationship to as many

springs as we wish by simply adding on the reciprocal of their spring constants. That is, for n springs,

$$\frac{1}{K_T} = \frac{7}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots + \frac{1}{K_n} \qquad (20.10)$$

It is perhaps worth noting that for two springs only we can add the right-hand fractions by using the common denominator:

$$\frac{1}{K_T} = \frac{K_1 + K_2}{K_1 K_2}$$

Then, taking the reciprocal of both sides, we have

$$\boxed{K_T = \frac{K_1 K_2}{K_1 + K_2}} \qquad (20.11)$$

which is a convenient relationship for the equivalent spring constant of two series springs.

Springs may also be arranged in parallel as shown in figure 20-4. In this arrangement, the two springs share the load so that

$$F_T = F_1 + F_2$$

and since

$$F_1 = K_1 (\Delta L_1) \text{ and } F_2 = K_2 (\Delta L_2)$$

we may write the force equation in the form,

$$F_T = K_1(\Delta L_1) + K_2(\Delta L_2)$$

Then, if we have arranged the springs and load so that both springs stretch the same amount,

$$\Delta L_1 = \Delta L_2 = \Delta L$$

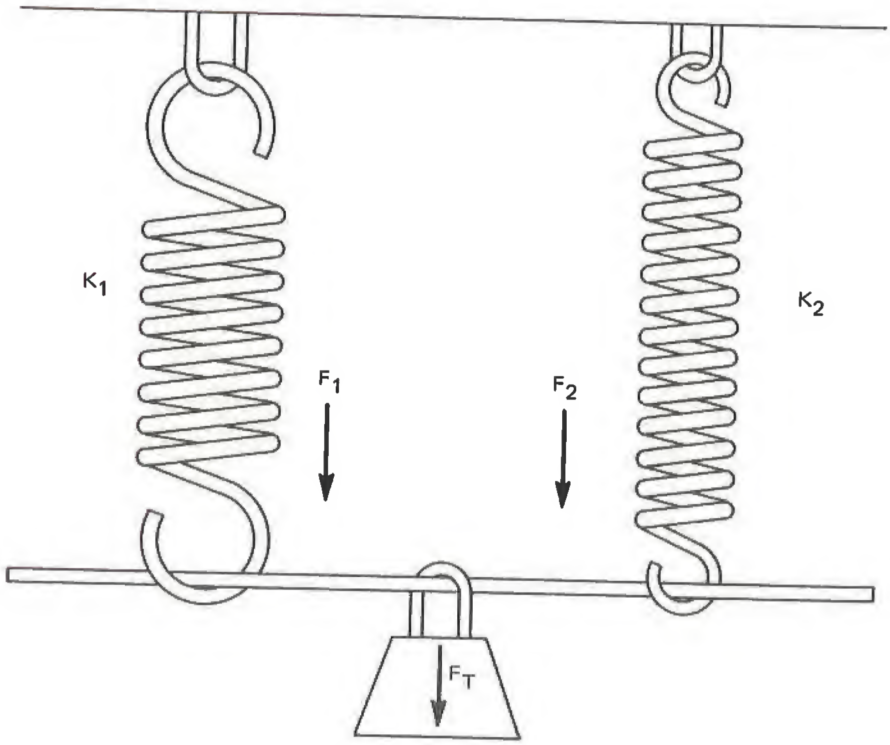


Fig. 20-4 A Parallel Spring System

and we can divide through by ΔL, giving

$$\frac{F_T}{\Delta L} = K_1 + K_2$$

But since $F_T/\Delta L$ is the effective spring constant of the parallel system, we have

$$\boxed{K_T = K_1 + K_2} \qquad (20.12)$$

for a parallel spring arrangement in which both springs stretch the same amount. As in the case of series springs, we can extend this equation to include any number of springs,

$$K_T = K_1 + K_2 + K_3 + \ldots + K_n \qquad (20.13)$$

The material discussed up to this point has pertained to extension type coil springs only. Actually, virtually everything covered is also appropriate for use with compression type coil springs as well.

Torsion type coil springs also react in much the same way as extension and compression types, but the reaction is along a circular direction rather than in a straight line. Figure 20-5 shows a torsion spring under load. The load force tends to rotate the wheel counterclockwise with a torque of

$$T_L = F_L R_w$$

where R_w is the radius of the wheel.

The spring is held at one end by the fixed pin (P), while the other end of the spring works the movable pin (Q) generating a clockwise torque of

$$T_s = F_s R_p$$

where R_p is the moment arm from the center of the wheel to the force F_s. F_s is the force produced at pin Q by the spring. If applying the load caused the wheel to turn through an angular displacement of θ, then the work done was

$$W = \frac{1}{2} T_L \theta = F_L (\Delta L)$$

And since the torque produced by the spring must equal the load torque, the energy stored by the spring is

$$PE = \frac{1}{2} T_L \theta = \frac{1}{2} T_s \theta = F_L (\Delta L)$$

The spring constant of a torsion spring is defined in much the same way as that of an extension or compression spring. For a torsion spring we use

$$\boxed{K = \frac{T}{\theta}} \qquad (20.14)$$

to find the value of the spring constant for corresponding values of T and θ.

MATERIALS

- 2 Extension springs of approx. equal length but different diameters.
- 1 Spring balance with posts and clamps
- 1 Breadboard with legs
- 1 Dial caliper
- 1 Short length of waxed string

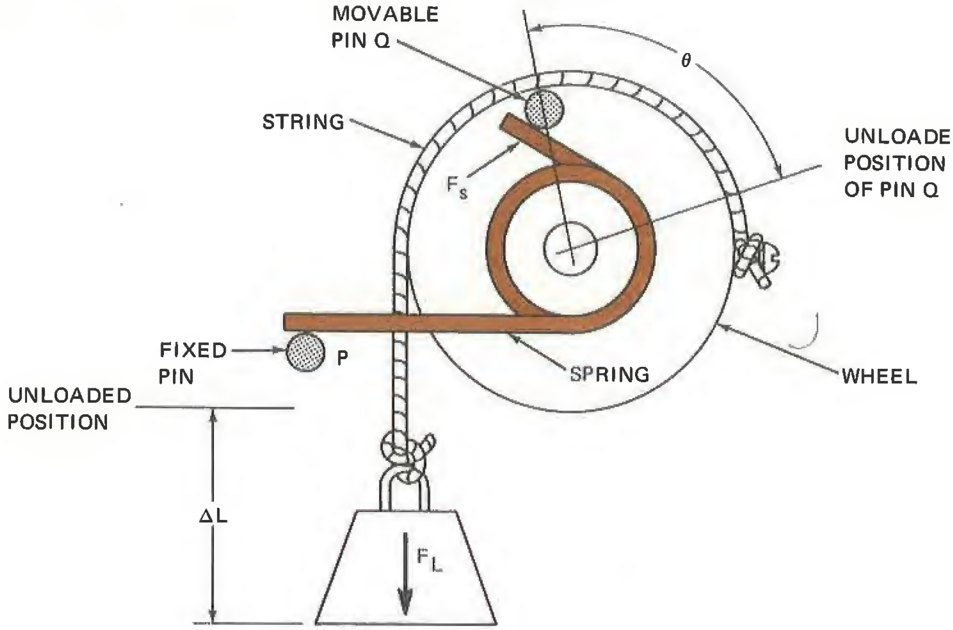


Fig. 20-5 A Torsion Spring Under Load

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Measure and record the wire diameter of each spring (d).
3. Measure and record the OD of each spring.
4. Compute the mean diameter of each spring and record the results in the Data Table, figure 20-7, as (D).
5. Compute and record the spring index of each spring (C).
6. Measure the overall unstretched length of each spring.
7. Construct the setup shown in figure 20-6 using one of the springs.

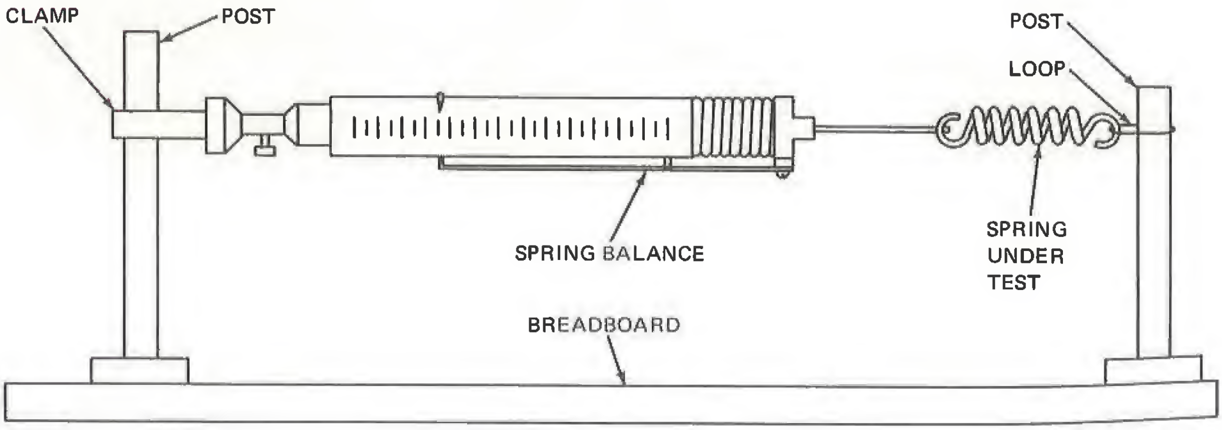


Fig. 20-6 The Experimental Setup

8. Adjust the post spacing so that the spring under test is stretched to about 1-1/4 times its original length.
9. Carefully measure and record the overall length of the spring under test (L_T).
10. Record the load force on the spring (F_L).
11. Increase the length about 1/4 in. and record F'_L and L'_T.
12. Compute the amount of extension caused by the change in load. Record ΔL and ΔF.
13. Compute and record the spring constant of the spring under test (K).
14. Repeat steps 7 through 13 with the other spring.
15. Repeat steps 7 through 13 with the two springs in series.

First Spring

d	OD	D	C	L_T	L'_T	ΔL	F_L	F'_L	ΔF	K

Second Spring

d	OD	D	C	L_T	L'_T	ΔL	F_L	F'_L	ΔF	K

Series Combination

K_T = ______________

(Computed Value)

Measured Values

L_T	L'_T	ΔL	F_L	F'_L	ΔF	K_T

Parallel Combination

K_T = ______________

(Computed Value)

Measured Values

L_T	L'_T	ΔL	F_L	F'_L	ΔF	K_T

Fig. 20-7 The Data Table

16. Similarly repeat steps 7 through 13 with the two springs in parallel. *It may be necessary to use loops of different lengths to insure that ΔL for both springs is the same.*
17. Using the appropriate expression from the discussion, compute the equivalent spring constant for the two springs in series and parallel. Use your measured values of the *individual* spring constants in making this computation.

ANALYSIS GUIDE. In this experiment we have examined the operating characteristics of coil springs. In analyzing your results there are several points that you should not overlook. Among these points are:

1. Did the equivalent spring constants for the series and parallel arrangements that you measured agree with those that you computed?
2. Based on your results, can you make a generalization about the relationship between spring constants and spring indices?
3. Do your results agree in general with the points covered in the discussion?

In addition to these points there are many others that you may wish to include in your analysis.

PROBLEMS

1. Using sketches, show how you would repeat this experiment using compression type springs. Explain how each setup (individual springs, series springs and parallel springs) could be done.
2. Repeat problem 1 using torsion type springs.
3. Repeat problem 1 using leaf springs.
4. A load of 70 lb. stretches a spring 6.2 in. How much of a load is required to stretch it 1.0 in.?
5. What is the spring constant in problem 4?
6. A compression type spring is found to change its length by 0.73 in. When a load of 85 lb. is applied, what is the spring constant?
7. A spring having a spring constant of 50 lb/in. is used in series with two others having spring constants of 30 lb/in. What will be the total deflection if a load of 75 lb. is applied?

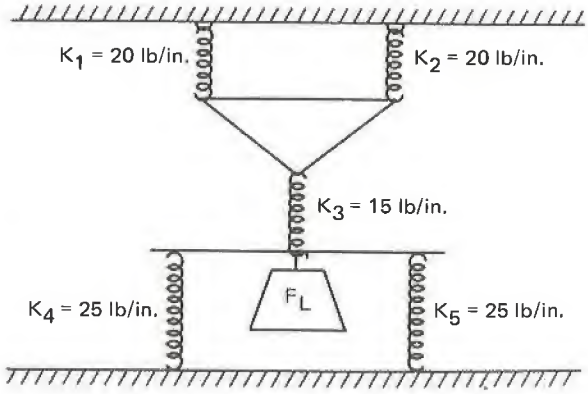


Fig. 20-8

8. What is the total spring constant of the arrangement shown in figure 20-8?
9. If the total deflection under load in problem 8 is 2.75 in., what is the weight of the load?
10. Eight compression springs in parallel support a load of 950 lb. If all of the springs are identical, what is their spring constant if the load deflects them 1.2 in.?

experiment 21 INERTIAL EFFECT

INTRODUCTION. All rotary components exhibit the property of *inertia* or flywheel effect. In this experiment we will examine this property and ways in which it affects mechanical performance.

DISCUSSION. The kinetic energy stored by a translating object is given by

$$K\,E = 1/2\ Mv^2 \qquad \textbf{(21.1)}$$

where M is the mass of the object and v is the velocity. In most cases, it is convenient to work with the weight of the object rather than with its mass. Weight and mass are related by

$$M = \frac{w}{g} \qquad \textbf{(21.2)}$$

where g is the acceleration due to gravity and is equal to 32.2 ft/sec^2 or 386.4 in/sec^2. Substituting this relationship into equation 21.1, we have

$$\boxed{K\,E = \frac{1}{2}\frac{w}{g}\,v^2} \qquad \textbf{(21.3)}$$

In working with rotating mechanical components (gears, pulleys, sprockets, shafts, etc.), we are normally interested in the angular velocity (ω) in RPM. Consequently, we would prefer to express kinetic energy in terms of RPM.

In order to develop such a relationship, let us consider a small ball swinging at the end of a very light string as shown in figure 21-1. The kinetic energy of the ball is the same as that given by equation 21.3.

The linear velocity of the ball is related to the angular velocity by

$$v = \omega_r R \text{ in/sec}$$

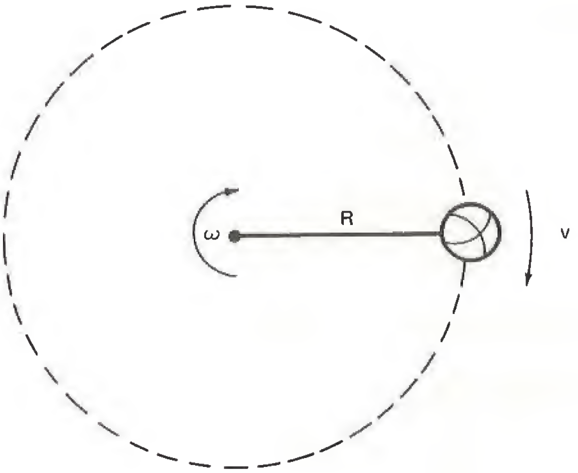


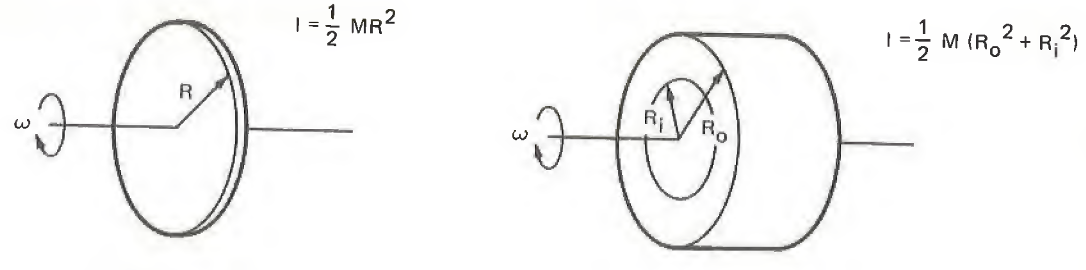
Fig. 21-1 A Spinning Ball

where R is in inches and ω_r is in radians per second. Consequently, the kinetic energy of the ball and string system is

$$\boxed{K\,E = \frac{1}{2}Mv^2 = \frac{1}{2}(MR^2)\,\omega^2}$$
$$\boxed{= \frac{1}{2}I\omega^2} \qquad \textbf{(21.4)}$$

In this equation the quantity (MR^2) is called the *moment of inertia* (I) of the system and is an expression of how much it resists changes in angular velocity. Notice that the moment of inertia of a rotating object is analogous to the mass of a linearly moving object.

Different geometric shapes have different moments of inertia. Figure 21-2 shows two



(a) A Solid Cylinder

(b) A Hollow Cylinder

Fig. 21-2 Moments of Inertia of Cylinders

commonly encountered bodies and their moments of inertia.

Rotating components can often be considered to approximate one or both of these geometric shapes. Shafts and small gears, for instance, are usually approximated by a solid cylinder, while flywheels and large gears may frequently be approximated by a hollow cylinder. Sometimes both must be used for an adequate approximation.

As a result, we can determine the kinetic energy of a spur gear using

$$KE = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

where R is the pitch radius, M is the mass, and ω is angular velocity in radians per second. Converting ω to revolutions per minute, we have

$$KE = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{60}{2\pi}\omega\right)^2$$

$$= 225\, MR^2 \frac{\omega^2}{\pi^2} \approx 22.75\, MR^2\omega^2$$

Then, since $M = w/g$ and $g = 386.4$ in/sec^2, we may write

$$KE \approx 0.059\, wR^2\omega^2$$

for the kinetic energy of a rotating spur gear. W is the weight of the gear wheel, R is the radius, and ω is the angular velocity in RPM. In terms of pitch diameter, this equation becomes

$$KE \approx 0.015\, wD^2\omega^2 \qquad \textbf{(21.5)}$$

This quantity is the amount of energy stored by a rotating spur gear. If we are attempting to start or stop the gear's rotation, this is the amount of energy we must contend with. For example, to get the gear to rotate at ω RPM, we must supply this energy in addition to any load and losses. Similarly, this is the amount of energy we must dissipate to stop the gear.

Because the kinetic energy of a rotating component must be changed in order to change its angular velocity, the flywheel effect tends to oppose changes in velocity. This effect is often used to help regulate rotating machinery against sudden small changes in velocity.

Let us suppose that a gear wheel is rotating and the driving force is suddenly lost. Because the gear has stored the amount of kinetic energy given by equation 21.5, it cannot change its velocity instantaneously. Consequently, it will continue to turn. However, since the load on the gear (as well as any bearing friction) is still present, the gear will start

giving up its kinetic energy. As the kinetic energy of the gear is reduced, its angular velocity will also reduce, and the system will "coast" to a stop. The greater the load on the gear, the quicker it will stop.

In this experiment we will vary the load on a rotating system and examine the length of time required for it to slow down a given amount.

MATERIALS

- 1 Motor and mount
- 1 Power supply
- 1 Stroboscope
- 1 Breadboard with legs and clamps
- 2 Hangers
- 2 Bearings
- 2 Collars
- 1 Dial with 1/4 in. bore hub
- 1 Pulley approx. 1 in. OD
- 1 Pulley approx. 1 1/2 in. OD
- 1 Pulley approx. 2 in. OD
- 1 Shaft approx. 4" X 1/4"
- 1 Flexible shaft coupling
- 1 Wrist watch with sweep second hand (to be supplied by student)
- 1 Spring balance
- 1 Dial Caliper
- 1 Sheet of graph paper

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Weigh each of the pulleys (w_1, w_2, w_3) and measure their outside diameters D_1, D_2, D_3).
3. Assemble the setup shown in figure 21-3 using the 1-in. pulley.
4. Carefully turn up the power supply until the pulley is running 6000 RPM.

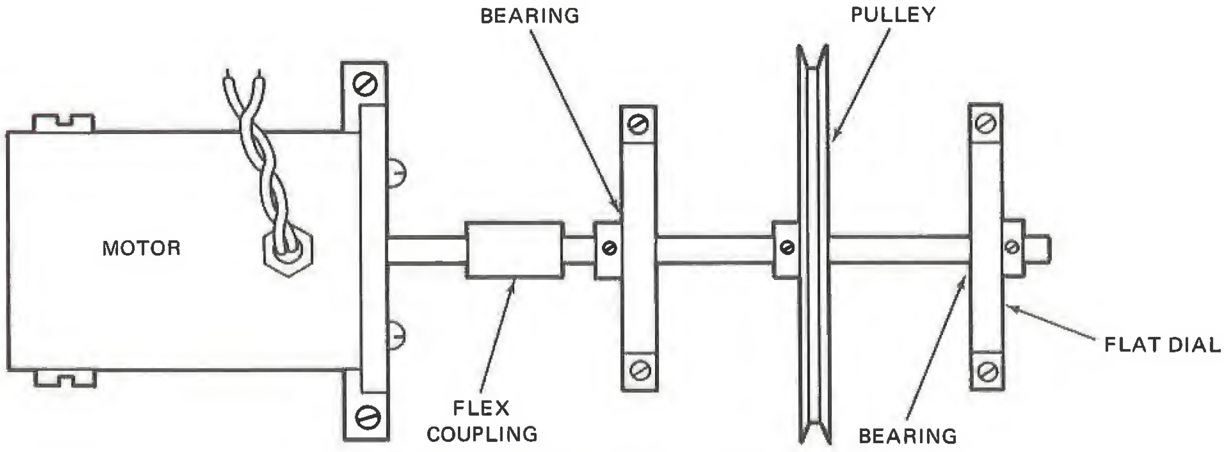


Fig. 21-3 The Experimental Mechanism

5. Quickly uncouple the motor from the shaft and measure the length of time required for the pulley to slow to a stop (t_1).
6. Repeat steps 4 and 5 using the following pulley arrangements:

   (a) The 1 1/2-in. pulley alone (t_2)
   (b) The 2-in. pulley alone (t_3)
   (c) The 1-in. and 1 1/2-in. pulleys together (t_4)
   (d) The 1-in. and 2-in. pulleys together (t_5)
   (e) The 1 1/2-in. and 2-in. pulleys together (t_6)
   (f) All three pulleys (t_7)

7. Compute the approximate K E for each case using equation 21.5.
8. Plot a curve of K E versus the time values measured in steps 5 and 6.

W_1	W_2	W_3	D_1	D_2	D_3

Trial	t	KE
1		
2		
3		
4		
5		
6		
7		

Fig. 21-4 The Data Table

ANALYSIS GUIDE. In the analysis of these data, you should discuss any trend observed in the various timed periods. Also, you should explain what happens to the kinetic energy stored in the rotating components.

PROBLEMS

1. A 96-tooth 32-pitch steel gear weighs 6.4 ounces. What would be the kinetic energy stored in the gear at 7000 RPM?

2. The gear in problem 1 is used in a compound train as the output gear. The input gear has 30 teeth. On the compound shaft the follower and driver have 72 and 24 teeth, respectively. If the output gear turns 275 RPM, what is the total kinetic energy in the system? The weights of the gears are:

   N_1 = 30 teeth ------------------- 1.3 oz.
   N_2 = 72 teeth ------------------- 5.7 oz.
   N_3 = 24 teeth ------------------- 1.0 oz.
   N_4 = 96 teeth ------------------- 6.4 oz.

3. A flywheel has a very light frame and a rim that weighs 22 lb. The rim is in the shape of a cylinder with an ID of 12 in. and an OD of 14 in. What is the moment of inertia of the flywheel?

4. How much kinetic energy is stored in the flywheel in problem 3 if its angular velocity is 3500 RPM?

5. In what units should kinetic energy be expressed?

6. How could the K E in problem 4 be converted into useful work?

experiment 22 BELT DRIVES

INTRODUCTION. Belts and pulleys are used in a wide variety of mechanical drive systems. In this experiment we shall examine some of the basic belt drive characteristics.

DISCUSSION. Belt drives are commonly used between shafts that are too far apart to be coupled economically with gear wheels. Moreover, since belt drives are usually somewhat elastic, they are very effective in absorbing shock and vibration.

Perhaps the most common types of belts in use at present are the flat, round, and vee-shaped ones. Figure 22-1 shows sketches of these types of belts.

Each type of belting has its own type of pulleys and applications to which it is particularly well suited.

Flat belting is used with very large pulley diameters and very long center distances. The belts are usually either leather or fabric and can be spliced. Pulleys for flat belt application may be either *flanged* or *crowned.* The crowned pulley is usually considered to be superior to the flanged one because it causes less wear at the edges of the belt. Figure 22-2 shows two types of flat belt pulleys.

Round belts are usually made from neoprene or some other synthetic material and are used only for light duty applications. Pulleys for round belt applications have a concaved face to fit the belt cross-section.

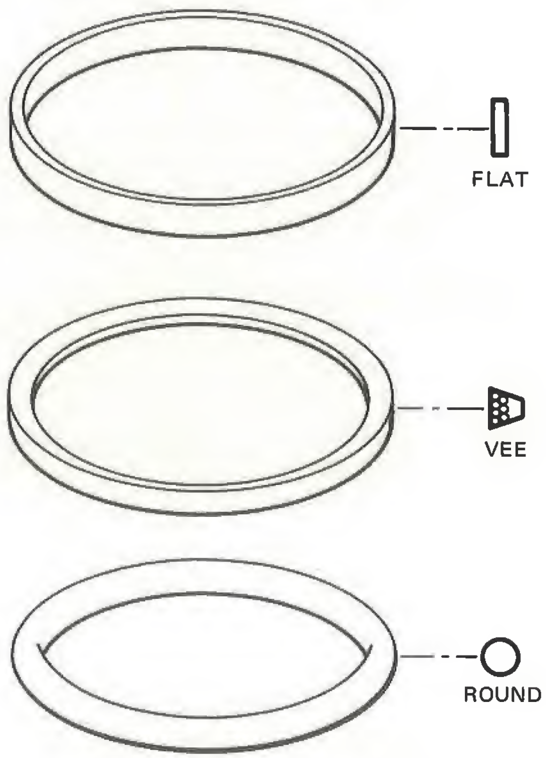


Fig. 22-1 Types of Belting

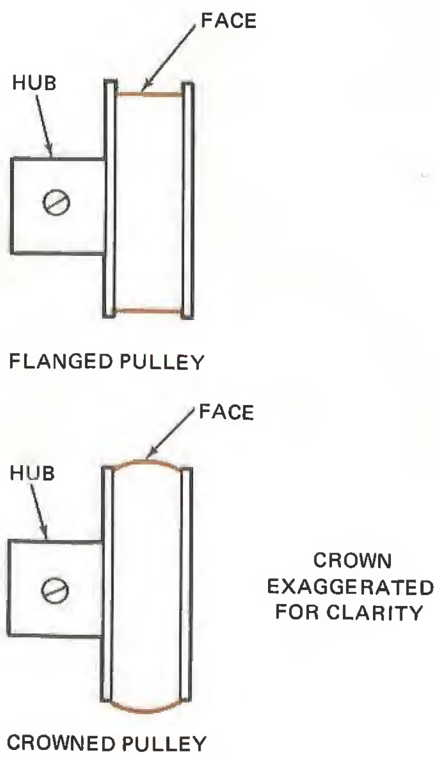


Fig. 22-2 Flat Belt Pulleys

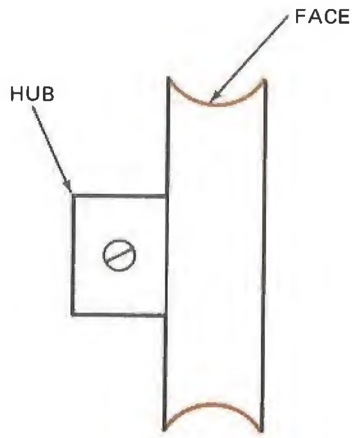


Fig. 22-3 A Round Belt Pulley

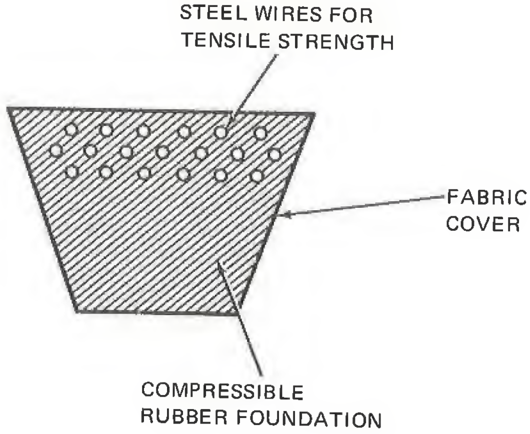


Fig. 22-4 Vee-belt Construction

Vee belts are probably the most popular type because of their ruggedness and dependability. Their construction is somewhat more complex than flat or round belts. Figure 22-4 shows a vee-belt cross-section.

Vee-belts are used with pulleys like the one shown in figure 22-5.

Belt drives are most frequently used in the *open belt* configuration shown in figure 22-6.

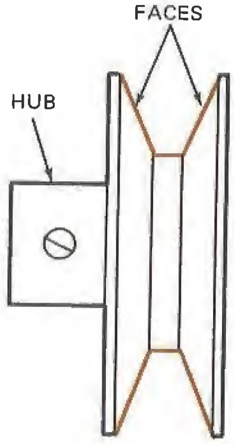


Fig. 22-5 A Vee Pulley

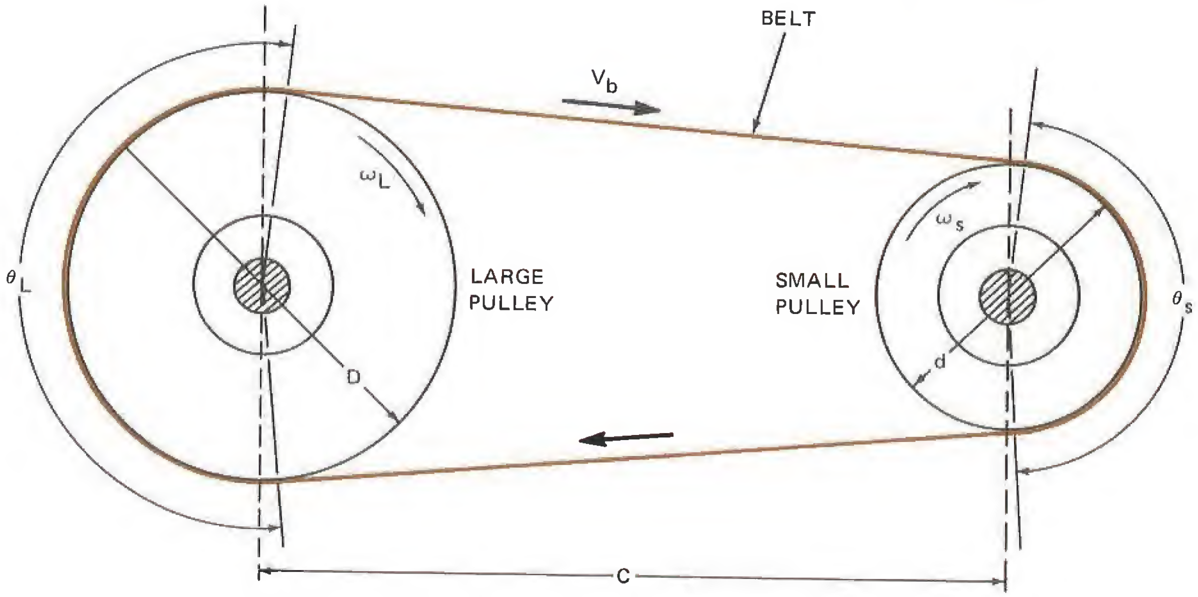


Fig. 22-6 Open Belt Drive

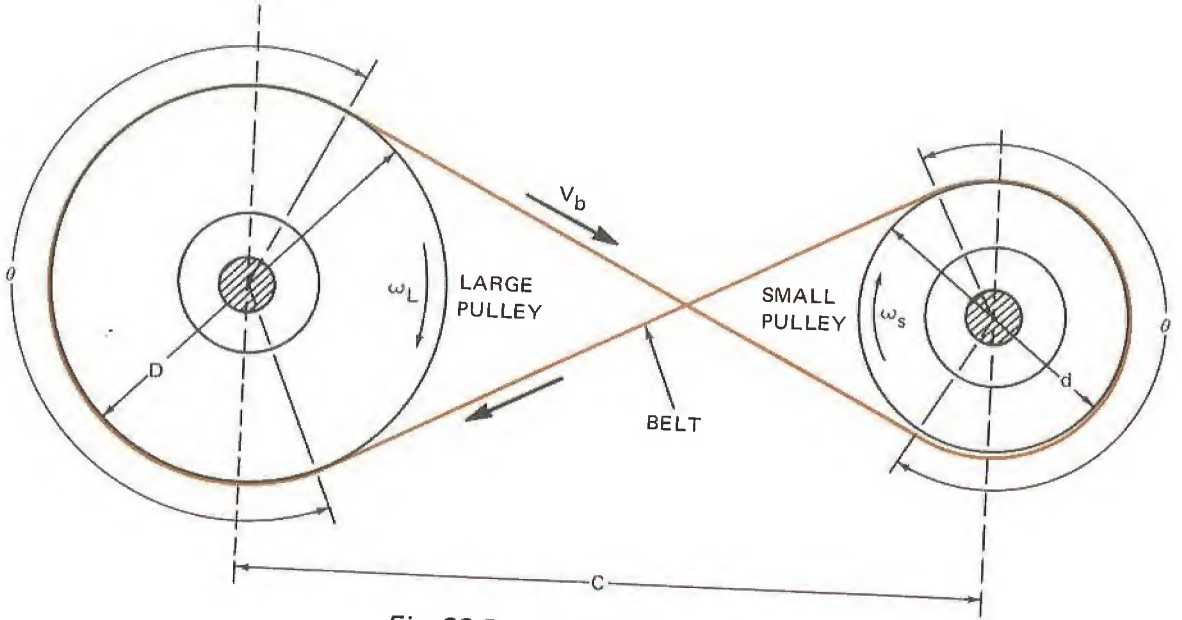


Fig. 22-7 Crossed Belt Drive

If we assume that there is no slippage in the belt, then the linear velocity of each pulley rim is equal to the belt velocity. Therefore, the rim velocities (linear) of the two pulleys are equal. The rim velocity in each case is

$$V_b = \omega R$$

where ω and R are the angular velocity and radius of the appropriate pulley. Or, in other words,

$$V_b = \omega_L R = 1/2\ \omega_L D$$

and at the same time

$$V_b = \omega_s r = 1/2\ \omega_s d$$

Equating these two quantities gives us

$$1/2\ \omega_L D = 1/2\ \omega_s d \quad \text{or} \quad \boxed{\frac{\omega_L}{\omega_s} = \frac{d}{D}} \quad 2.1$$

This equation gives us the pulley angular velocity ratio as a function of the diameters. Notice that this equation is very similar to the one for a gear but that both pulleys turn in the same direction.

You will recall that we assumed no belt slippage in deriving this relationship. In most practical cases there will be some slippage. It is usually reasonable to expect the follower pulley to run about 5% slower than the value indicated by equation 22.1.

In addition to the velocity ratio, we frequently need to know the belt length. It is normally practical to measure the pulley diameters and the center distance. With these quantities known the belt length can be found:

$$\boxed{L = \sqrt{4C^2 - (D - d)^2} + 1/2\,(D\theta_L + d\theta_s)} \quad \textbf{(22.2)}$$

Where θ_L and θ_s are the belt contact angles,

$$\theta_L = \pi + 2 \arcsin \frac{D - d}{2C} \quad \textbf{(22.3)}$$

and

$$\theta_s = \pi - 2 \arcsin \frac{D - d}{2C} \quad \textbf{(22.4)}$$

Pulleys are also frequently used with the *crossed belt* configuration shown in figure 22-7.

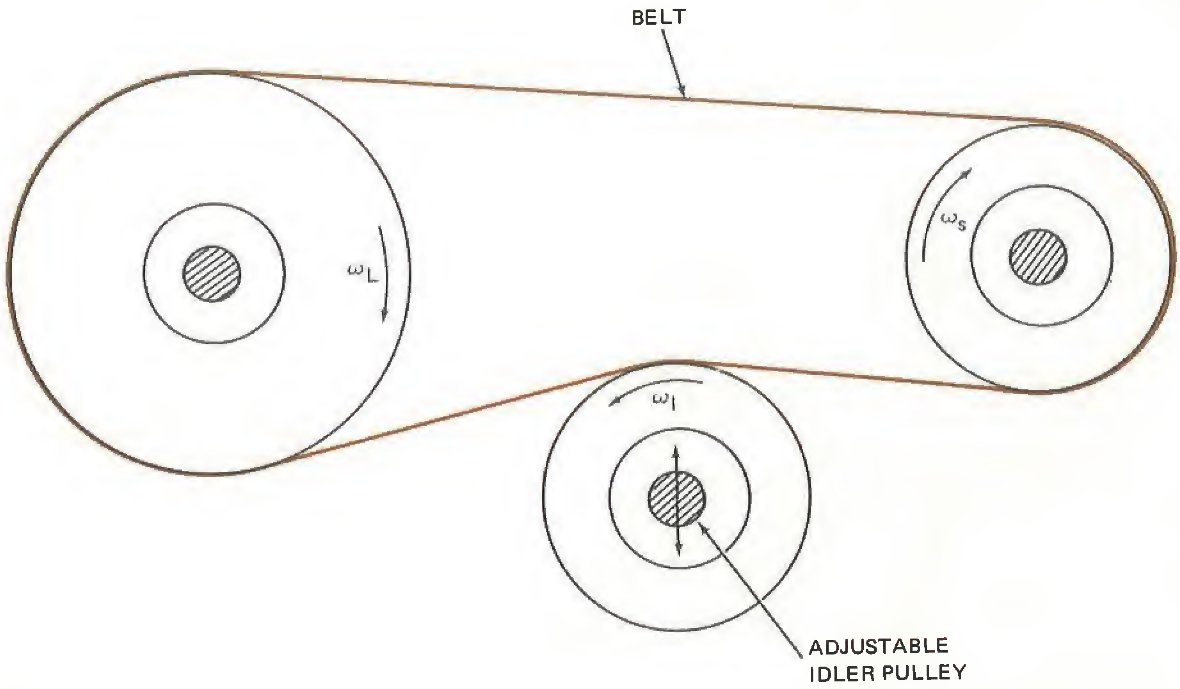


Fig. 22-8 Tightening a Belt with an Adjustable Idler Pulley

With a crossed belt, the velocity ratio is the same value, but since the pulleys turn in opposite directions we have

$$\boxed{\frac{\omega_L}{\omega_S} = -\frac{d}{D}} \qquad (22.5)$$

Also notice that the contact angles of both pulleys are greater for the crossed belt arrangement than they are for an open belt. As a result of the greater contact angle, the crossed belt configuration is able to handle somewhat larger loads.

The belt length required for a crossed belt drive is

$$\boxed{L = \sqrt{4C^2 - (D-d)^2} + 1/2\ \theta\ (D+d)} \qquad (22.6)$$

Where θ is the contact angle,

$$\theta = \pi + 2 \arcsin \frac{D+d}{2C} \qquad (22.7)$$

All belts tend to stretch with use. Therefore, a provision for tightening the belt must be included in belt drive designs. If the pulley center distance can be varied, then it may be used for belt tightening. Another common method is to use an adjustable idler pulley as shown in figure 22-8.

This arrangement has the additional advantage of increasing the contact angles. The idler pulley must, of course, be of a type suitable for use on the outside of the belt. It will normally be a flat pulley for flat or vee-belts and a round pulley for round belts.

MATERIALS

2 Bearing plates with spacers

4 Bearing mounts

4 Bearings

2 Shafts approx. 4″ X 1/4″

4 Collars

1 Shaft coupling

1 Large pulley, approx. 1 1/2 in. OD

1 Small pulley, approx. 1 in. OD

1 Suitable pulley belt

1 Breadboard with legs and clamps

1 Motor and mount

1 Power supply

1 Stroboscope

1 Dial caliper

1 Sheet of graph paper

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the bearing plate assembly shown in figure 22-9.
3. Mount the whole bearing plate assembly on the spring balance stand for additional stability.
4. Place the belt over the pulleys using the open belt configuration.

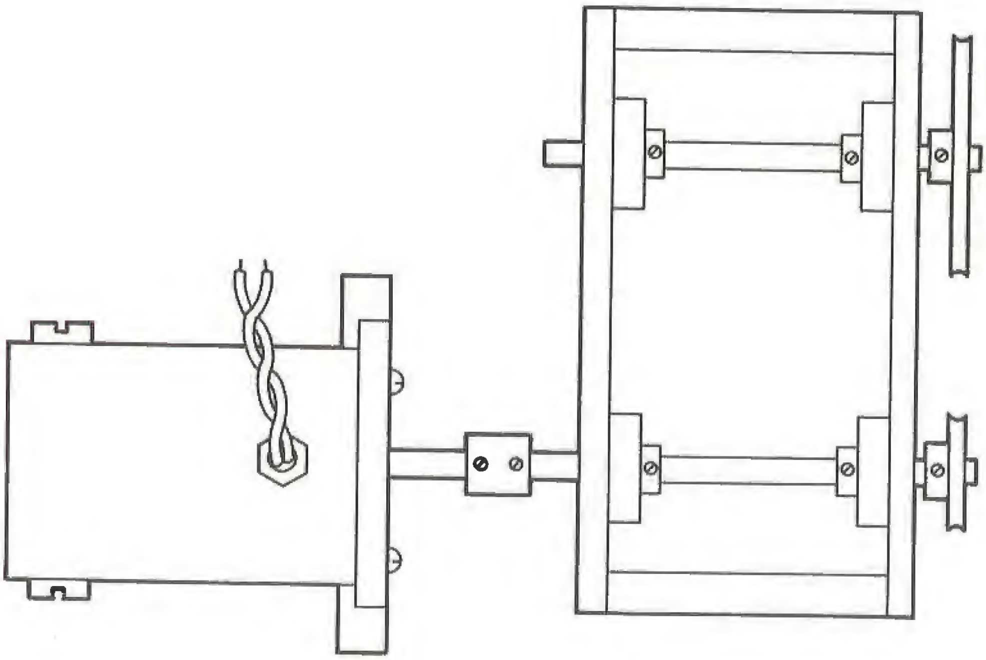


Fig. 22-9 The Experimental Mechanism

5. Adjust the center distance by moving the large pulley shaft until the belt is snug.
6. Connect the DC motor to the DC power supply and set the voltage to 28 volts.
7. Strobe the speed of both pulleys and record them (ω_S, ω_L).
8. Repeat step 7 for power supply voltages of 26, 24, 22, 20, 18, 16, 14, 12, and 10 volts.
9. Turn off the power supply and measure the diameter of each pulley. Record the results (D, d).

Motor Voltage (Approx.)	OPEN BELT		CROSSED BELT	
	ω_S	ω_L	ω_S	ω_L
28				
26				
24				
22				
20				
18				
16				
14				
12				
10				

d = ____________ D = ____________

Fig. 22-10 The Data Table

10. On a sheet of graph paper, plot the equation $\omega_L = \omega_s \frac{d}{D}$, using ω_s as the independent variable and ω_L as the dependent variable.
11. On the same sheet of graph paper plot the data taken in steps 7 and 8.
12. Remove the belt from the pulleys and reinstall it using the crossed belt configuration. Re-adjust the large pulley as necessary.
13. Repeat steps 6, 7, and 8. Record your results in the data table.
14. On the same sheet of graph paper, plot these new data points. Clearly identify each plot.

ANALYSIS GUIDE. In analyzing the results from this experiment, you should compare the measured data plots to that of the equation. Explain why they are different.

PROBLEMS

1. Pulley having diameters of 2 inches and 3.5 inches are mounted with their centers 8 inches apart. The smaller pulley is rotating at 1725 RPM. How fast is the larger pulley rotating? (ignore slippage)
2. What would be the result in problem 1 if the larger pulley is the follower and there is 5% slippage?
3. What would be the result in problem 1 if the smaller pulley is the follower and there is 5% slippage?
4. What is the length of the belt in problem 1 if the pulleys are being used in the open belt configuration?
5. What is the length of the belt in problem 1 if the pulleys are being used in the crossed belt configuration?
6. What are the contact angles in problem four?
7. What is the contact angle in problem five?
8. Discuss in your own words the effects that loose and tight belts would have on a belt drive.

experiment 23 PULLEY BLOCKS

INTRODUCTION. Pulley blocks are used in a variety of applications to lift a heavy load. In this experiment we shall examine the principles of operation of the simple pulley block.

DISCUSSION. Let's consider the simple pulley arrangement shown in figure 23-1.

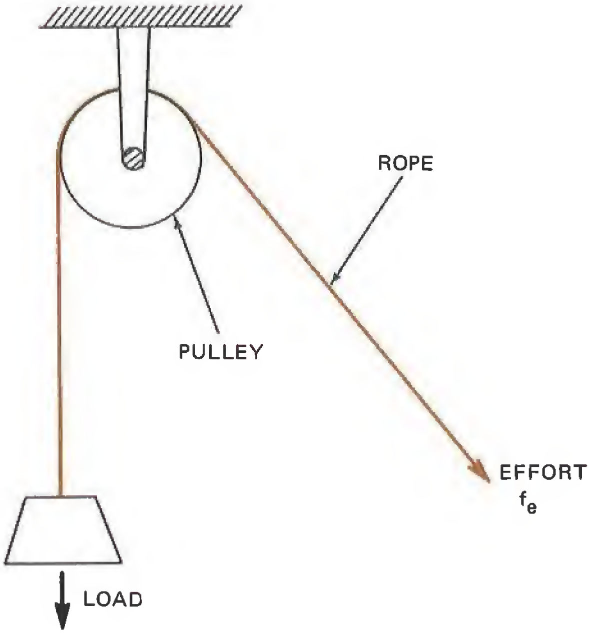


Fig. 23-1 A Simple Pulley and Load

In this simple arrangement, the effort pulling on the rope is equal to the load force (ignoring bearing friction),

$$f_e = F_L$$

and if we wish to move the load one foot, we must move the effort one foot. That is, the distance moved by the load (S_L) is equal to the distance moved by the effort S_e.

$$S_e = S_L$$

Moreover, since the two distances are covered in the same time interval (T), we have:

$$\frac{S_e}{T} = \frac{S_L}{T}$$

And since S divided by T is velocity we see that

$$V_e = V_L$$

That is, the load and effort move with equal velocities.

It would seem, then, that this simple pulley arrangement does nothing except change the direction of the action. That is, the load force and the effort are acting in different directions.

With this in mind, let's turn our attention to the simple two-pulley block shown in figure 23-2.

If we wish to lift the load one foot, then we must shorten the rope on *each* side of the lower pulley by a foot. To do this we must move the effort a total of two feet. Therefore, the distance relationship is:

$$S_e = 2S_L$$

And since the time of movement is the same for the load and the effort, the velocity relationship becomes

$$V_e = 2V_L$$

If we ignore any losses in the pulleys, the input and output work must be the same.

$$W_e = W_L$$

or

$$f_eS_e = F_LS_L$$

But since $S_e = 2S_L$, we may write:

$$2f_eS_L = F_LS_L$$

or

$$f_e = 1/2\ F_L$$

In other words, the effort force is only half of the load force. Comparing this result to the simple pulley we see that the pulley block not only changes direction, but also changes the force required to move the load.

It is possible to build a pulley block with any reasonable number of pulleys. Figure 22-3 shows several possibilities.

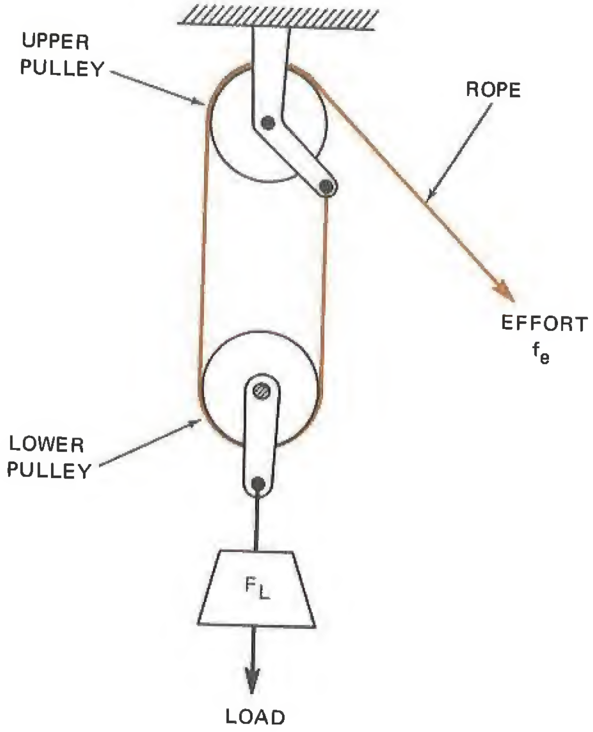
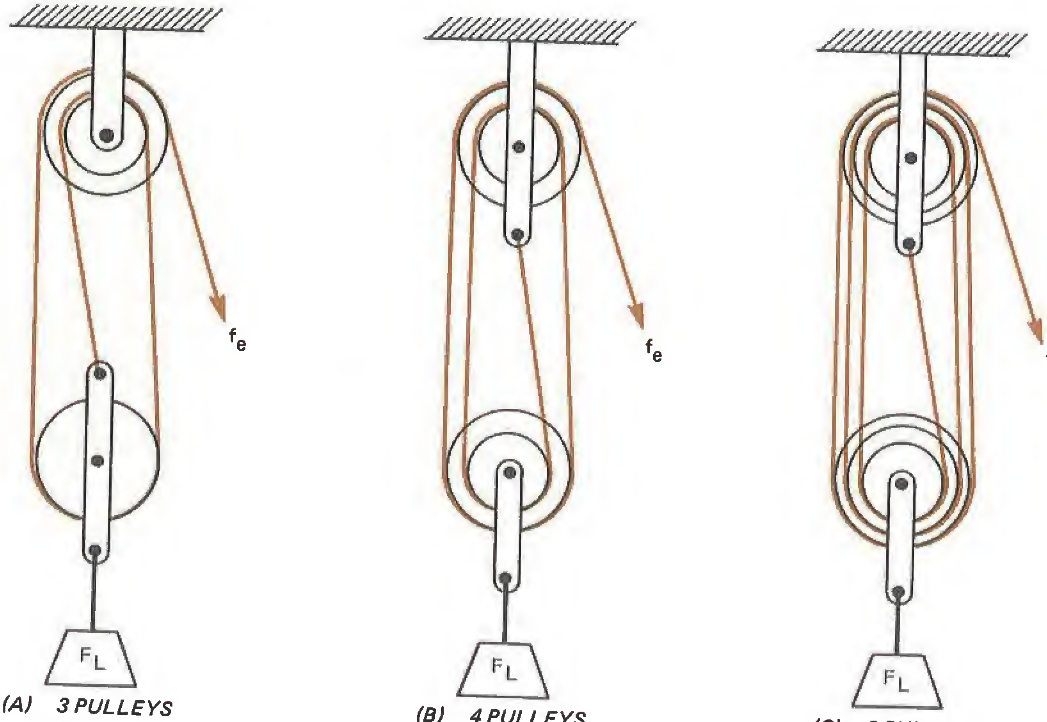


Fig. 23-2 A Two-Pulley Block Assembly



NOTE: ALL PULLEYS ARE USUALLY THE SAME DIAMETER. DIFFERENT DIAMETERS ARE USED HERE FOR DIAGRAMMATIC PURPOSES ONLY.

Fig. 23-3 Block With Multiple Pulleys

It is also possible to work out general relationships for distances, velocities, and forces. To do this we observe that, in this case, the number of ropes that must be shortened to move the load upward is always equal to the number of pulleys. Therefore, we have

$$\boxed{S_e = NS_L} \tag{23.1}$$

where N is the number of pulleys in the block. Also

$$\boxed{V_e = NV_L} \tag{23.2}$$

and

$$\boxed{f_e = \frac{1}{N} FL} \tag{23.3}$$

But the force relationship only holds for frictionless pulleys.

Actually, in most pulley blocks the pulley friction cannot be completely ignored. It can usually be accounted for most readily by measuring the input and output forces. For example, suppose that we have a six-pulley block and we wish to lift a 540-pound load. According to equation 23.3, the effort should be:

$$f_e = \frac{1}{N} FL = \frac{1}{6} \times 540 = 90 \text{ lb.}$$

Now further suppose that we find that it actually takes 100 pounds to raise the load. Knowing this we can calculate the efficiency of the block.

$$\eta = \frac{\text{Work done at load}}{\text{Work done by effort}}$$

$$\eta = \frac{F_L S_L}{f_e S_e}$$

$$\eta = \frac{F_L}{f_e} \times \frac{S_L}{S_e}$$

However, we see from equation 23.1 that

$$\frac{S_L}{S_e} = \frac{1}{N}$$

So the efficiency becomes

$$\boxed{\eta = \frac{F_L}{Nf_e}}$$

or in our example,

$$\eta = \frac{540}{6 \times 100} = 0.90$$

That is to say, the block in question is 90 percent efficient. In other words, 10 percent of the input work is lost in pulley friction.

Another way to look at equation 23.4 is to solve for the effort.

$$f_e = \frac{F_L}{N\eta} \tag{23.5}$$

which is a more practical relationship for effort in terms of the load, number of pulleys, and efficiency. Of course, this relationship only applies when we are raising the load.

When we are lowering the load, the friction works in the opposite direction. In such case the effort becomes the output and the load the input. We therefore have:

$$\eta = \frac{\text{Work done by effort}}{\text{Work done at load}}$$

$$\eta = \frac{f_e S_e}{F_L S_L}$$

$$\eta = N \frac{f_e}{F_L}$$

or

$$\boxed{f_e = \eta \frac{F_L}{N}} \qquad (23.6)$$

for the effort required to lower the load.

In the case of our example, this would be:

$$f_e = 0.90 \left(\frac{540}{6}\right) = 81 \text{ lb.}$$

Pulley blocks of various types have been in use for literally thousands of years. Today they are still used in a great variety of applications.

MATERIALS

- 2 Bearing plates with spacers
- 2 Bearing mounts
- 2 Bearings
- 2 Shafts, approx. 4″ X 1/4″
- 6 Collars
- 2 Pulleys, approx. 1 in. OD
- 2 Pulleys, approx. 2 in. OD
- 1 Piece of waxed string approx. 3 ft. long
- 1 Breadboard with legs and clamps
- 2 Spring balances with posts and clamps

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the bearing place assembly shown in figure 23-4.

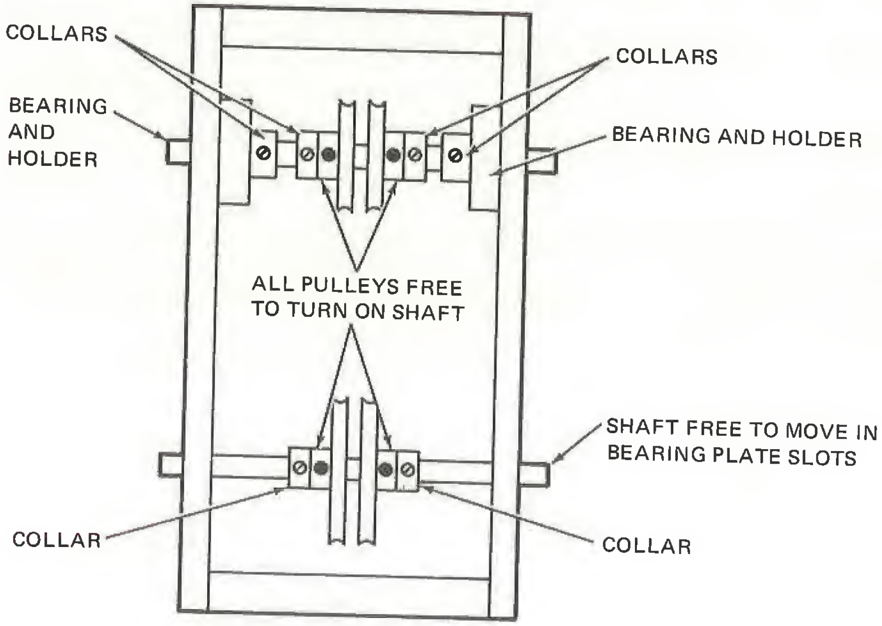


Fig. 23-4 The Bearing Plate Assembly

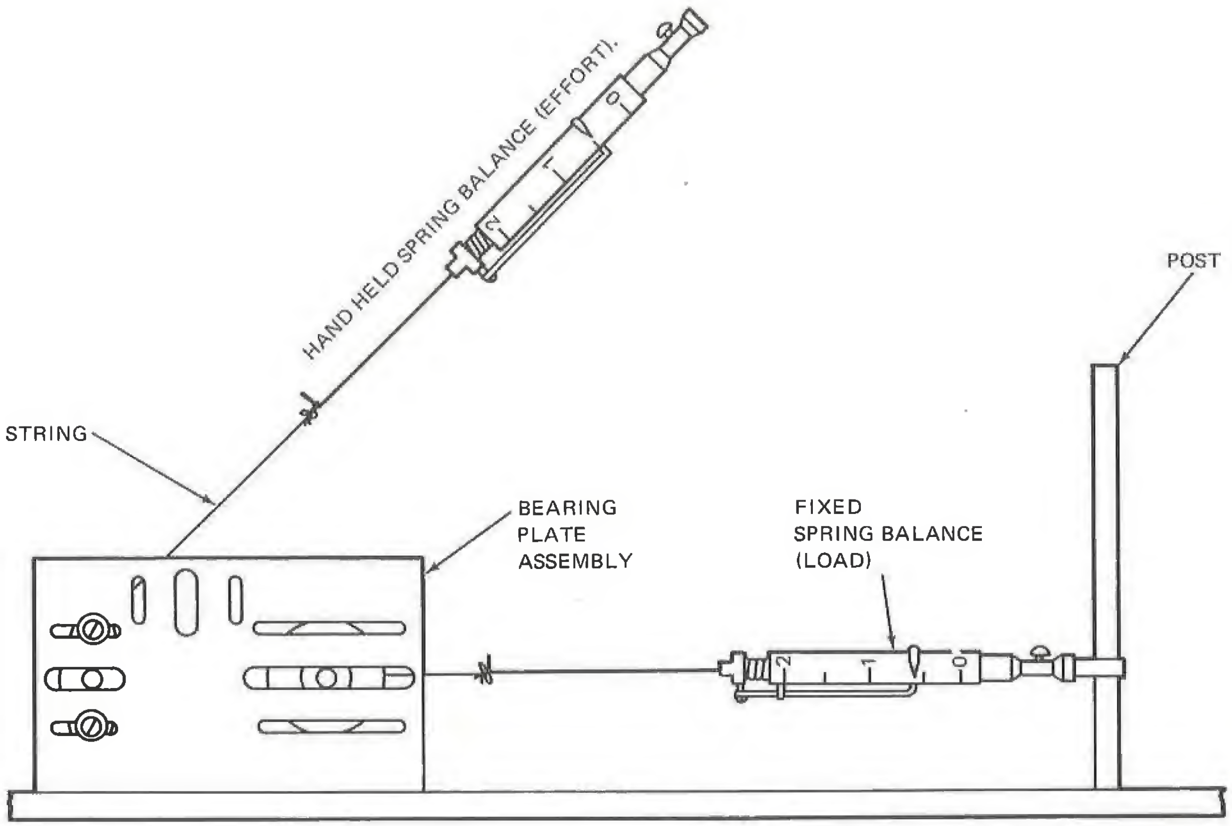


Fig. 23-5 The Experimental Setup

3. Mount the bearing plate assembly on the spring balance stand as shown in figure 23-5.
4. With a piece of string, rig the pulleys as a two-pulley block.
5. Pull the hand-held spring balance until the fixed spring balance reads about 8 oz. Record the readings of both spring balances (f_e, F_L).
6. Repeat step 5 for loads of about 6 oz, 4 oz, and 2 oz.
7. Now rig the pulleys as a three-pulley block and repeat steps 5 and 6.
8. Finally, rig the pulleys as a four-pulley block and repeat steps 5 and 6.
9. Calculate the ratio of load force (F_L) to effort (f_e) for each data set.
10. Record the number of pulleys used to get each data set.
11. Compute and record the pulley block efficiency for each set of data.

ANALYSIS GUIDE. In the analysis of these data you should discuss the differences observed between the various pulley blocks. Also discuss the efficiency of the various blocks. Do you feel that this is an accurate measure of efficiency? Why? Give several specific practical applications of a pulley block.

Two-Pulley Block					Three-Pulley Block					Four-Pulley Block				
F_L	f_e	$\frac{F_L}{f_e}$	N	η	F_L	f_e	$\frac{F_L}{f_e}$	N	η	F_L	f_e	$\frac{F_L}{f_e}$	N	η

Fig. 23-6 The Data Tables

PROBLEMS

1. A six-pulley block is lifting a 95-lb. load. The effort required is 40 lb. What is the efficiency of the pulley block?

2. How much effort must be supplied to lower the load in problem 1? Discuss the meaning of your results.

3. The effort supplied to a three-pulley block is 240 lb. If the block is 80 percent efficient, what is the load?

4. Suppose that we wish to build a five-pulley block which will not unwind when a one-pound effort is applied unless the load *exceeds* 100 lbs. What should the efficiency be?

5. We wish to lift a 790-lb. load using a 100-lb. effort. What is the minimum number of pulleys that we can use if the efficiency is to be 80 percent?

6. If the load in problem 5 must be lifted 3 feet and the pulleys all have a diameter of 4 in., what length of rope is necessary to make the block?

7. A pulley block is to be made from a group of pulleys. *Each* pulley added reduces the efficiency 5 percent. How many pulleys are needed to make the effort exceed the load force? (There are two answers.)

experiment 24 DIFFERENTIAL HOISTS

INTRODUCTION. Differential hoists have found wide acceptance in lifting applications. In this experiment we shall examine the principles of operation of the most common types of differential hoists.

DISCUSSION. Probably the most familiar differential hoist is the so-called *chain* hoist shown diagrammatically in figure 24-1.

In a differential hoist of this type the differential pulley at the top is composed of two parts: A large diameter (D) pulley, and a small diameter (d) pulley. The two pulleys are fixed together so that they turn together.

A chain is normally used in this type of differential hoist and the pulleys are "toothed" so that the chain cannot slip.

To raise the load we pull on the side of the chain that is looped over the large upper pulley. This causes both upper pulleys to turn through an angle θ. In doing this the effort moves through a distance S_e. This distance is related to the upper pulley rotation by

$$S_e = R\,\theta = 1/2\,D\,\theta \qquad \textbf{(24.1)}$$

At the same time the other side of the chain passes over the smaller upper pulley. And since this pulley also turns through the angle

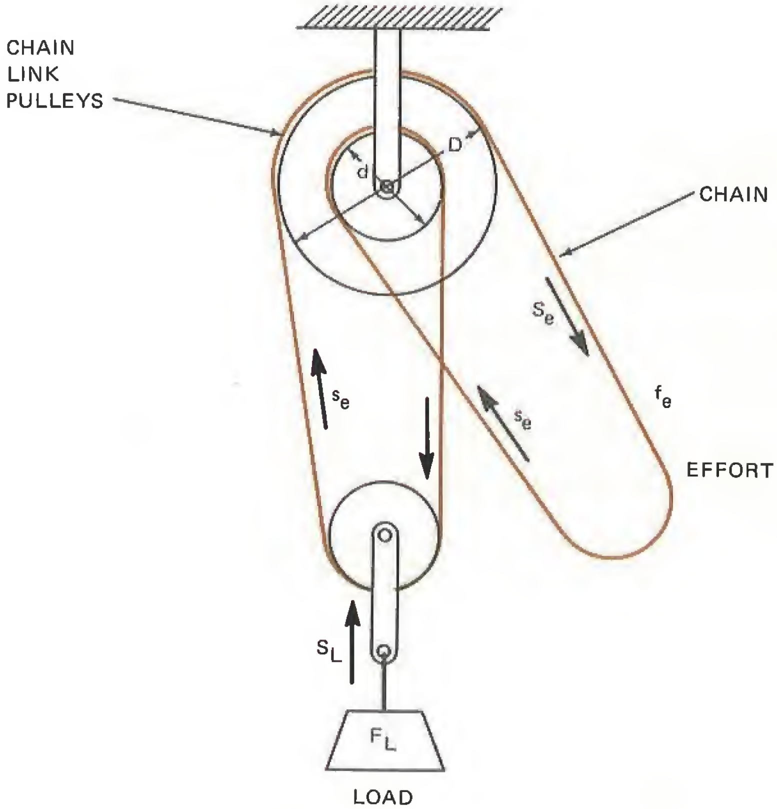


Fig. 24-1 Differential Chain Hoist

θ, the lower half of the chain loop moves a distance s_e given by

$$s_e = r\,\theta = 1/2\,d\theta \qquad (24.2)$$

Both of these movements are transmitted to the load. It *tends* to move up a distance S_e and down a distance s_e simultaneously. The total load movement (S_L) is half the difference in these two values since the total difference must be divided equally between the two supporting lines.

$$S_L = \frac{1}{2}(S_e - s_e) \qquad (24.3)$$

With these three equations (24.1, 2, 3) we can determine the ratio of S_e to S_L starting with the identity

$$\frac{S_e}{S_L} = \frac{S_e}{S_L}$$

Then substituting equation 24.1 for S_e and equation 24.3 for S_L on the right gives us

$$\frac{S_e}{S_L} = \frac{\frac{1}{2}D\,\theta}{\frac{1}{2}(S_e - s_e)}$$

And substituting equation 24.1 and equation 24.2 for S_e and s_e respectively on the right side, we have

$$\frac{S_e}{S_L} = \frac{D\,\theta}{\frac{1}{2}D\theta - \frac{1}{2}d\,\theta}$$

And if we factor and cancel the common terms, we will get

$$\boxed{\frac{S_e}{S_L} = \frac{2D}{D - d}} \qquad (24.4)$$

It is usually easier to count the chain link teeth than to measure pulley diameters. But since the number of teeth and the diameters are proportional, equation 24.4 may be written as

$$\boxed{\frac{S_e}{S_L} = \frac{2N}{N - n}} \qquad (24.5)$$

with equal validity.

In equations 24.4 and 24.5 you should notice that if the diameters (or tooth counts) are nearly equal, the ratio of S_e to S_L becomes very large.

Chain hoists of this type tend to be rather inefficient. Efficiencies of 30 to 40 percent are quite typical. We can calculate efficiency (η) by recalling that

$$\eta = \frac{\text{work done at load}}{\text{work done by effort}}$$

when we are raising the load. And since work is force times the distance in each case,

$$\eta = \frac{F_L S_L}{f_e S_e}$$

or

$$\eta = \left(\frac{F_L}{f_e}\right)\frac{N - n}{2N} \qquad (24.6)$$

Solving for f_e gives us

$$\boxed{f_e = \frac{1}{2}\frac{F_L}{\eta}\left(\frac{N - n}{N}\right)} \qquad (24.7a)$$

or, in terms of the diameters,

$$\boxed{f_e = \frac{1}{2}\frac{F_L}{\eta}\left(\frac{D - d}{D}\right)} \qquad (24.7b)$$

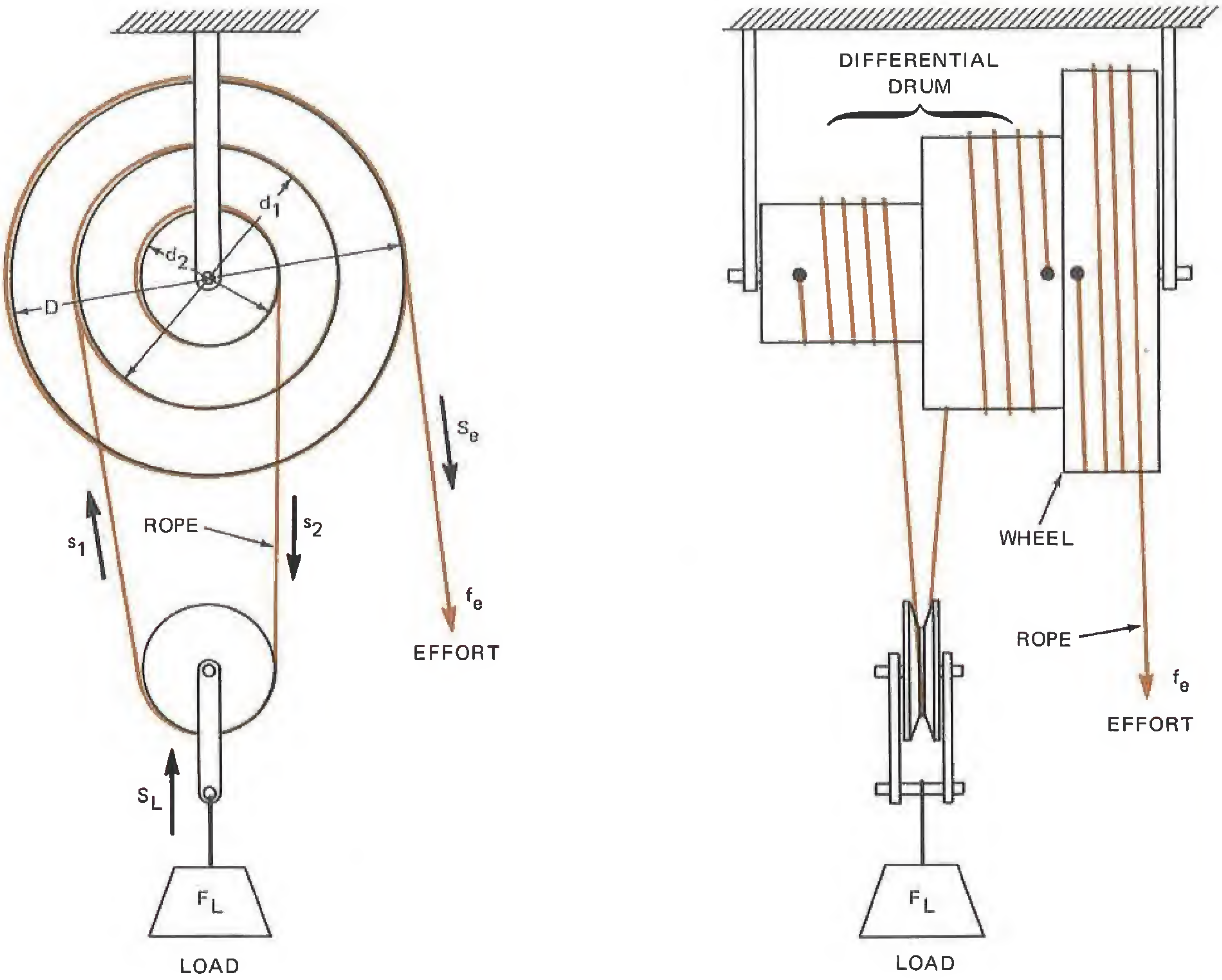


Fig. 24-2 A Wheel and Differential Drum

The hand-operated chain hoist is used very widely for lifting loads up to about 1000 pounds. However, because of their inefficiencies, they are most widely used for lighter loads.

Electric hoists are probably the "lifting" work horse of industry. These hoists usually employ the wheel and differential drum mechanism. Figure 24-2 shows a simplified diagram of such a mechanism. In this diagram the wheel is shown being driven by a rope. In a practical hoist, this wheel would be a gear driven by an electric motor.

The operation of the differential drum is the same as for the differential chain hoist. The distance moved by the effort line is

$$S_e = \frac{1}{2} D \, \theta$$

while the distance moved by the load is

$$S_L = \frac{1}{2}(s_1 - s_2)$$

where

$$s_1 = \frac{1}{2} d_1 \, \theta \text{ and } s_2 = \frac{1}{2} d_2 \theta$$

Combining these relationships as before, we have

$$\frac{S_e}{S_L} = \frac{2D}{d_1 - d_2} \qquad (24.8)$$

and the efficiency is

$$\eta = \frac{F_L}{f_e}\left(\frac{d_1 - d_2}{2D}\right) \qquad (24.9)$$

while the effort is

$$f_e = \frac{1}{2}\frac{F_L}{\eta}\left(\frac{d_1 - d_2}{D}\right) \qquad (24.10)$$

Hoists of this type may be found for use with loads from 500 pounds to well over 20,000 pounds. They are usually mounted on overhead trolleys and can be used for lifting and conveying loads from point to point.

MATERIALS

- 2 Bearing plates with spacers
- 2 Bearing mounts
- 2 Bearings
- 2 Shafts approx. 4" X 1/4"
- 2 Pulley approx. 1 in. OD
- 1 Pulley approx. 1 1/2 in. OD
- 1 Pulley approx. 2 in. OD
- 1 Breadboard with legs and clamps
- 2 Spring balances with posts and clamps
- 1 Dial caliper
- 1 Piece of waxed string approx. 3 feet long
- 2 Collars

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Measure and record the diameter of each flat pulley.
3. Assemble the bearing plate assembly as indicated in figure 24-3.

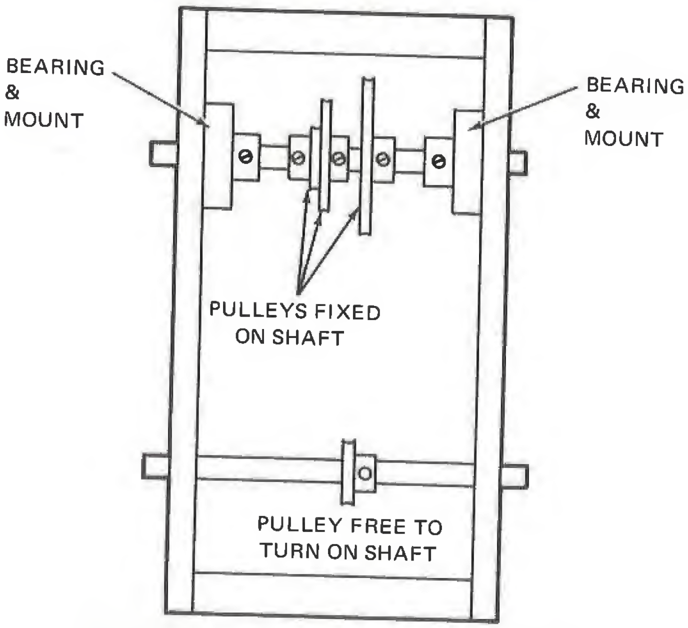


Fig. 24-3 The Bearing Plate Assembly

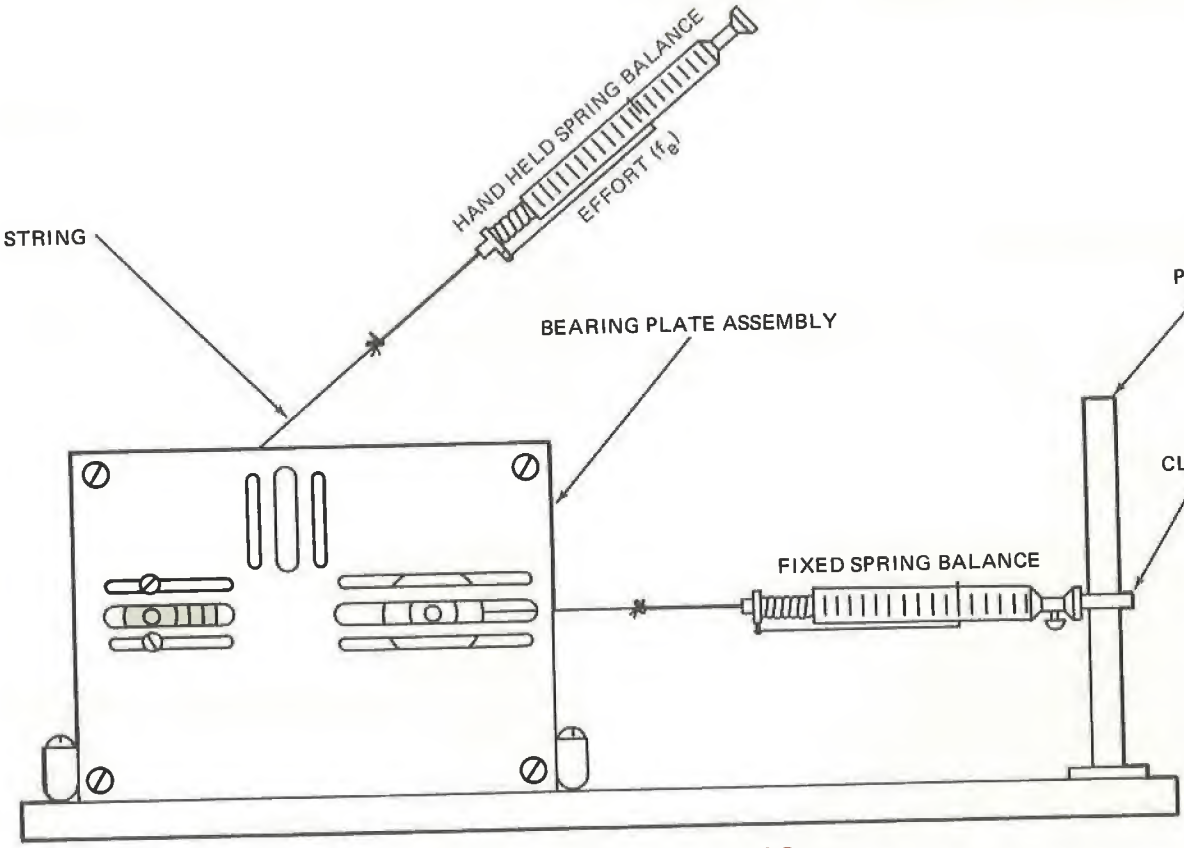


Fig. 24-4 The Experimental Setup

4. Mount the bearing plate assembly on the spring balance stand as shown in figure 24-4.
5. Using the string, rig the pulleys as a wheel and differential drum.
6. Pull on the hand-held spring balance until the fixed spring balance reads about 1 ounce. Record the reading of both spring balances (f_e and F_L).
7. Repeat step 6 for loads of about 203, 303, and 403 lbs.
8. Calculate the ratio of F_L to f_e for each of the pairs of measured data.
9. Compute the efficiency of the mechanism for each data point.
10. Starting from some convenient reference, measure the distance (S_e). The effort must move to cause a load movement (S_L) of about 1/2 inch. Record both values.
11. Compute the ratio of S_e to S_L.
12. Using the measured diameters, compute the value of $20/(d_1 - d_2)$.

ANALYSIS GUIDE. In the analysis of these data you should discuss any variations you observed in the values of efficiency. Also, compare the ratio of the distances to the differential diameter ratio. Finally, you should consider the extent to which your observations agree with the material presented in the discussion.

F_L	f_e	$\frac{F_L}{f_e}$	η

D	d_1	d_2	S_e	S_L	$\frac{S_e}{S_L}$	$\frac{2D}{d_1-d_2}$

Fig. 24-5 The Data Table

PROBLEMS

1. A differential chain hoist has 42 and 38 teeth respectively on its upper pulleys. What effort is required to raise a 790-pound load if the efficiency is 35 percent?
2. In problem 1, what effort is required to lower the load?
3. Explain why the answers to problems 1 and 2 are not the same.
4. A wheel and differential drum hoist has diameters of 12, 8, and 6 inches. What is the efficiency of a 10-pound effort that is needed to lift a 96-pound load?
5. Explain the effect of using a 1/2 inch diameter wire rope on the hoist in problem 4.
6. An effort of 175 pounds is just able to support a load on a differential chain hoist. If the effective pulley diameters are 12 and 10 inches while the efficiency is 33 percent, what is the size of the load?

experiment 25 CHAIN DRIVES

INTRODUCTION. Chain drives are employed in applications where the center distances are too large for economical gearing, and belt slippage cannot be tolerated. In this experiment we shall examine the basic principles of chain drives.

DISCUSSION. There are several different types of drive chains. Each type has its own special features and is well suited to its own particular application. Figure 25-1 shows a few of the most common types of drive chains.

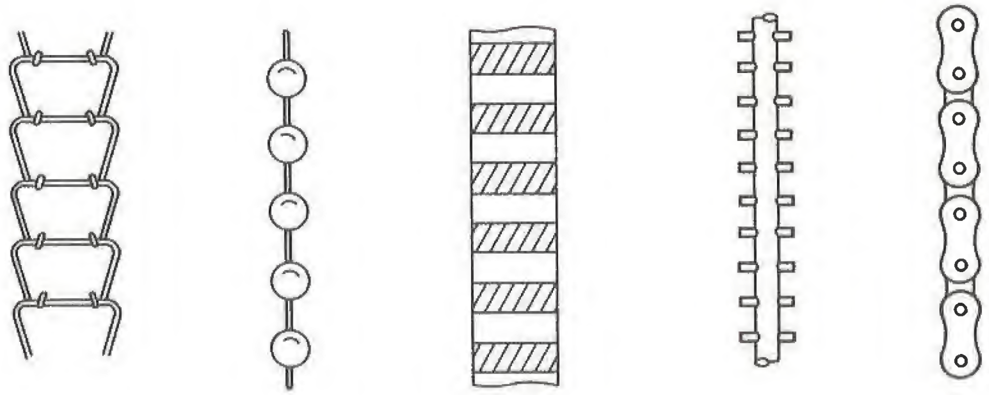
The wire ladder chain has the advantage of being very economical. It is, however, quite difficult to splice and is normally not considered suitable for precision applications. Sprockets for ladder chains may be either metal or plastic.

Bead chains normally have metal stamped beads. They can also be made with molded plastic beads. While the bead chain is very economical, the dimpled sprocket wheels used with them can be quite expensive. The sprockets may be either metal or plastic.

Cog belts may be constructed from metal links or molded in flexible rubber-like materials. Molded cog belts cannot normally be spliced and must, therefore, be purchased in exact sizes. Some means of preventing the belt from "walking" off of the side of the sprocket wheels is necessary.

Toothed belts are suitable for precision drives and have the added advantage that the sprockets can be meshed directly with a spur gear. These belts can be meshed with a sprocket on either side of the belt; and like the bead chain, they can bend in any direction. They cannot normally be spliced and must, therefore, be purchased in exact sizes. Sprockets may be of either metal or plastic.

Roller chains are perhaps the most commonly encountered chain drives. They may be used for virtually any kind of application from precision instruments to power transmission. The chains are made up of alternate roller and pin links. A sketch of each type of link is shown in figure 25-2.



(A) LADDER CHAIN (B) BEAD CHAIN (C) COG BELT (D) TOOTHED BELT (E) ROLLER CHAIN

Fig. 25-1 Common Types of Drive Chains

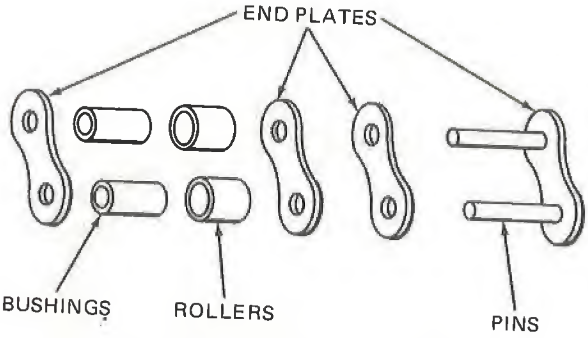


Fig. 25-2 Roller Chain Parts

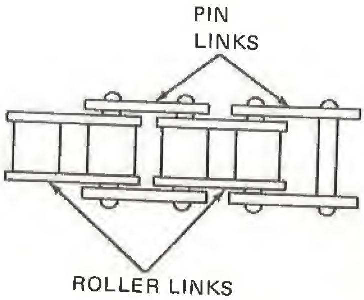


Fig. 25-3 Roller Chain Assembly

When the chain is assembled, the links are connected alternately as shown in figure 25-3.

The ends of a roller chain are usually joined with a special link called a *master link.* Figure 25-4 shows a sketch of such a link.

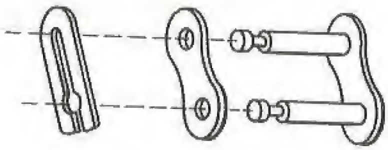


Fig. 25-4 A Roller Chain Master Link

The pitch (p) of a roller chain is the distance between the centers of the rollers. This value, as well as the roller width and diameter, has been standardized. American standard roller chain dimensions are available in most mechanical engineering textbooks and handbooks. They will not, therefore, be included here.

When a roller chain meshes with a sprocket, the pitch diameter (D) can be defined as the diametrical distance across the circle inscribed through the centers of completely seated rollers. Figure 25-5 shows the pitch diameter of a roller chain and sprocket. Figure 25-5 also shows the relationship between the pitch (p), pitch angle (γ), and pitch diameter (D). From the right triangles we see that

$$\sin\frac{r}{2} = \frac{P/2}{D/2} = P/D \qquad \textbf{(25.1)}$$

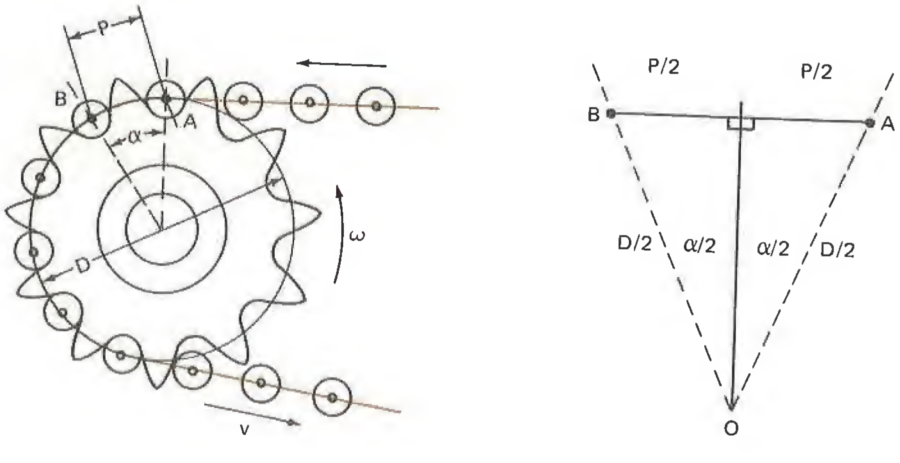


Fig. 25-5 A Roller Chain and Sprocket

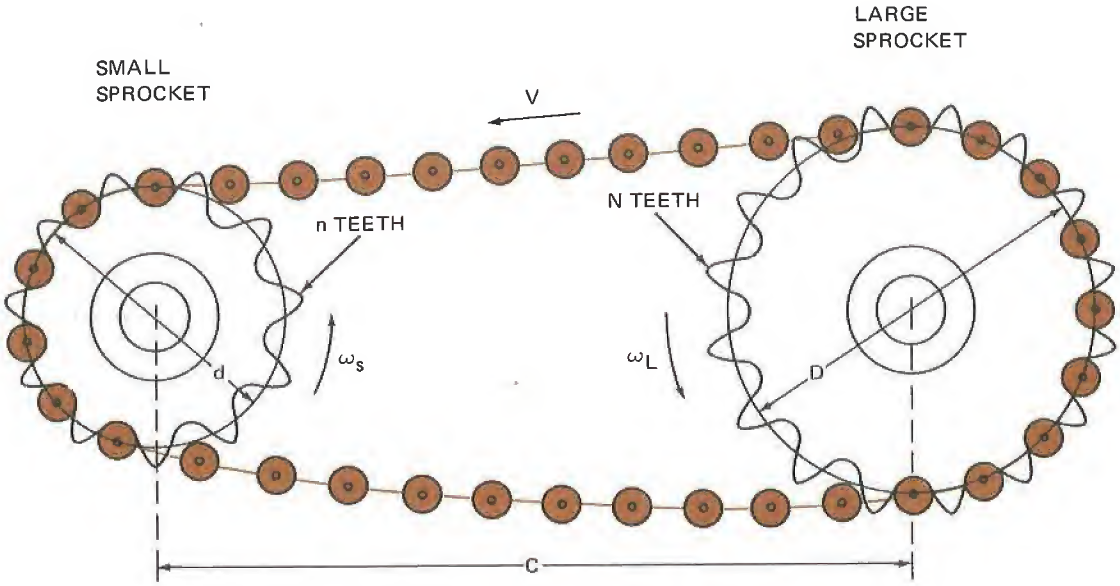


Fig. 25-6 A Chain Drive

However, we notice that the pitch angles (γ) is simply the angular displacement between two adjacent teeth. It must therefore be equal to

$$\gamma = \frac{360^\circ}{N}$$

where N is the number of teeth. As a result, $\gamma/2$ must be

$$\gamma/2 = \frac{1}{2}\left(\frac{360^\circ}{N}\right) = \frac{180^\circ}{N}$$

and equation 25.1 becomes

$$\sin\frac{180^\circ}{N} = P/D$$

If we solve for the pitch diameter we have

$$\boxed{D = \frac{P}{\text{Sin}\,\frac{180^\circ}{N}}} \qquad \textbf{(25.2)}$$

which is a very useful expression for finding the pitch diameter of a sprocket.

When two sprockets are coupled by a chain, as shown in figure 25-6, we can find the angular displacement and velocity ratios in much the same way as we do with spur gears.

Let us suppose that the small sprocket in figure 25-6 is the input and the large sprocket is the output. Each time the small sprocket rotates one tooth width, the large sprocket must also rotate one tooth width. In the case of the small sprocket, the tooth moves through an arc length of

$$s = r\ \theta_s = \frac{d}{2}\,\theta_s$$

Similarly, the large sprocket moves through an arc length of

$$s = R\ \theta_L = \frac{D}{2}\,\theta_L$$

Since these two arc lengths are equal, we have

$$\frac{d}{2}\,\theta_s = \frac{D}{2}\,\theta_L$$

or

$$d\ \theta_s = D\ \theta_L \qquad (25.3)$$

Therefore,

$$\boxed{\frac{d}{D} = \frac{\theta_L}{\theta_s}} \qquad (25.4)$$

Then, since both the sprockets rotate for the same length of time, we can divide equation 25.3 by t and have

$$d\ \frac{\theta_s}{t} = D\ \frac{\theta_L}{t}$$

However, since θ/t is the same as angular velocity, we may write

$$d\ \omega_s = D\ \omega_L$$

or

$$\boxed{\frac{d}{D} = \frac{\omega_L}{\omega_s}} \qquad (25.5)$$

If the small sprocket in figure 25-6 rotates through an angle θ_s, then the large sprocket must turn through an angle θ_L such that

$$\theta_L = \theta_s \frac{n}{N}$$

where n and N are the small and large sprocket tooth counts respectively. We can rewrite this relationship as

$$\boxed{\frac{n}{N} = \frac{\theta_L}{\theta_s}} \qquad (25.6)$$

If we apply a load torque to the output shaft of T_L, then it is reflected as a pitch circle force of

$$f_L = \frac{T_L}{D/2}$$

This force is transmitted through the chain to the smaller sprocket's pitch circle. At the input, the torque supplied by the source must be such that

$$f_s = \frac{T_s}{d/2}$$

If we ignore chain losses, the value of f_L and f_s must be equal,

$$f_L = f_s$$

Consequently,

$$\frac{T_L}{D/2} = \frac{T_s}{d/2}$$

or

$$\boxed{\frac{d}{D} = \frac{T_s}{T_L}} \qquad (25.7)$$

Summarizing equations 25.4 through 25.7, we have

$$\boxed{\frac{d}{D} = \frac{n}{N} = \frac{\theta_L}{\theta_s} = \frac{\omega_L}{\theta_s} = \frac{T_s}{T_L}} \qquad (25.8)$$

as the performance ratios of a chain drive.

Determining the length of chain for use with a given center distance and sprocket ratio presents a number of interesting problems. The chain should, whenever possible, have an even number of links because of the alternate roller, pin link construction. Moreover, there should be some small amount of slack to prevent binding and unnecessary wear.

An approximate length (L) can be obtained with

$$\boxed{\frac{L}{P} = \frac{2C}{P} + \frac{n+N}{2} + \frac{P}{C}\left(\frac{N-n}{2\pi}\right)^2} \qquad (25.9)$$

where
L = length of the chain
P = chain pitch
C = center distance
N = large sprocket tooth count
n = small sprocket tooth count

It is a sensible practice to purchase roller chains with several more links than is indicated by equation 25.9. Notice that the ratio L/P is, in fact, the number of links in the chain. It is often convenient to order chain by the number of links desired rather than by length in inches.

Also, it is good practice to make the center distance adjustable whenever possible. Finally, an idler sprocket can be used to provide some chain length adjustment.

MATERIALS

2 Bearing plates with spacers

4 Bearing mounts

4 Bearings

2 Shafts approx. 4" X 1/4"

4 Collars

1 Motor and mount

1 Shaft coupling

1 Power supply

1 Stroboscope

1 Dial caliper

1 Sprocket wheel approx. 1 in. OD

1 Sprocket wheel approx. 2 in. OD

1 Roller chain approx. 10 in. long

1 Breadboard with legs and clamps

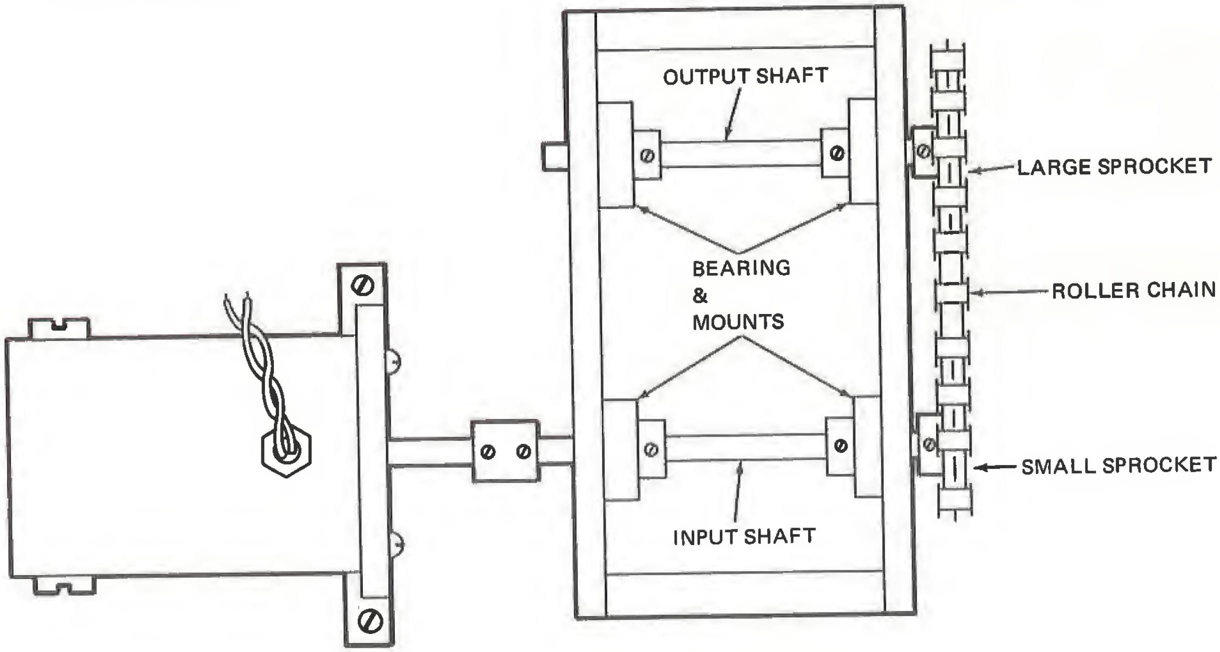


Fig. 25-7 The Bearing Plate Assembly

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Count the number of teeth on each sprocket wheel and record the results.
3. Measure or otherwise determine the chain pitch. Record the results.
4. Assemble the bearing plate assembly shown in figure 25-7.
5. Adjust the location of the output shaft so that the chain has about 1/4 inch of freedom.
6. Mount the whole bearing plate assembly on the spring balance stand for additional stability.
7. Connect the motor to the DC power supply.
8. Set the motor voltage to approximately 12 volts. Strobe both sprockets and record the results. (ω_S and ω_L)
9. Repeat step 8 for power supply voltages of about 14, 16, 18, 20, 22, 24, and 26 volts.
10. Turn off the power supply and measure the sprocket center distance (c).

P	n	N	$\frac{N}{n}$

c	d	D	$\frac{D}{d}$

L Meas.	L Comp.

ω_S Meas.	ω_L Meas.	$\frac{\omega_S}{\omega_L}$
Average $\frac{\omega_S}{\omega_L}$ →		

Fig. 25-8 The Data Table

11. Remove the chain and measure its total length (L).
12. Calculate the pitch diameter of each sprocket wheel. (d and D)
13. Calculate the ratio of the sprocket pitch diameters.
14. Compute the ratio of the sprocket tooth counts.
15. Calculate the approximate chain length using equation 25.9.
16. For each set of measured data, compute the ratio of ω_S to ω_L.
17. Compute the average value of all the measured velocity ratios.

ANALYSIS GUIDE. In the analysis of these data you should investigate the extent to which your results agreed with the material presented in the discussion. To do this you can compare the pitch diameter ratio and the tooth count ratio to the average velocity ratio. You can also compare the two chain length values.

PROBLEMS

1. Use an American standard roller chain table to find the pitch of American standard chains number 25, 40, 80, and 120.
2. A 21-tooth sprocket is used to drive a 38-tooth sprocket with miniature pitch (0.1475 in.) chain. If the large sprocket is turning 940 RPM, how fast is the small sprocket turning?
3. What is the pitch diameter of each of the sprockets in problem 2?
4. The center distance in problem 2 is 2.45 in. What is the approximate chain length?
5. If the output torque in problem 2 is 165 in.-oz., what is the input torque? (Ignore chain losses).
6. What is the input power in problem 2? (Give your answer in horsepower).
7. Which of the chains in figure 25-1 can be crossed to make the sprockets turn in opposite directions?

experiment 26 TOOTHED BELTS

INTRODUCTION. Some mechanical drive applications require some of the characteristics of a chain or gear wheel *and* the flexibility of a belt. The toothed belt is one way to satisfy both these requirements. In this experiment we shall examine the operating characteristics of one type of toothed belt.

DISCUSSION. Perhaps the main shortcoming of a belt drive is its inherent slippage. The toothed belt shown in figure 26-1 effectively overcomes this problem.

Pulleys suitable for use with toothed belts must, of course, be specially designed for it. As you can see in figure 26-1, the pulleys are a sort of compromise between a round belt pulley and a spur gear.

In fact, the pulley teeth are involute-shaped and will mesh satisfactorily with a spur gear of appropriate pitch. Figure 26-2 shows a toothed belt drive that is, in fact, being driven by a spur gear.

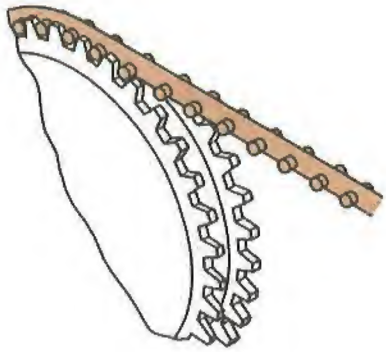


Fig. 26-1 A Toothed Belt and Pulley

Also shown in figure 26-2 is an idler pulley being used to take up belt slack. It is worth mentioning that the idler pulley can be used to drive a light load. With ordinary belt drives this is usually not the case.

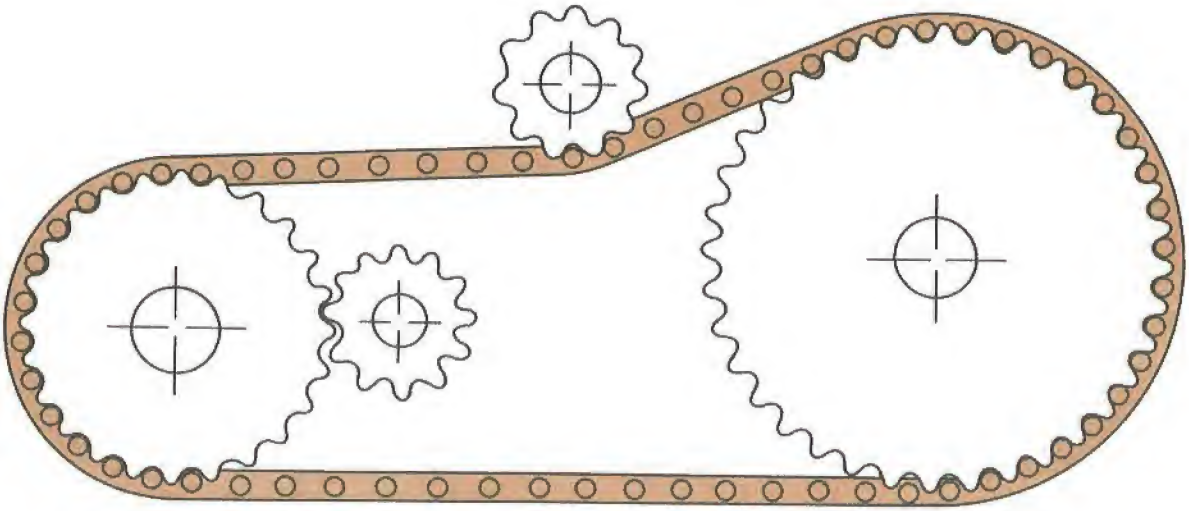


Fig. 26-2 A Toothed Belt Drive

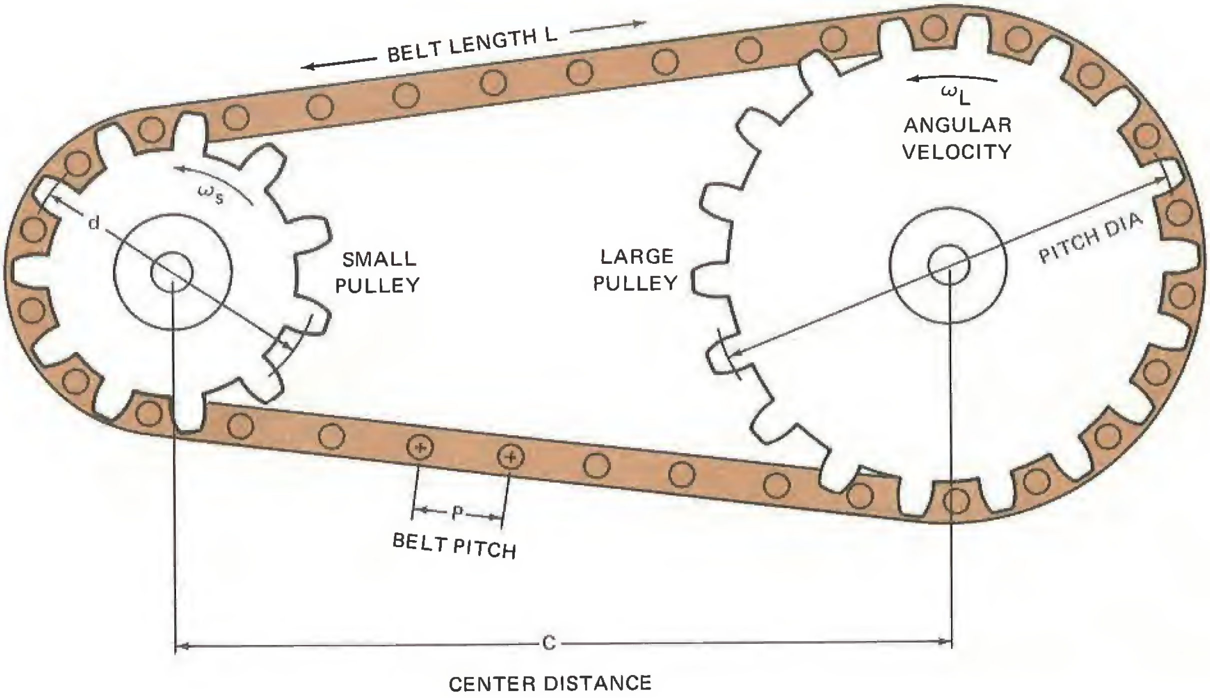


Fig. 26-3 Open Belt Configuration

At this point let's turn our attention to the open belt configuration shown in figure 26-3. When one of the pulleys, say the smaller one, turns through an angle θ_s, the belt moves through an arc length, given by

$$\ell = r\theta_s = \frac{d}{2}\theta_s$$

Since the belt is *positively coupled* to the larger pulley, its pitch circle moves through this same arc length. As a result, the large pulley turns through an angle θ_L such that

$$\ell = R\theta_L = \frac{D}{2}\theta_L$$

Equating these two relationships for ℓ gives us

$$\frac{d}{2}\theta_s = \frac{D}{2}\theta_L$$

or

$$\boxed{\frac{d}{D} = \frac{\theta_L}{\theta_s}} \qquad (26.1)$$

We should observe two things at this point. First, both pulleys turn in the same direction. Second, the quantities d and D are the *pitch* diameters of the pulleys, not the ODs.

Since the teeth on the pulleys are involute-shaped, pitch diameter may be found in the way as for a spur gear.

$$d = D_o \frac{N}{N+2} \qquad (26.2)$$

where d is the pitch diameter.
D_o is the outside diameter.
N is the tooth count.

Returning to equation 26.1, we observe that both pulleys turn through their respective angles during the same time interval. We may, therefore, divide each angle by the time interval t.

$$\frac{d}{D} = \frac{\theta_L/t}{\theta_s t}$$

And since θ/t is, by definition, the angular velocity of the respective pulleys, we have

$$\boxed{\frac{d}{D} = \frac{\omega_L}{\omega_s}} \qquad \textbf{(26.3)}$$

Any torque T_L acting on the larger pulley results in a pitch circle force of

$$f = \frac{T_L}{R} = 2\frac{T_L}{D}$$

This force is transmitted directly to the smaller pulley where it produces a torque T_s of

$$T_s = f_r = f\frac{d}{2}$$

Solving for the force in terms of torque renders

$$f = 2\frac{T_s}{d}$$

Since the two forces are identical we may equate the force relationships,

$$2\frac{T_L}{D} = 2\frac{T_s}{d}$$

or

$$\boxed{\frac{d}{D} = \frac{T_s}{T_L}} \qquad \textbf{(26.4)}$$

Summarizing the relationships from equations 26.1, 26.3, and 26.4, we have

$$\boxed{\frac{d}{D} = \frac{\theta_L}{\theta_s} = \frac{W_L}{W_s} = \frac{T_s}{T_L}} \qquad \textbf{(26.5)}$$

as the performance ratios for the toothed-belt drive.

The length of the toothed belt may be found in the usual way with the equation

$$\boxed{\begin{aligned} L &= \sqrt{4C^2 - (D-d)^2} \\ &+ \frac{1}{2}(D\theta_1 - d\theta_2) \end{aligned}} \qquad \textbf{(26.6)}$$

Where: L is the belt length.
C is the center distance.
d is the smaller pitch diameter.
D is the larger pitch diameter.
θ_1 is the larger belt contact angle.

$$\theta_1 = \pi + 2 \text{ arc sin } \frac{D-d}{2C}$$

θ_2 is the smaller belt contact, angle.

$$\theta_2 = \pi - 2 \text{ arc sin } \frac{D-d}{2C}$$

Toothed belts come in a variety of stock sizes. The equation given provides the minimum length to be used. In most cases it will be necessary to use the next larger stock size. Slack may be taken up either by adjusting the center distance or with an idler pulley. The use of an idler for this purpose is shown in figure 26-2.

Toothed belts may be used in a crossed belt drive as shown in figure 26-4.

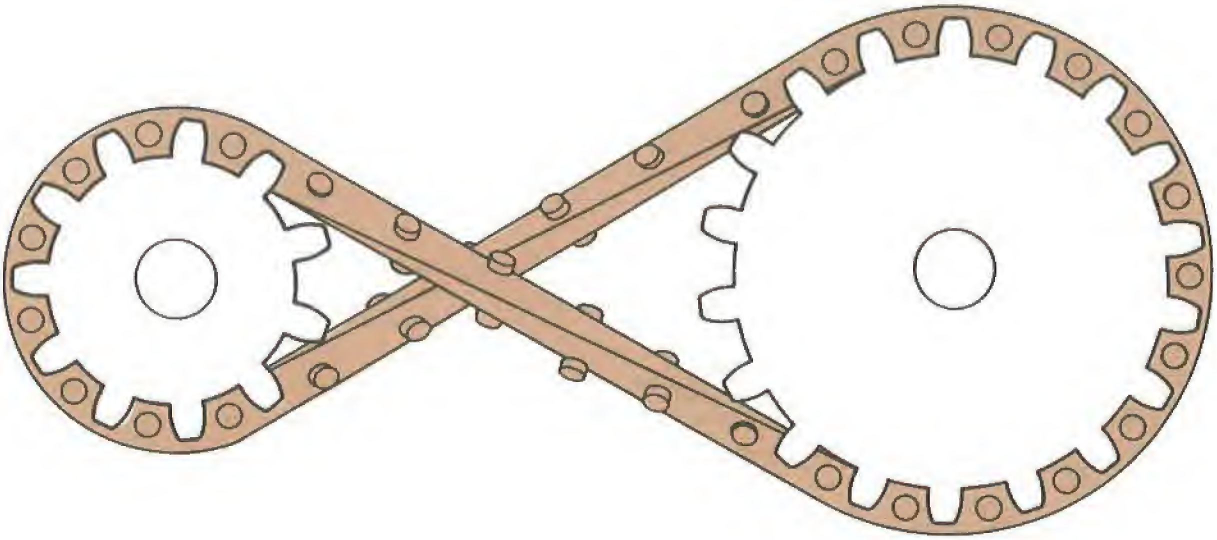


Fig. 26-4 A Crossed Belt Drive

The pulley ratios remain the same for the crossed belt with the exception that the direction of the driven pulley is reversed.

$$\boxed{\frac{d}{D} = -\frac{\theta_L}{\theta_s} = -\frac{\omega_L}{\omega_s} = -\frac{T_s}{T_L}} \quad \textbf{(26.7)}$$

The minimum crossed belt length is given by

$$\boxed{L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}\theta_1 (D+d)} \quad \textbf{(26.8)}$$

where θ_1 this time is the belt contact angle on each pulley,

$$\theta = \pi + 2 \text{ arc sin } \frac{D+d}{2C}$$

One of the important advantages of a toothed belt drive is its flexibility. The belt may be twisted to fit many difficult applications. Figure 26-5 shows just a few of the possibilities.

Overall, the toothed belt combines many of the advantages of other drive systems with very few of their limitations.

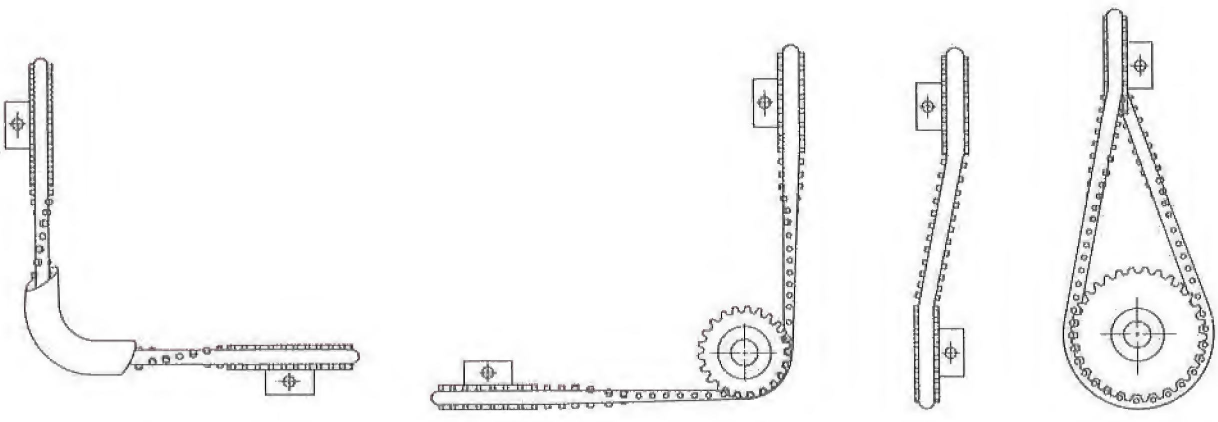


Fig. 26-5 Various Toothed Belt Configurations

MATERIALS

2 Bearing plates with spacers

6 Bearing mounts

6 Bearings

1 Toothed pulley, approx. 1 in. OD

1 Toothed pulley, approx. 1 1/2 in. OD

1 Toothed pulley, approx. 2 in. OD

1 Toothed belt, approx. 12 in. long

1 Motor mount

1 Shaft coupling

6 Collars

3 Shafts, approx. 4" X 1/4"

1 Power supply

1 Stroboscope

1 Dial caliper

1 Breadboard with legs and clamps

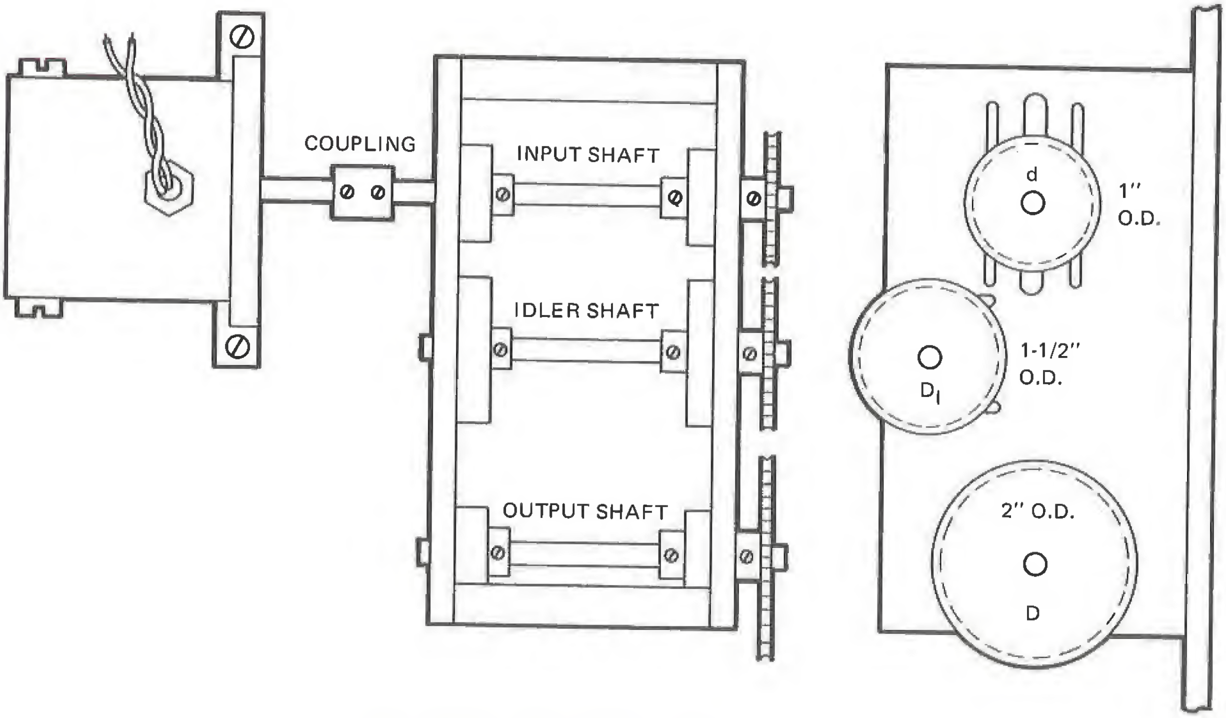


Fig. 26-6 The Bearing Plate Assembly

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Construct the bearing plate assembly shown in figure 26-6.
3. Mount the bearing plate assembly on the spring balance stand for additional stability.
4. Measure and record the OD of each pulley (d_o, D_o, and D_{oI}).
5. Measure and record the total length of the belt (L).

d_o	D_o	D_{oI}	η	N	N_I	L	C

d	D	D_I	ω_s	ω_L	ω_I	L'
Second Run						

$\frac{d}{D}$	$\frac{\omega_s}{\omega_L}$	% Diff.	$\frac{d}{D_I}$	$\frac{\omega_s}{\omega_I}$	% Diff.	$\frac{D_I}{D}$	$\frac{\omega_I}{\omega_L}$	% Diff.

Fig. 26-7 The Data Table

6. Count and record the number of teeth on each pulley (η, N, and N_I).
7. Compute and record the pitch diameter of each pulley (d, D, and D_I).
8. Install the belt between the 1-inch and 2-inch pulleys with the 1 1/2-in. pulley as an idler outside the belt.
9. Adjust the center distance and idler position so that the belt is snug but not tight.
10. Measure and record the center distance (C).
11. Connect the DC motor to the power supply and set the voltage to about 12 volts.
12. Strobe each pulley and record the results (ω_s, ω_L, and ω_I).
13. Compute and record the three diameter ratios (d/D, d/D_I, and D_I/D).
14. Compute and record the three velocity ratios (ω_s/ω_L, ω_s/ω_I, and ω_I/ω_L).

15. Compute and record the percent difference between each diameter ratio and the corresponding velocity ratio.
16. Repeat steps 12 through 16 for a power supply voltage of about 24 volts.
17. Using your measured values of C, d, and D, compute the minimum belt length (L′).

ANALYSIS GUIDE. In the analysis of these data you should consider the effectiveness of the toothed belt as a positive drive. Does your data indicate the presence of belt slippage? Did your value of L′ come out to be less than L? Why? What applications can you think of for toothed-belt drives?

PROBLEMS

1. What was the diametrical pitch of the pulleys you used in this experiment?
2. What was the circular pitch of the pulleys used in this experiment?
3. What was the pitch of the belt used in this experiment?
4. Compute the tooth ratios η/N, η/N_I, and N_I/N for the pulleys used in the experiment.
5. How do the tooth ratios computed in problem 4 compare to the diameter ratios computed in step 14 of the procedures?
6. Show algebraically that $\eta/N = d/D$ for a toothed pulley.
7. Two toothed pulleys are 32-pitch and have OD s for 0.500 in. and 1.000 in. with a center distance of 3.75 in. What is the minimum length belt that should be used for the crossed belt configuration?
8. What is the velocity ratio in problem 7?

experiment 27 *DISK DRIVES*

INTRODUCTION. Friction disk drives are used in a variety of low torque systems from phonograph turntables to lawn mowers. In this experiment we shall examine the principles of this type of drive train.

DISCUSSION. Disk drives have been used for a very long time. It is possible that they predate gear wheels and may, in fact, have been their ancestors.

Anyone who has lifted the turntable off of a phonograph has probably seen a disk drive somewhat like figure 27-1.

The idler wheel and the drive wheels are rimmed with a rubbery friction pad. The idler wheel is spring-loaded so that it is pressed snugly against one of the drive wheels and the motor shaft. The largest drive wheel, in turn, is pressed against the inside of the turntable rim.

If there is little or no slippage, the motor shaft drives the turntable in a manner similar to a geared drive.

Let's look at a single disk pair of the type shown in figure 27-2.

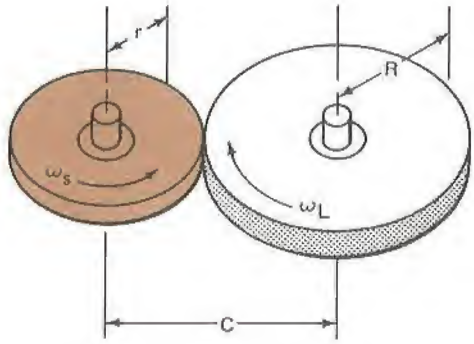


Fig. 27-2 A Simple Disk Drive

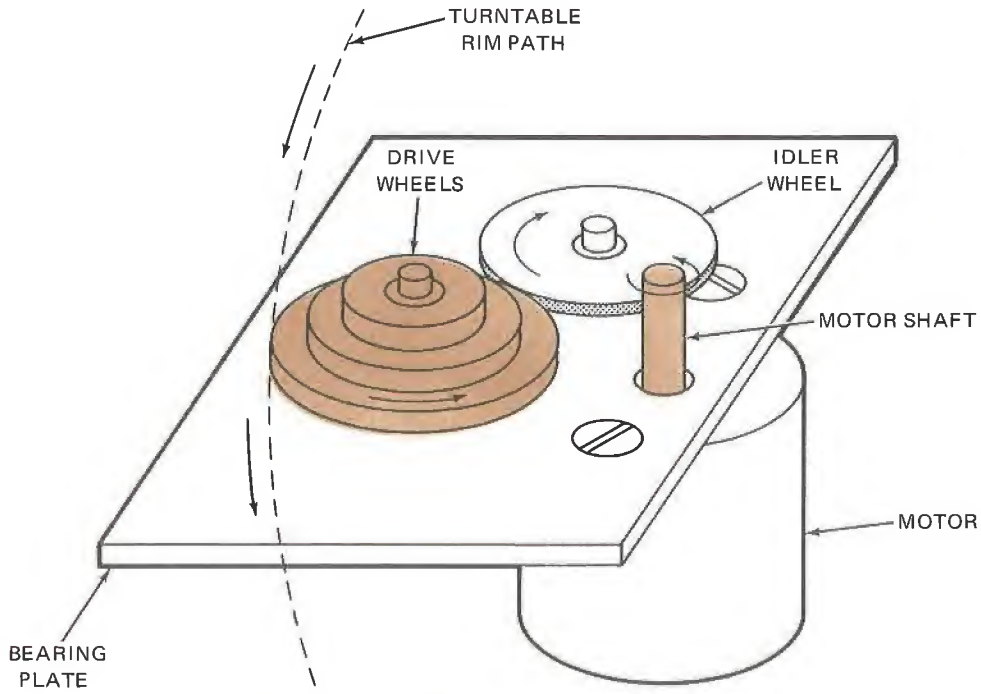


Fig. 27-1 A Phonograph Drive System

Suppose that these two disks are made of (or rimmed with) a hard rubber material and that there is no slippage between them. This being the case, they will act very much like a gear mesh. The velocity ratios will be

$$\boxed{-\frac{\omega_S}{\omega_L} = \frac{R}{r} = \frac{D}{d} = -\frac{\theta_S}{\theta_L} = -\frac{T_L}{T_s}} \qquad \textbf{(27.1)}$$

where ω_S is the small disk angular velocity.
ω_L is the large disk angular velocity.
R is the large disk radius.
r is the small disk radius.
D is the large disk diameter.
d is the small disk diameter.
θ_S is the small disk angular displacement.
θ_L is the large disk angular displacement.
T_L is the large disk torque.
T_s is small disk torque.

and the negative signs reflect a change in the direction of the action.

In a practical case these ratios will be in error by about 5 percent due to two factors. These factors are: First, there is normally some slippage in a friction contact. Second, when the two wheels are pressed together, they will be smashed a little, thereby changing their *effective* diameters. The actual amount of slippage usually depends quite a lot on the amount of torque being transmitted and on the contact force.

To illustrate how an allowance can be made for slippage, let's assume that the disks in figure 27-2 have diameters of D = 1.25 in. and d = 0.75 in. Let's also suppose that the smaller disk is rotating at 1225 RPM and is driving the larger disk which has a load of 190 in.-oz. Based on these values the angular velocity of the larger disk can be found as follows:

1. Assuming *no* slippage

$$\omega_L = -\omega_S \frac{d}{D} = -1225 \frac{0.75}{1.25} = -735 \text{ RPM}$$

2. Assuming 5% slippage

$$\omega_L = 0.95\,(-735) = -698 \text{ RPM}$$

Similarly, the torque transmitted by the smaller disk can be found.

1. Assuming no slippage

$$T_s = -T_L \frac{d}{D} = -190 \frac{0.75}{1.25} = -114 \text{ in.-oz.}$$

2. Assuming 5% slippage

$$T_s = \frac{(-114)}{0.95} = -120 \text{ in.-oz.}$$

In both cases the negative signs reflect the change in direction.

In figure 27-1 the disk ratio can be changed in three steps by moving the idler disk from one drive wheel to another.

One of the important advantages of disk drives is that it is possible to have *continuously* variable ratios. Figure 27-3 shows one type of variable ratio disk drive.

The velocity ratios of this type of drive are still

$$\boxed{\frac{r_1}{r_2} = \frac{d_1}{d_2} = -\frac{\omega_2}{\omega_1} = -\frac{\theta_2}{\theta_1} = -\frac{T_1}{T_2}} \qquad \textbf{(27.2)}$$

However, if we can move the smaller disk along its own axis we can change r_2. If r_2 changes, the velocity ratio also changes. When this type of drive is used, the smaller wheel is usually the driver. A splined shaft is the usual method used to allow movement of the driver to change the ratio.

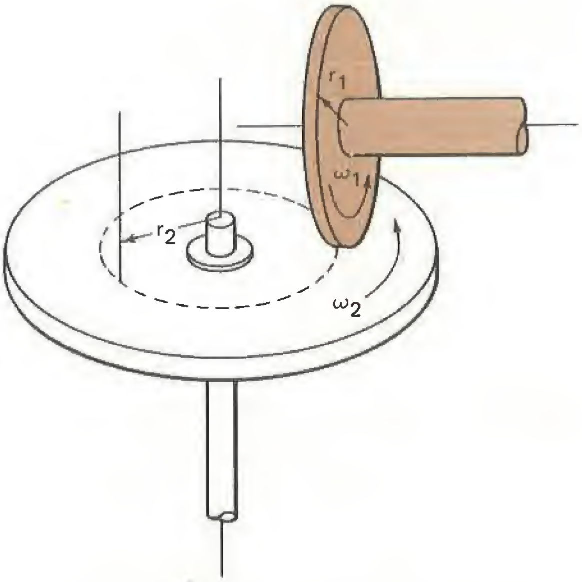


Fig. 27-3 A Variable Ratio Disk Drive

Another type of variable ratio disk drive is shown in figure 27-4. In this case the input disk drives the idler disk which, in turn, drives the output disk. Equation 27.3 gives the ratios. However, by moving the idler shaft, we can change both r_1 and r_2.

$$\boxed{\frac{r_1}{r_2} = \frac{d_1}{d_2} = \frac{\omega_2}{\omega_1} = \frac{\theta_2}{\theta_1} = \frac{T_1}{T_2}} \qquad \textbf{(27.3)}$$

Notice that there is no change in direction in this case. Also, while there are two chances for slippage, notice that slight idler deformation does not alter the ratios. As a result, the total error is still normally about 5 percent.

There are a great many different types of disk drives in use today. Since space is limited we can't hope to discuss them all here. The ones presented are quite common and should serve to introduce you to the operation of this type of drive assembly.

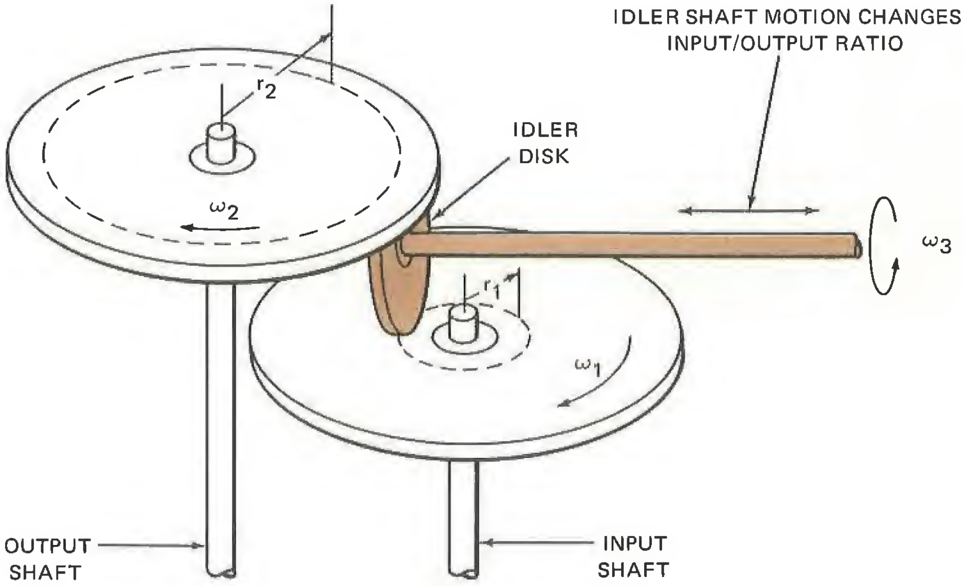


Fig. 27-4 A Variable Speed Drive

MATERIALS

2 Bearing plates with spacers'
2 Shaft hangers with bearings
2 Bearing mounts
2 Bearings
2 Shafts approx. 4" X 1/4"
1 Motor and mount
1 Shaft coupling
1 Power supply
2 Dials with 1/4 in. bore hubs
1 Pulley approx. 2 in. OD
1 Pulley approx. 1 in. OD
1 O-ring approx. 1" X 1/8"
1 Stroboscope
1 Dial caliper
3 Collars
1 Breadboard with legs and clamps

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Construct the bearing plate assembly shown in figure 27-5.
3. Snap the pulley belt onto the smaller of the two pulleys. It should fit quite tightly.

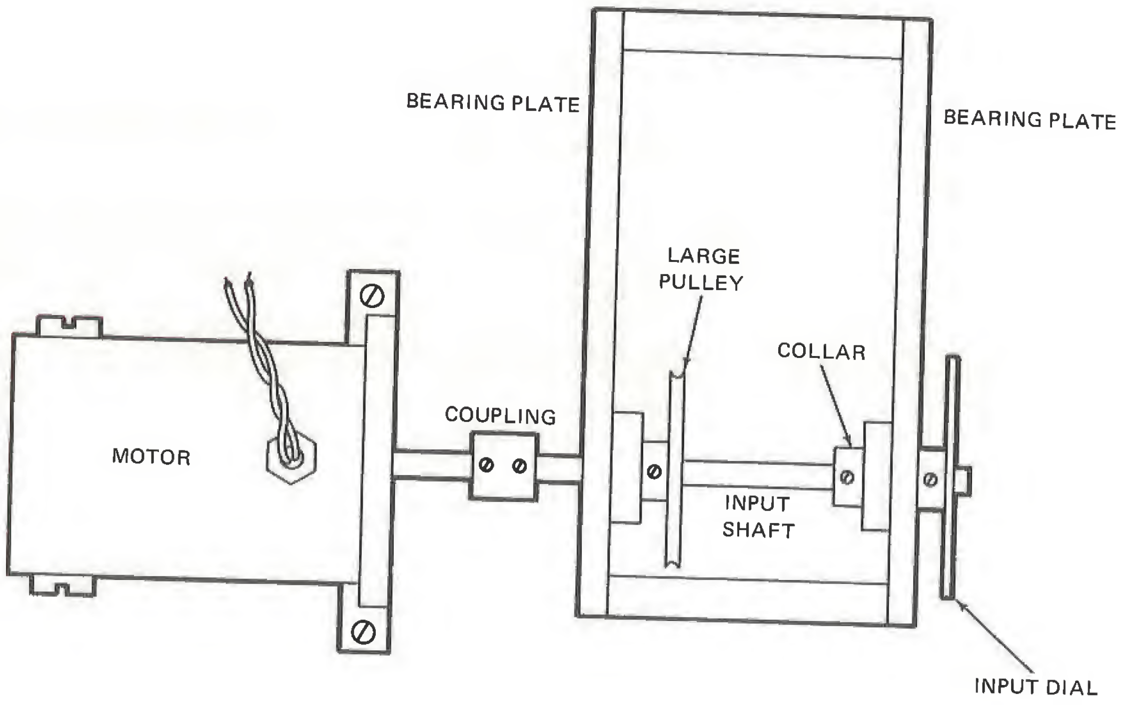


Fig. 27-5 The Bearing Plate Assembly

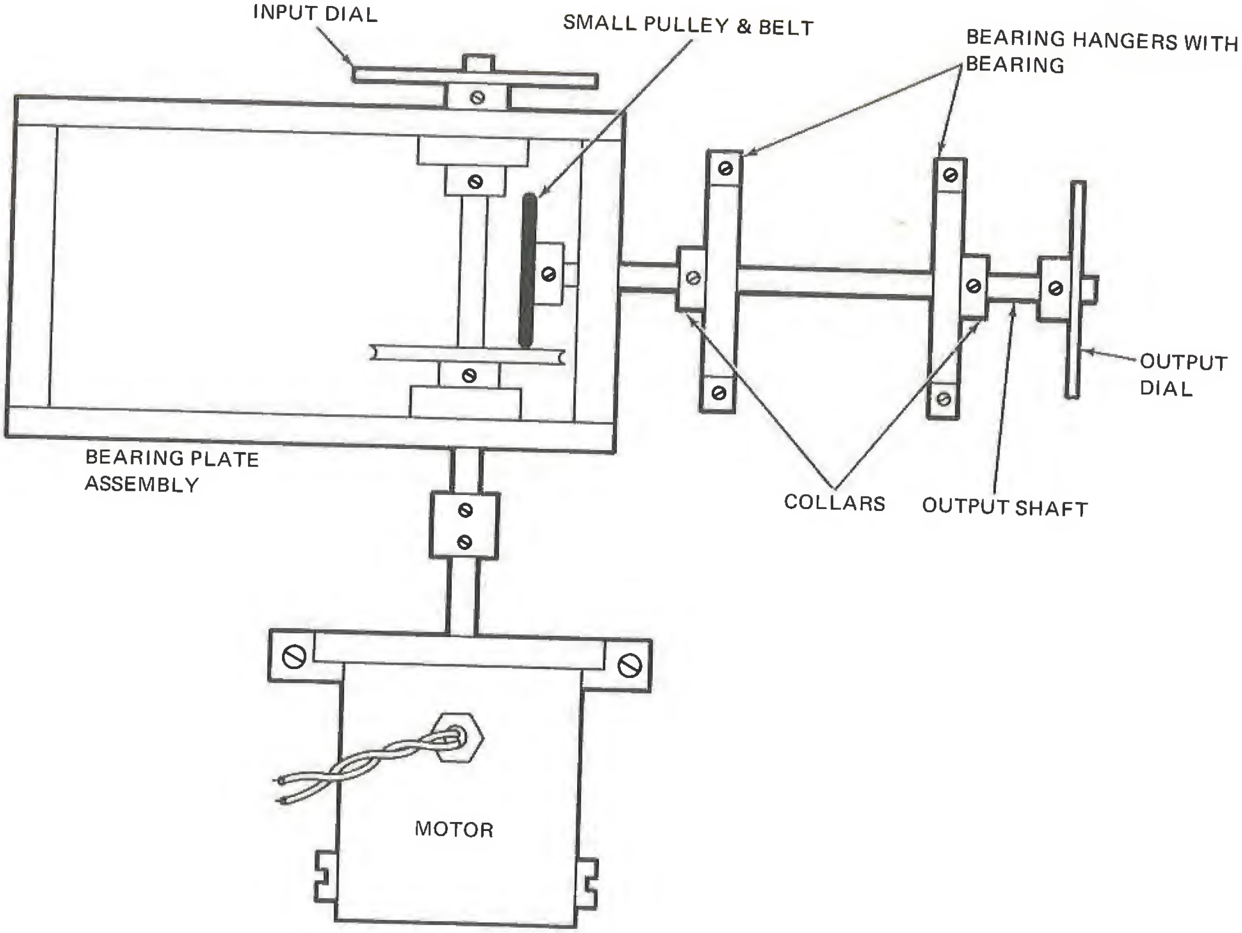


Fig. 27-6 The Experimental Mechanism

4. Mount the bearing plate on the Breadboard as shown in figure 27-6. Also mount the other components.
5. Loosen the clamps and slide the whole bearing plate assembly up snugly (not tightly) against the small pulley with the belt on it. *NOTE: The metal parts of the two pulleys should contact the belt only.* Retighten the clamps.
6. Connect the DC motor to the DC power supply.
7. Adjust the collars on the output shaft so that the small pulley with the belt contacts the large pulley approximately 1/4 in. from the large pulley rim.
8. Measure and record the distance (r_S) from the center of the output shaft to the point of contact of the two disks.
9. Measure and record the distance (r_L) from the center of the input shaft to the point of contact of the two disks.
10. Set the power supply voltage to about 10 volts.
11. Strobe both dials and record the results (ω_S and ω_L).

		$r_s =$			
$r_L =$		$r_L =$		$r_L =$	
ω_s	ω_L	ω_s	ω_L	ω_s	ω_L
$\frac{r_L}{r_s} =$		$\frac{r_L}{r_s} =$		$\frac{r_L}{r_s} =$	

Fig. 27-7 The Data Table

12. Repeat step 11 for power supply voltages of about 14, 18, 22, and 26 volts.
13. On a sheet of graph paper plot ω_L (horizontally) versus ω_s (vertically).
14. Move the small pulley with the belt in to about 1/2 in. from the rim of the larger pulley. Try this both with the mechanism running and with it still. Do you feel any difference?
15. Repeat steps 9 through 13. (Use a separate sheet of graph paper for your plot).
16. Similarly, move the small pulley with the belt in to about 3/4 in. from the rim of the larger pulley.
17. Again repeat steps 9 through 13. (Plot the results on a third sheet of graph paper).
18. Compute the ratio r_L/r_s for each set of data.
19. On each of your graphs plot the equation $\omega_s = \omega_L(r_L/r_s)$. Do not use measured values of ω_s and ω_L in making this plot.

ANALYSIS GUIDE. In the analysis of these data you should examine your plots of the measured data and compare them to the plots of $\omega_S = \omega_L(r_L/r_S)$. Discuss any differences between the two in each case. Also compare your results for one velocity ratio to those of another. Why do you think you could feel a difference in step 14?

PROBLEMS

1. The phonograph mechanism in figure 27-1 has a 13-in. ID turntable. The motor shaft is 1/4-in. OD and the largest drive wheel is 1 1/2-in. OD. The motor turns at 1725 RPM; at what speed will the turntable turn? (Ignore slippage)
2. What would be the results in problem 1 if there is 5% slippage in *each* disk contact?
3. What should be the diameter of the middle size drive wheel if the turntable speed is to be 45 RPM? (Notice that the idler drives the middle disk but the large disk still drives the turntable.)
4. Find the size of the small size drive wheel if the turntable speed is to be 78 RPM.
5. A disk drive of the type shown in figure 27-4 has input and output disks that are 6 in. OD. The centers of the input and output disks are 3.5 in. apart and the idler disk is 1 in. OD. What is the range of the output velocity if the input velocity is 500 RPM?
6. Make a sketch of the relationship between idler position (horizontal axis) and output velocity (vertical axis) for the mechanism in problem 5.

experiment 28 ROTARY CAMS

INTRODUCTION. Cams are used in a great variety of mechanical applications. In this experiment we shall deal with one of the most basic types of rotary cam, the single lobe timing cam.

DISCUSSION. Consider the rotary cam shown in figure 28-1.

When it is operating properly the cam rotates with its shaft. The follower roller rolls over the cam face. The cam follower then has a motion which is determined by the cam profile. During the portion of a revolution when the smaller cam radius (r) is under the follower roller, the follower arm is in its lower position. As the cam rotates, the *lobe region* having the larger radius (R) eventually forces the follower up to its upper position. The distance traveled by the follower arm in making the transition is

$$S = R - r \text{ in.} \qquad \textbf{(28.1)}$$

This follower travel is frequently used to operate some other mechanism such as an electrical switch.

Advertising signs, for example, are frequently "blinked" on and off in this way.

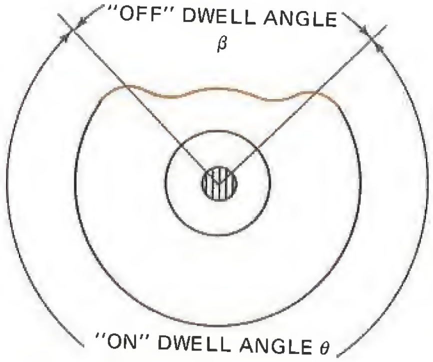


Fig. 28-2 Cam Dwell Angles

By controlling the lobe angle and angular velocity of the cam, we can control the on and off time of such "blinking" operation. Figure 28-2 shows the cam again with the on and off angles marked. These angles are often referred to as *dwell angles.* When the follower is up, we say it is *"ON"* the cam. When it is down, we say it is *"OFF"* of the cam. The follower is not shown in figure 28-2 so that you can see the angles more clearly.

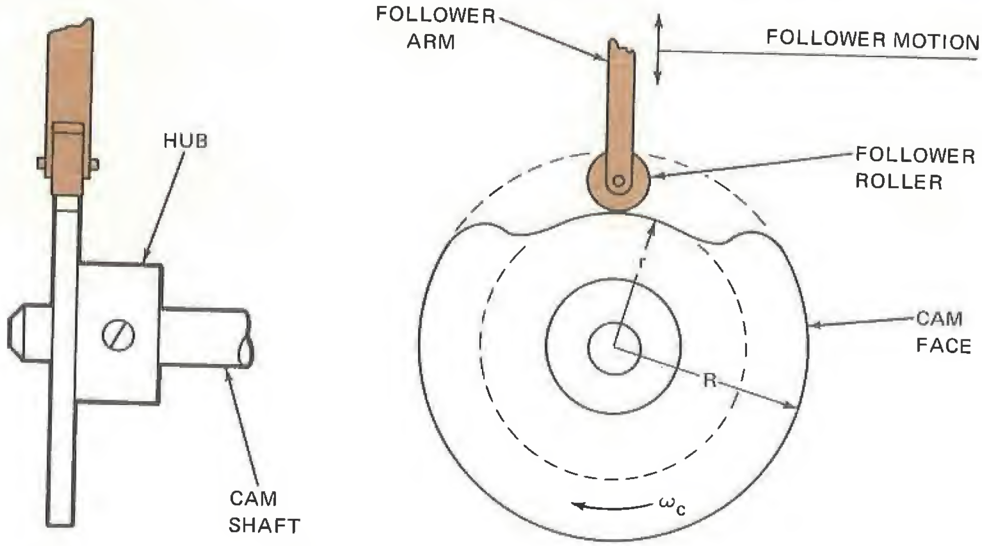


Fig. 28-1 A Simple Rotary Cam

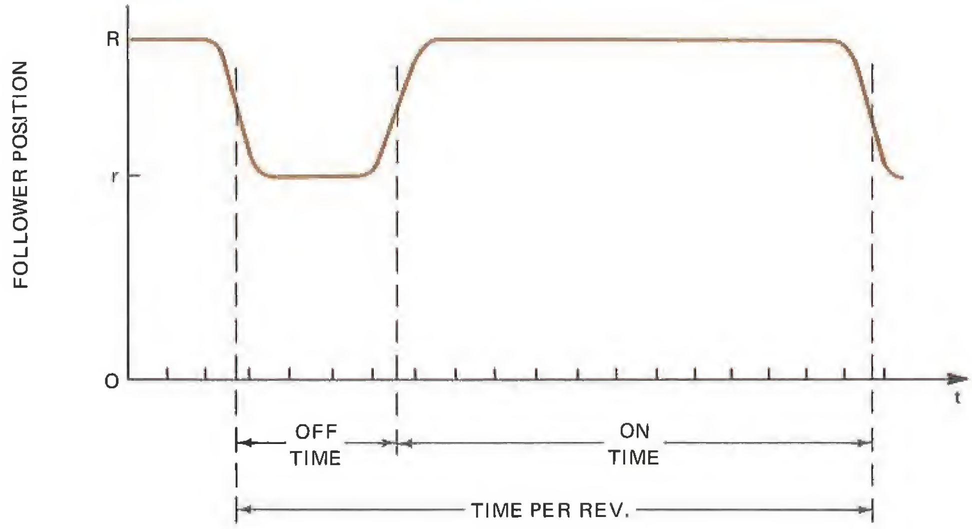


Fig. 28-3 Follower Position vs Time

There are, of course, also two angles during which the follower is neither on nor off of the cam. These are called the *transition angles.* In low speed electrical switching the transition angles are of only small importance. In high speed cam operations these transitions become extremely important.

If we choose to ignore the transition periods, then the follower will be on the cam for an angle θ and off of it for an angle β. Moreover the sum of these two angles is

$$\theta + \beta = 2\pi \text{ radians}$$

or one complete revolution.

If we plot the follower position versus time, the result is somewhat like that shown in figure 28-3.

Now, the time required per revolution is simply

$$T = \frac{1}{\omega_c} \text{ min/rev.}$$

while the on and off times are

$$t_{on} = \frac{\theta}{2\pi} T = \frac{\theta}{2\pi\omega_c} \text{ minutes}$$

and

$$t_{off} = \frac{\beta}{2\pi} T = \frac{\beta}{2\pi\omega_c} \text{ minutes}$$

where β and θ are in radians.

It is occasionally more convenient to express the angles in degrees. Thus, the on and off times become

$$t_{on} = \frac{\theta}{360\omega_c} \text{ minutes}$$

and

$$t_{off} = \frac{\beta}{360\omega_c} \text{ minutes}$$

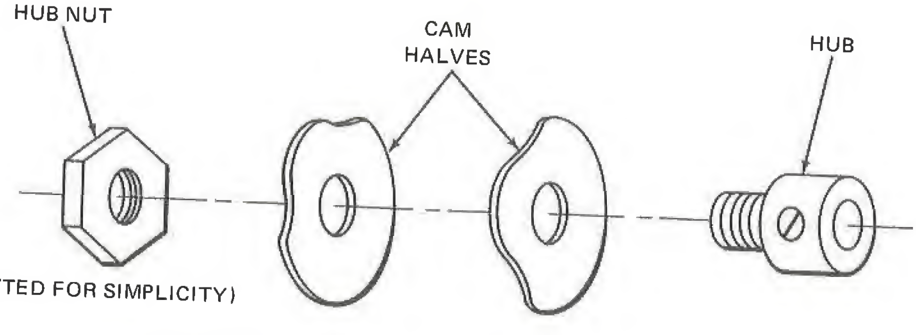


Fig. 28-4 An Adjustable Lobe Cam

In most practical cases the on and off times will be small fractions of a minute. Consequently it is frequently convenient to express them in seconds,

$$t_{on} = 60 \frac{\theta}{360\omega_c} = \frac{\theta}{6\omega_c} \qquad \textbf{(28.2a)}$$

and

$$t_{off} = 60 \frac{\beta}{360\omega_c} = \frac{\beta}{6\omega_c} \qquad \textbf{(28.2b)}$$

We can make the on and off times adjustable by building the cam out of several pieces. Figure 28-4 shows the construction of a typical adjustable cam. When the cam is assembled, the two cam halves can be rotated in relationship to one another to provide the desired on and off times.

As the rotational speed of a cam is increased, it becomes more and more difficult for the cam follower to maintain contact with the cam face. Figure 28-5 shows the path of a roller center at low and high cam speeds. The dotted line represents the *relative* position of the follower center as the cam rotates. Notice that the cam follower floats and bounces at relatively high cam speeds.

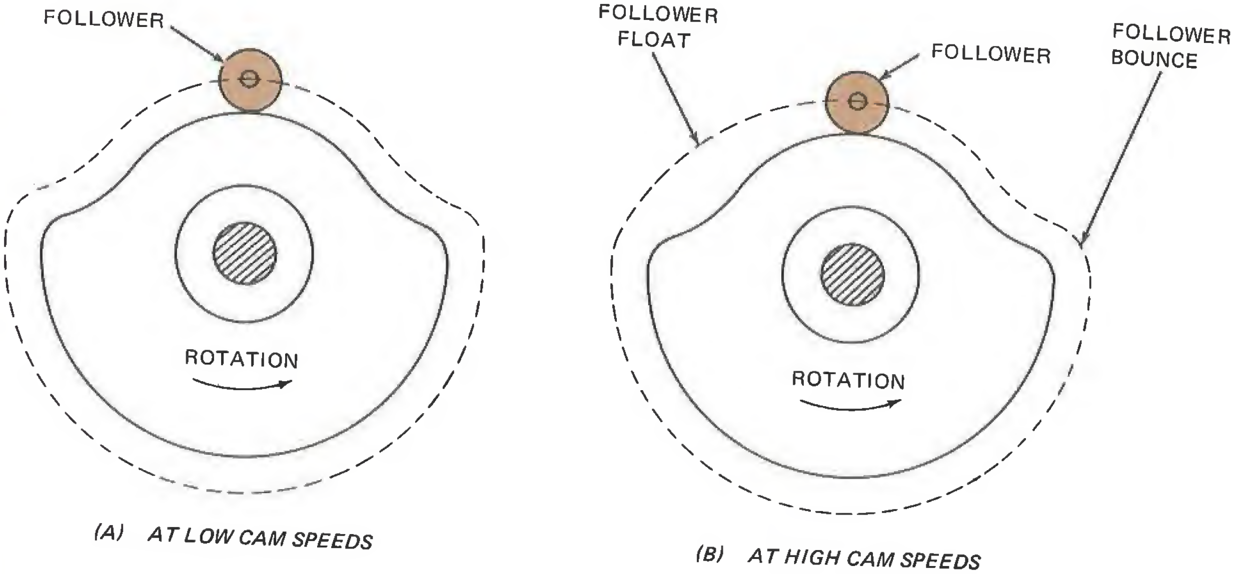


Fig. 28-5 Cam Follower Paths

Float and bounce can be controlled by advanced cam design. Such designs are beyond the scope of this experiment. But we will at least observe the problem.

MATERIALS

- 2 Bearing plates with spacers
- 3 Shafts approx. 4" X 1/4"
- 4 Bearing mounts
- 4 Bearings
- 2 Shaft hangers with bearings
- 6 Collars
- 1 Shaft coupling
- 1 Adjustable cam with 1/4 in. bore hub
- 1 Cam follower roller
- 2 Lever arms with 1/4 in. bore hubs
- 1 Rubber band
- 1 Worm wheel
- 1 Worm screw
- 1 Motor and mount
- 1 Power supply
- 1 Breadboard with legs and clamps
- 1 Stroboscope

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Construct the bearing plate assembly shown in figure 28-6.
3. Mount the bearing plate assembly and other components on the spring balance stand as shown in figure 28-7.
4. Adjust the worm wheel/cam shaft as necessary to mesh with the worm screw.
5. Adjust the rubber band tension so that the cam follower roller rests firmly (but not tightly) against the cam.

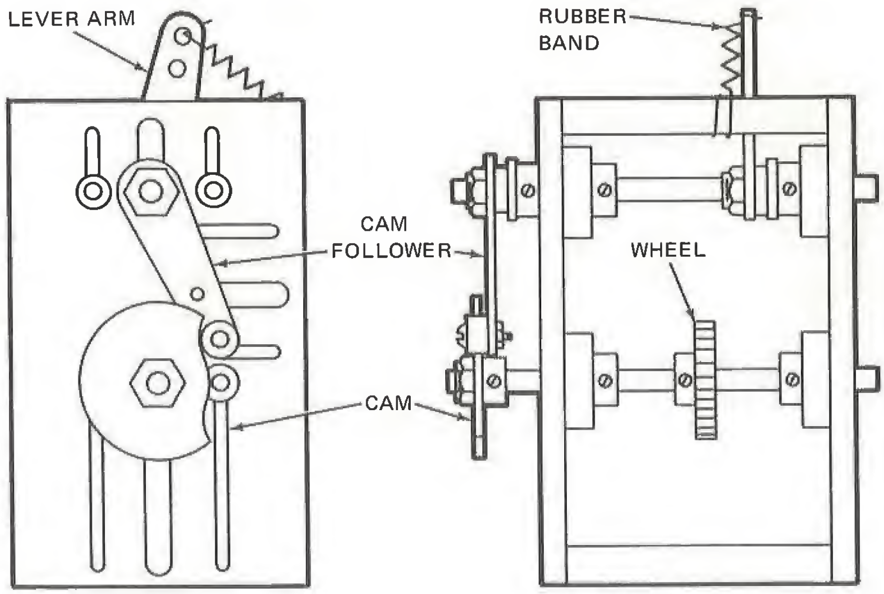


Fig. 28-6 The Bearing Plate Assembly

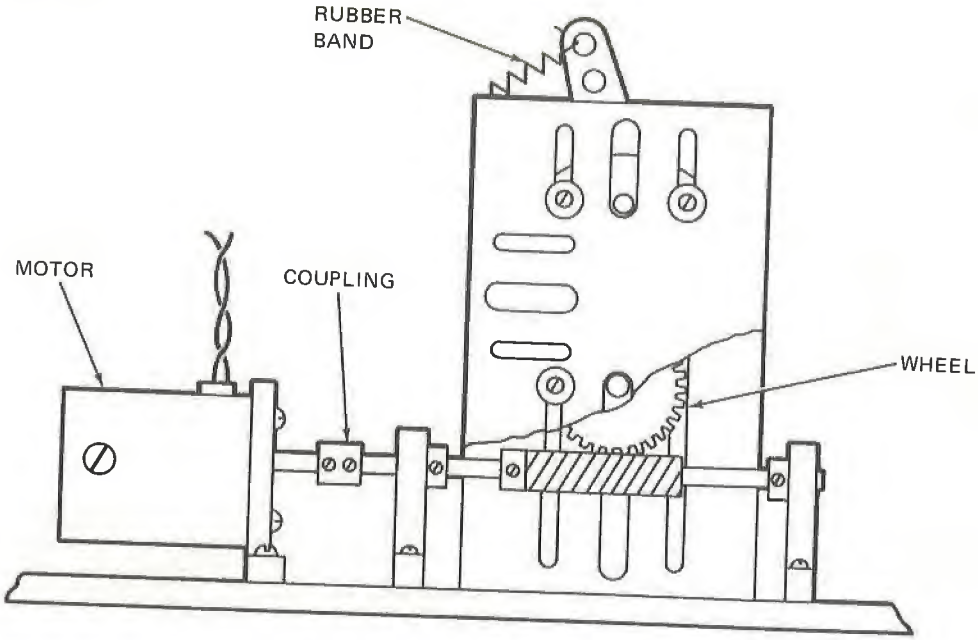


Fig. 28-7 The Experimental Mechanism

6. Connect the DC motor to the power supply. Set the voltage so that the cam runs very slowly. Make any necessary adjustments to insure that the whole mechanism is running smoothly.

7. Set the power supply voltage to approximately 10 volts.

8. Set up the stroboscope so that the cam *appears* to be rotating *very* slowly. Watch the follower roller very carefully. You should be able to see some follower float as it comes off the cam. If not, increase the motor speed slightly. By varying the motor voltage determine the cam speed at which the follower first starts to float. Record this speed (ω_1).

	ω_1	ω_2
First Tension Setting		
Second Tension Setting		
Third Tension Setting		

Fig. 28-8 The Data Table

9. Repeat step 8 for follower bounce as it comes on the cam. Record the result as ω_2.
10. Increase the rubber band tension on the follower to about twice its original amount and repeat steps 8 and 9.
11. Again increase the spring tension and repeat steps 8 and 9.

ANALYSIS GUIDE. Examine your data carefully, then discuss the effect of follower tension on bounce and float. Also discuss any differences between the values of ω_1 and ω_2: Did the follower begin to float and bounce at the same cam speed? Why?

PROBLEMS

1. A cam of the type shown in figure 28-1 is rotating at 4 RPM. This cam operates a blinking sign which is on 1/3 of the time. Assuming that the sign is on when the follower is on the cam, what are the dwell angles in degrees? Radians?
2. Make a sketch of the cam in problem 1.
3. Assume that the sign in problem 1 is on when the follower is off the cam. How would this affect the dwell angles?
4. Make a sketch of the cam in problem 3.
5. A certain timing cam has an on dwell angle of 90°. What is the RPM of the cam if the on dwell time is 3.5 sec.?
6. A single cam is to be used with a 1.0 RPM timing motor to switch an electric sign. The sign is to be on for 12 sec., off for 21 sec., then on for 16 sec., and off for 11 sec. each minute. Sketch a cam that would perform this job. Show the dwell times and angles.
7. Suppose that you have a 1/4-in. OD cam follower roller that is to be used with a cam having R = 3/4 in. and r = 1/2 in. The cam rotates at 2.0 RPM. Is it possible to have a dwell time so short that the roller cannot possibly follow the cam? Explain your answer using sketches to illustrate the conditions.
8. Discuss how follower float and bounce affect the on and off dwell times of a cam application.

experiment 29 UNIVERSAL JOINTS

INTRODUCTION. Universal joints are used to couple shafts which are angularly displaced one to another. In this experiment we shall examine the operating characteristics of single and double universal joints.

DISCUSSION. Universal joints come in many different sizes, types, and designs. Perhaps the simplest one is the hooke-type universal joint shown in figure 29-1.

This type of universal joint is constructed out of two yoke-shaped pieces and a cross-shaped spider.

If the input velocity ω_1 is constant, the output velocity is not. The output velocity varies instantaneously from a minimum value below ω_1 of

$$\boxed{\omega_{2\,min} = \omega_1 \cos\alpha} \qquad (29.1)$$

up to a maximum value above ω_1 given by

$$\boxed{\omega_{2\,max} = \frac{\omega_1}{\cos\alpha}} \qquad (29.2)$$

This instantaneous variation in output velocity is cyclic in nature. Figure 29-2 shows a sketch of ω_2 for a constant input plotted as a function of the input displacement θ_1.

In this sketch α is also held constant.

If we change the drive angle α, the values on the curve in figure 29-2 change, but the general cyclic nature of the plot remains the same.

Universal joints of this type can be effectively used for drive angles as large as 30 degrees or even more.

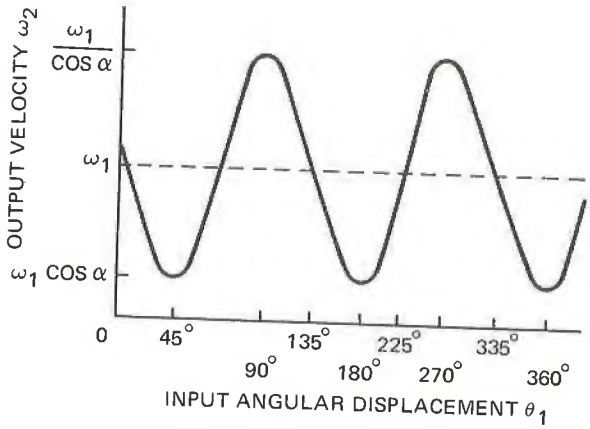


Fig. 29-2 Universal Joint Output Variation

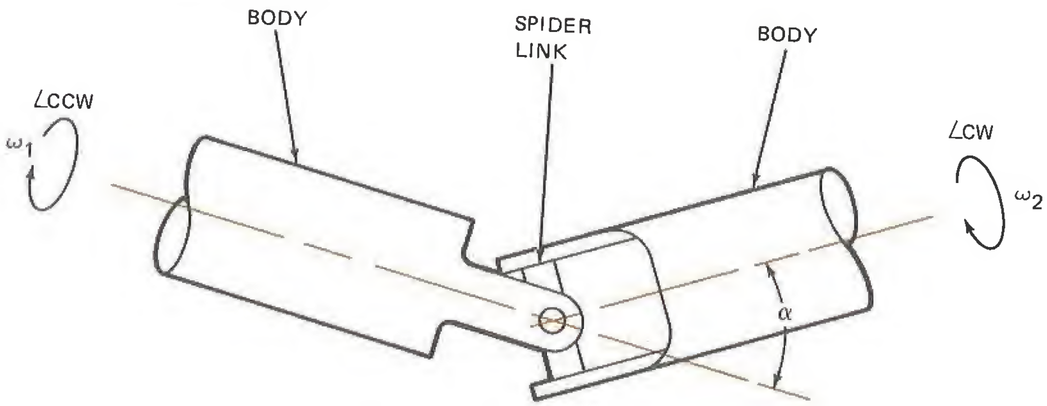


Fig. 29-1 A Hooke-Type Universal Joint

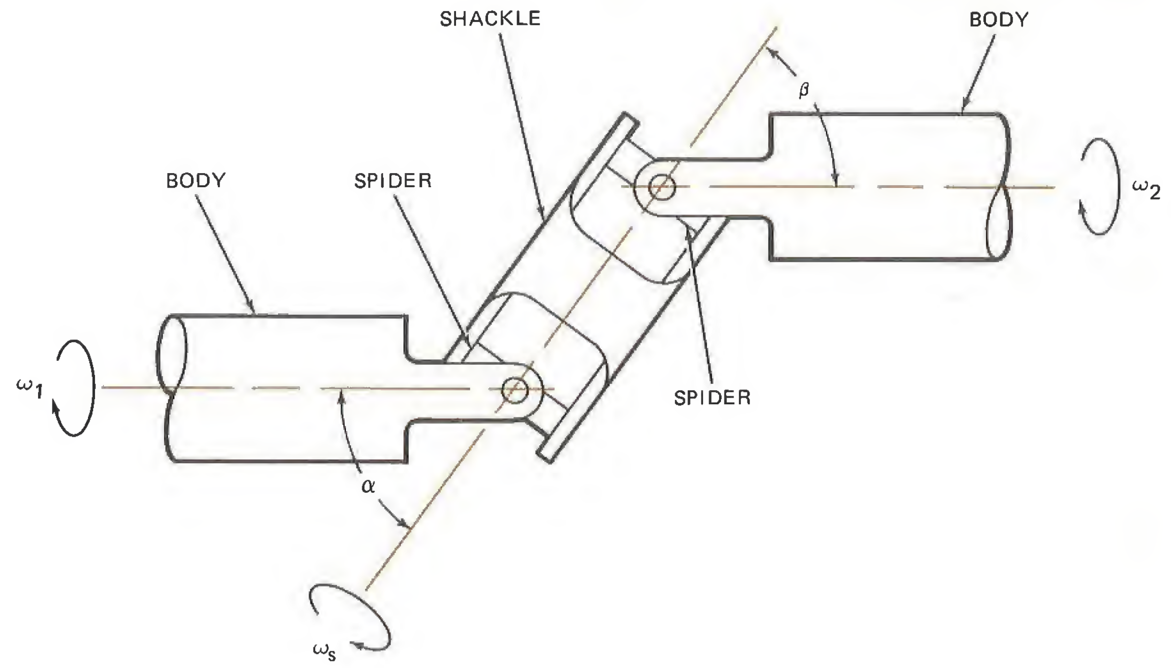


Fig. 29-3 A Double Universal Joint

The cyclic variations in output velocity can be very troublesome; therefore, in most applications two universal joints are employed to produce a constant velocity output. Figure 29-3 shows such an arrangement.

With this mechanism, if we hold the input velocity, ω_1, constant, the spider velocity (ω_s) will vary as in the case of a single U-joint. Then as the motion is transmitted through the second spider, just the reverse actions take place and the output velocity becomes constant again. However, we should observe that this only happens if the two displacement angles (α and β) are equal.

$$\alpha = \beta$$

When the two angles are equal, then the input and output velocities are also equal.

$$\omega_1 = \omega_2 \qquad \textbf{(29.3)}$$

If the two displacement angles are not equal, then the velocities are related in much the same manner as for a single universal joint. The minimum and maximum output velocity will be

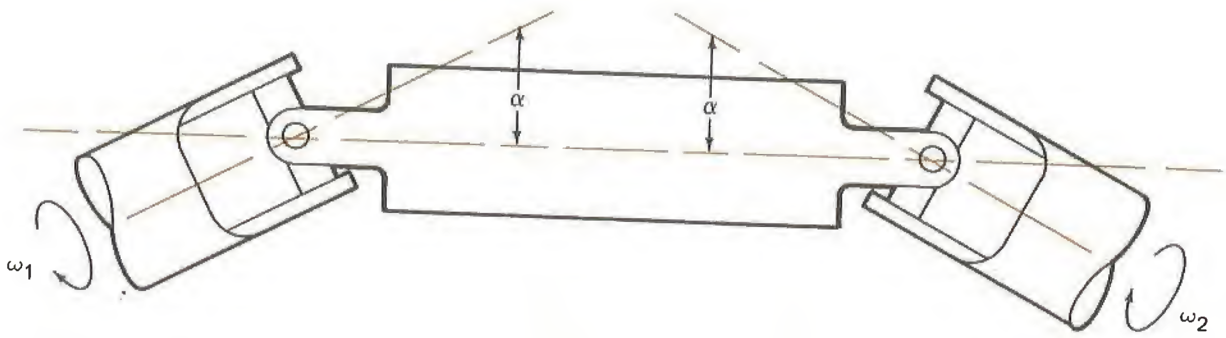
$$\boxed{\omega_{2\ min} = \omega_1 \cos(\beta - \alpha)} \qquad \textbf{(29.4)}$$

and

$$\boxed{\omega_{2\ max} = \omega_1 / \cos(\beta - \alpha)} \qquad \textbf{(29.5)}$$

In either case the forks of the spider *must* be in the same plane.

Figure 29-3 reveals that we will have constant velocity transmission if the input and output shafts are parallel to one another. It is not *necessary*, however, for the shafts to be parallel in order to have equal displacement angles. Figure 29-4 shows a drive line which has constant velocity and nonparallel shafts.



$\omega_1 = \omega_2$

Fig. 29-4 Nonparallel Shafts

Automobile drive shafts are connected to the transmission and rear end using a configuration very much like the one shown in Figure 29-4. Both of the universal joints in an automobile operate at approximately the same angle so the motion transmission is relatively uniform.

There are ball-coupled universal joints that transmit constant velocities through a single connection. However, a detailed consideration of these devices is beyond the scope of this experiment.

MATERIALS

2 Bearing plates with spacers

2 Shaft hangers with bearings

2 Bearing mounts

2 Bearings

2 Shafts, approx. 4″ X 1/4″

1 Single universal joint

1 Double universal joint

2 Dials with 1/4 in. bore hubs

2 Dial indices with mounting hardware

4 Collars

1 Breadboard with legs and clamps

1 Protractor

1 Sheet of graph paper

PROCEDURE

1. Inspect each of your components to insure that they are undamaged.
2. Assemble the mechanism shown in figure 29-5. Locate the components so that the universal joint displacement angle is approximately 30 degrees.

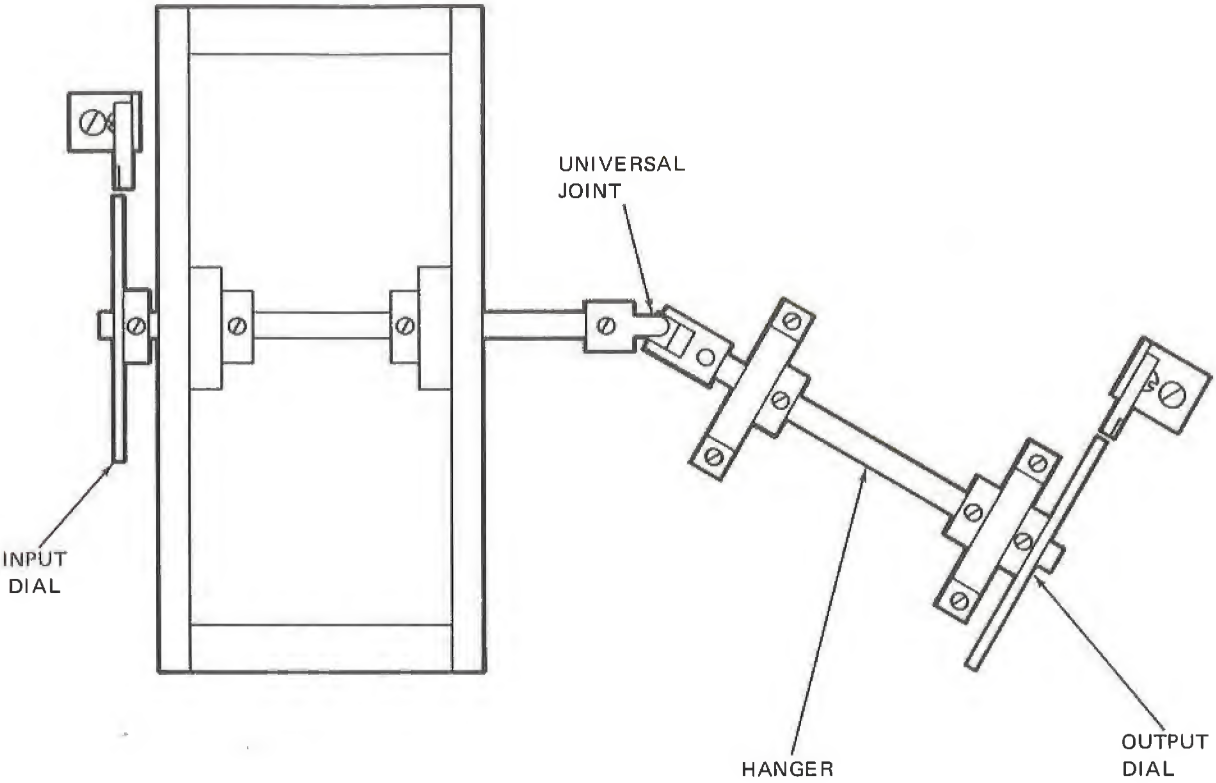


Fig. 29-5 The Experimental Mechanism

3. Set both dials to read zero.
4. Rotate the input dial 15° and record the output dial rotation (θ_2).
5. Repeat step 4 in 15° increments until you have completed one complete input dial revolution.
6. Measure the angle between the two shafts with a protractor and record the results (α).
7. On a sheet of graph paper plot the input rotations, θ_1, (horizontally) versus the output rotation, θ_2 (vertically).
8. Replace the single universal joint with the double universal joint, **but do not disturb the displacement angle.**
9. Repeat steps 3 through 7 and plot your curve on the same sheet of graph paper.

ANALYSIS GUIDE. In analyzing these data you should discuss the extent to which your results agreed with the material presented in the discussion. In particular you should explain how your values of θ_1 and θ_2 are related to ω_1 and ω_2. Discuss the differences between the two plots of θ_1 versus θ_2. What causes the difference?

α =		
Input θ_1	Single U-Joint θ_2	Double U-Joint θ_2
0		
15		
30		
45		
60		
75		
90		
105		
120		
135		
150		
165		
180		
195		

α =		
Input θ_1	Single U-Joint θ_2	Double U-Joint θ_2
210		
225		
240		
255		
240		
255		
270		
285		
300		
315		
330		
345		
360		

Fig. 29-6 The Data Table

PROBLEMS

1. If the single universal joint in this experiment were driven at 750 RPM, what would be the minimum output velocity?
2. What would be the maximum output velocity in problem 1?
3. What would be the results in problems 1 and 2 if the double universal joint were used?
4. Four universal joints can be used to transmit motion around an obstruction. Make a sketch showing how this is done.
5. What everyday applications of universal joints can you think of in addition to an automobile drive line?

experiment **30** *CROSSED HELICALS*

INTRODUCTION. In some applications it is necessary to drive shafts which are crossed. The use of helical gears is one way to accomplish this type of drive. In this experiment we shall examine some of the operating characteristics of crossed helical gears.

DISCUSSION. Helical gears like those shown in figure 30-1 have their teeth cut at an angle across the width of the gear blank.

Virtually all of the relationships which are appropriate for spur gears also apply to helical gears. Specifically the gear ratios are

$$\boxed{\frac{n}{N} = -\frac{\theta_g}{\theta_p} = -\frac{\omega_g}{\omega_p} = -\frac{T_p}{T_g}} \qquad \textbf{(30.1)}$$

where

n = Pinion tooth count
N = Gear tooth count
θ = Angular displacement (subscript p for pinion, g for gear)
ω = Angular velocity
T = Torque

The angle between the side of the gear blank and a line perpendicular to the teeth is called the *helix angle* ψ (psi) of the gear. This is, of course, the same angle as that between the teeth and the gear shaft center line. Figure 30-2 shows these angles.

Having the teeth set at an angle makes it possible to define pitch differently from that of a spur gear. You will recall that for a spur gear, circular pitch (P_c) is defined as the ratio of the pitch circumference to the number of teeth on the gear, where D is the pitch diameter of the gear.

$$P_c = \frac{\pi D}{N} \text{ in./tooth} \qquad \textbf{(30.2)}$$

In other words the circular pitch of a gear is the distance along the pitch circle between the teeth.

In the case of a helical gear we could measure this distance parallel to the side of

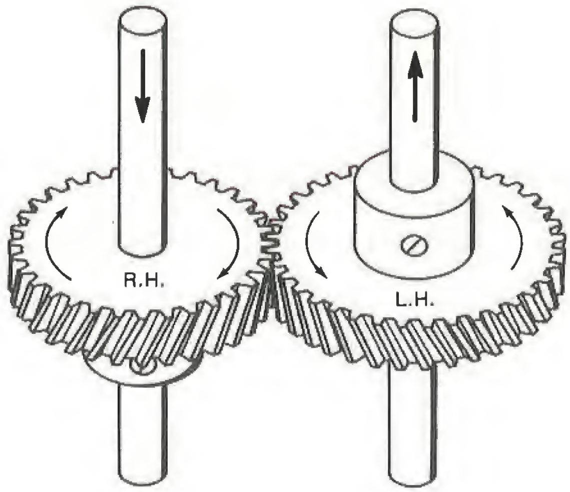


Fig. 30-1 Meshed Helical Gears

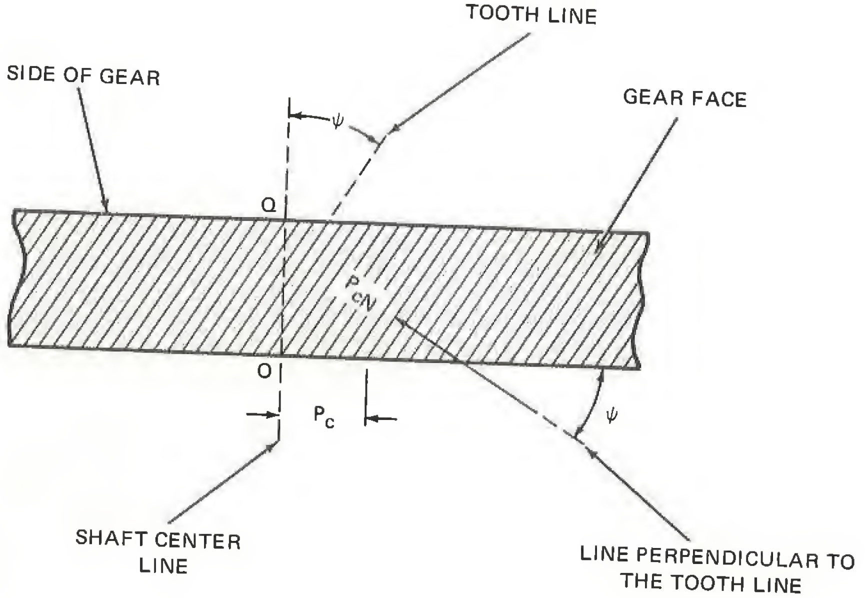


Fig. 30-2 Helix Angle and Normal Circular Pitch

the gear or perpendicular to the tooth line. In practice, both systems are used. The term *circular pitch* (P_c) refers to the pitch measured parallel to the gear side. This is the same as for a spur gear. The pitch measured perpendicular to the tooth line is called the *normal circular pitch* (P_{cN}). Examining figure 30-2 closely we see that P_c and P_{cN} form an angle equal to ψ as shown in figure 30-3. Moreover, we see that P_c, P_{cN}, and the tooth line form a right triangle with P_c as the hypotenuse, P_{cN} the adjacent side, and ψ the enclosed angle. For such a right triangle we know that

$$\cos \psi = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Consequently we may write

$$\cos \psi = \frac{P_{cN}}{P_c}$$

or

$$P_{cN} = P_c \cos \psi \qquad \textbf{(30.3)}$$

as the relationship between the circular and normal circular pitches.

In the case of spur gears, the circular and diametrical pitches are related by

$$P_c = \frac{\pi}{P_d}$$

This same relationship is used for the normal circular pitch and *normal diametrical pitch* (P_N) of a helical gear. That is

$$P_{cN} = \frac{\pi}{P_N} \qquad \textbf{(30.4)}$$

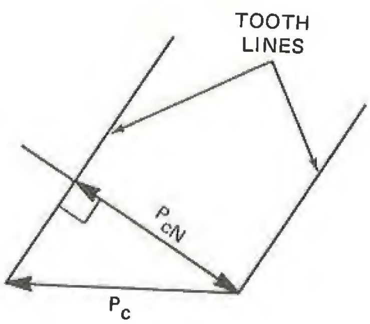


Fig. 30-3 The P_c, P_{cN} Triangle

Substituting this quantity into equation 30.3 for P_{cN} gives us

$$\frac{\pi}{P_N} = P_c \cos \psi$$

Then substituting π/P_d for P_c provides

$$\frac{\pi}{P_N} = \frac{\pi}{P_d} \cos \psi$$

or

$$\boxed{P_N = \frac{P_d}{\cos \psi}} \qquad \textbf{(30.5)}$$

which is a useful relationship between the two diametrical pitches.

The helix angle selected for a particular gear application is usually such that there is some overlap of a meshing gear from one tooth to another. This can occur if point Q in figure 30-2 is in line with, or to the right of, point O. The triangle formed by line OQ, the slide of the gear, and the tooth line is shown in figure 30-4. The distance OQ is the width of gear τ (tau).

From this triangle we see that

$$\frac{P_c}{\tau} = \tan$$

or

$$\psi = \arc\tan \frac{P_c}{\tau} \qquad \textbf{(30.6)}$$

Helical gears may be made with either a righthand or lefthand helix angle. A righthand helix angle makes the teeth run in the direction of a righthand screw thread. Figure 30-1 shows both right- and lefthanded helix angles.

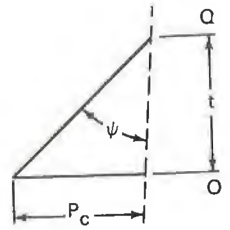


Fig. 30-4 Circular Pitch, Width, and Helix Angle

If helical gears are to be meshed between parallel shafts, they must have equal pitches, equal helix angles, and be of opposite "hand". That is, one must be righthanded and the other lefthanded as shown in figure 30-1.

In addition to being able to handle greater torque due to the increased contact ratio, helical gears also tend to run considerably quieter than do spur gears.

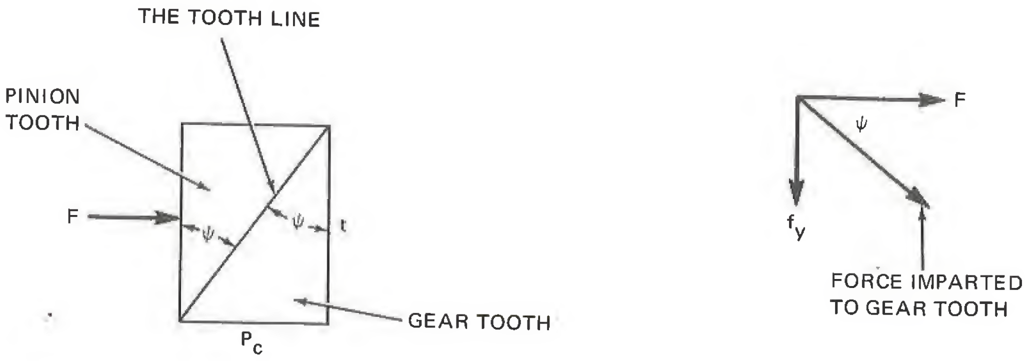
Unfortunately helical gears are more expensive to manufacture than are spur gears. An even more troublesome disadvantage of helical gears is their *side thrust.* To better understand this problem let's observe that if a pinion is delivering a torque (T), then force on the pinion teeth is

$$F = \frac{T}{r}$$

where r is the pitch radius of the pinion. In terms of the pitch diameter this is

$$F = \frac{T}{d/2} = \frac{2T}{d}$$

This force is transmitted to the teeth of the gear. However, since the helical gear's teeth are inclined at an angle ψ, we have a situation somewhat as shown in figure 30-5a. The force tends to do two things. Part of it is transmitted to the gear along a line perpendicular to the tooth line. At the same time the helix angle incline tries to force the pinion sideways.



(A) FORCE ON THE TOOTH LINE *(B) THE FORCE DIAGRAM*

Fig. 30-5 Force on Helical Teeth

From the force diagram we see that this side thrust force (f_y) has a value of

$$f_y = F \tan \psi$$

And since we have found that

$$F = \frac{2T}{d}$$

we have

$$\boxed{f_y = \frac{2T}{d} \tan \psi} \qquad \textbf{(30.7)}$$

as the equation for the side thrust on a helical gear. This side thrust effect can be very large and sometimes causes troublesome problems in mounting helical gears.

Helical gears may also be meshed with their shafts at right angles to each other as shown in figure 30-6. However, this arrangement is usually employed only with relatively light loads.

The gears shown are both lefthanded. They could also both be righthanded. In any case

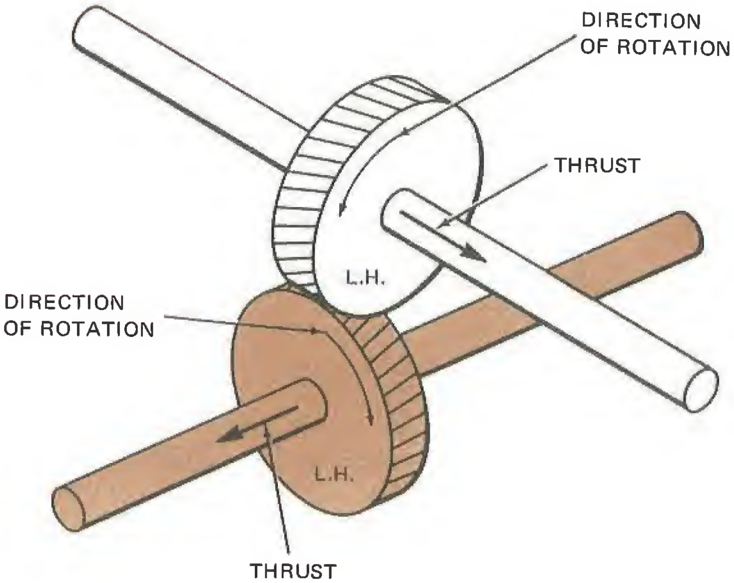


Fig. 30-6 Crossed Helical Gears

the angle between the two shafts is related to the helix angles by

$$\boxed{\theta = \psi 1 \pm \psi 2} \qquad \textbf{(30.8)}$$

where θ is the angle between the shafts.

The plus sign is used when both gears are of the same hand and the negative sign is used for gears of different hand.

There are no universally accepted standards for the tooth proportions of helical gears although a number of systems have been tried successfully. Since helical gears are normally purchased in pairs and are rarely meshed with others, the lack of standardization rarely causes problems. Because tooth proportions vary, pitch diameters should *not* be used in computing the gear ratio of a mesh. The ratio of the tooth counts may, however, be relied upon for these calculations.

MATERIALS

1 Righthand helical pinion, approx. 3/4 in. OD.

1 Lefthand helical pinion, same N as R.H. pinion.

1 Righthand helical gear, approx. 1-1/2 in. OD.

1 Lefthand helical gear, same N as R.H. gear.

2 Bearing plates with spacers

2 Bearing mounts

2 Bearings

2 Shafts approx. 4" X 1/4"

4 Collars

1 Shaft coupling

1 Motor and mount

1 Power supply

2 Shaft hangers with bearings

1 Breadboard with legs and clamps

2 Dials with 1/4 in. bore hubs

1 Stroboscope

PROCEDURE

1. Examine all of the components to insure that they are undamaged.
2. Identify the right- and lefthand helical gears. Count and record the number of teeth on each gear wheel.
3. Assemble the mechanism shown in figure 30-7. Use an instrument grade lubricant on the shafts at the bearing.
4. Rotate the mechanism by hand to insure that it turns freely.
5. Connect the motor wires to the DC power supply. Starting at zero volts increase the voltage slowly until the pinion is turning a few hundred RPM. The pinion dial should be turning clockwise as viewed from the right side of figure 30-7.
6. With the pinion running clockwise, slowly increase the speed until the power supply is set for about 10 volts. Strobe the pinion dial to determine the angular velocity of the motor (ω_p).

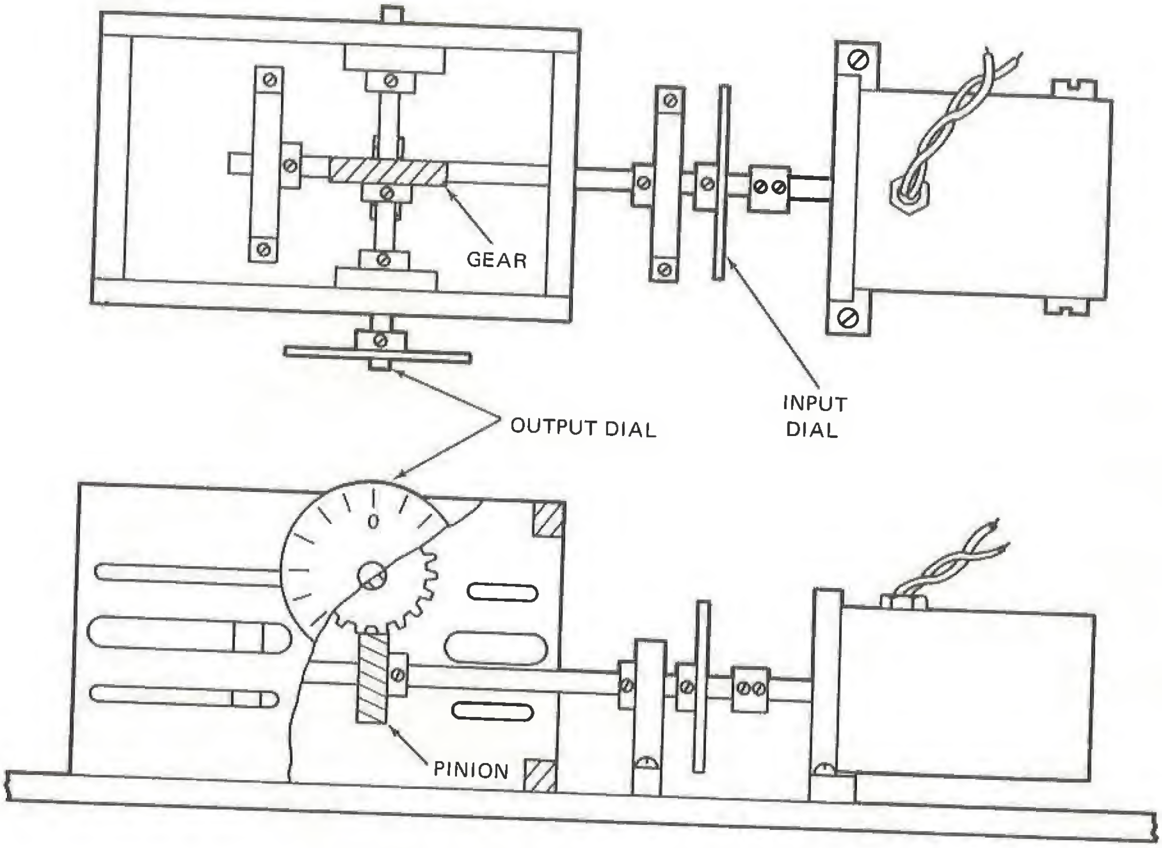


Fig. 30-7 The Experimental Mechanism

7. Strobe the gear dial and record the angular velocity (ω_g).
8. If the presence of side thrust is not already apparent, carefully reach in and touch the gear shaft with your finger. This should increase the load and side thrust. Make a note of the gear's reaction.

n (R.H.)	n (L.H.)	N (R.H.)	N (L.H.)

n/N	ω_g/ω_p	ω'_g/ω'_p

ω_g	V (Volts)	ω'_p	ω'_g	ω'_p
	10			
	12			
	14			
	16			

Fig. 30-8 The Data Table

9. Repeat steps 6, 7, and 8 for voltages of 12, 14, and 16 volts.
10. Exchange the pinion and gear pair for one of the opposite hand.
11. Repeat steps 6 through 9 and record your results in the Data Table as ω'_g, ω'_p.
12. Compute the tooth ratio n/N and record the results.
13. Compute the velocity of the single mesh ω_g/ω_p.
14. Compute the velocity ratio of the opposing pair mesh ω'_g/ω'_p.

ANALYSIS GUIDE. In the analysis of these results there are three main points that you should consider. They are:

1. Did the tooth ratio and the single mesh velocity ratio agree? How closely?
2. Did using the opposite hand gear set significantly change the velocity ratio?
3. Did using the opposite hand gear set significantly change the side thrust? Why?

In addition to these points you should discuss the general accuracy of the experiment and any difficulties you encountered.

PROBLEMS

1. A 30-tooth R.H. helical pinion is meshed at an angle with a 72-tooth R.H. gear. The helix angle is 30° and the normal diametrical pitch is 4. What is the diametrical pitch?
2. What is the circular pitch of the gears in problem 1?
3. What is the normal circular pitch in problem 1?
4. What should be the minimum tooth width in problem 1?
5. If the gear in problem 1 turns 75 degrees clockwise, through what angle will the pinion rotate?
6. What is the velocity ratio in problem 1?
7. If 940 in.-lb. of torque is transmitted by the gear in problem 1, what is the side thrust?
8. In problem 7 what torque is transmitted by the pinion?

LABORATORY REPORT WRITING

There are a number of different forms that a technical laboratory report may take. The forms proposed here are intended to meet the needs of these experiments and should not be considered to be universally applicable.

I. THE INFORMAL REPORT. In reporting the results of these experiments, it may be convenient to write an informal type of laboratory report. Informal reports are normally due at the start of the laboratory period following the period during which the experiment was performed. A report outline which you may wish to follow is given below.

A. **Cover Page.** The cover page should include:
   1. Your name.
   2. Your partner's name.
   3. Date the experiment was performed.
   4. Experiment title and number.

B. **Data Section.** The data section should include:
   1. A neat drawing of the experiment setup.
   2. A list of equipment used, including the manufacturer's name, model number, and serial number.
   3. Measured and calculated data in tabular form.
   4. Curves.

C. **Analysis Section.** The analysis section should contain a satisfactory technical discussion of the data. It should, in general, include brief discussions of each of the points mentioned in the "Analysis Guide" and the solutions to any problems given at the end of the experiment.

II. THE FORMAL REPORT. You may be required to write and submit a formal laboratory report on some of the experiments that you have performed. All formal reports should be submitted in a satisfactory report folder, and are normally due about 1 week after the time that the experiment was performed. The formal report should include the following:

A. **Title Page.** The title page should contain the following:
   1. Title of the experiment.
   2. Name of the person making the report.
   3. Date the experiment was performed.

B. **Introduction Section.** The introduction should consist of a paragraph which sets forth the technical objective of the experiment.

C. **Theory Section.** The theory section should include a brief discussion of the theory which is pertinent to the particular experiment.

D. **Method of Investigation Section.** The method of investigation should include the following:
1. A neat drawing of the experimental setup.
2. A brief outline of the experimental procedure.
3. A brief outline of the calculations to be made.
4. A brief discussion of how the calculations and measurements are to be compared.

E. **Equipment List.** The equipment list should contain every item of equipment used. It should show the manufacturer's name, the model number, and the serial number of every item.

F. **Data Section.** The data section should include a smooth copy of the following:
1. All measured values in tabular form.
2. All computed values in tabular form.
3. All curves.

G. **Sample Computations Section.** This section should include a smooth sample of each type of calculation made.

H. **Analysis Section.** The analysis section should include a discussion of each of the following points:
1. How valid is the data?
2. What are the probable sources of error?
3. What are the probable magnitudes of the different errors?
4. How could the errors be reduced?

I. **Rough Data Section.** This section is provided to contain all work not presented elsewhere in the report. It should contain such items as:
1. Notes taken from reference material.
2. The actual calculations performed.
3. The actual rough experimental data.

As you have no doubt already concluded, the writing of a formal laboratory report is by no means quick or easy. You should remember however that a technician is frequently judged on the quality of his reports. Therefore, it is wise to make each report as good as possible.

EXPERIMENT 1 ______ Name ______

Date: ______ Class ______ Instructor ______

EXPERIMENT 2 ________ Name ________

Date: ________ Class ________ Instructor ________

Qty	d_o	n	D_o	N	d_s	X	d	D	r	R
Value										

C

C

Fig. 2-6 The Data Table

EXPERIMENT 3

Date: ____________________

Name ____________________

Class __________ Instructor ________________

Qty	D_o	d_o	N	n	X	d_s	C	P_d	p_d
Value									

Qty	D	d	a (3-16)	a (3-12)	b	c	w	a′
Value								

Fig. 3-9 The Data Table

EXPERIMENT 4

Name ____________________

Date: ____________________ Class __________ Instructor ____________________

D_o	d_o	N	n	D	d	d/D	n/N	% Diff.

MEAS. NO.	θ_p	θ_g	% DIFF.
1			
2			
3			
4			
5			
6			
7			

Fig. 4-5 The Data Table

EXPERIMENT 5

Date: ____________________ Name ____________________

Class __________ Instructor ____________________

Measured Values

d_o	D_o	n	N	ω_p	ω_g	ω_1	ω_2

Computed Values

d	D	$\frac{\omega_g}{\omega_p}$	$\frac{n}{N}$	$\frac{d}{D}$	v_p	v_g	ω_p'

Percent Difference

between $\frac{\omega_g}{\omega_p}$ and $\frac{n}{N}$	between $\frac{\omega_g}{\omega_p}$ and $\frac{d}{D}$	between v_p and v_g	between ω_p and ω_p'

Fig. 5-4 The Data Tables

EXPERIMENT 6 Name ____________________

Date: ____________________ Class ____________ Instructor ____________

Measured Values

f_p	f_g	r_p	r_g	d_o	D_o	n	N

Computed Values

T_p	T_g	n/N	d	D	d/D	T_p/T_g

Fig. 6-6 The Data Table

EXPERIMENT 7

Date: ______________ Name ______________

Class ________ Instructor ______________

Measured Values

D_{o1}	D_{o2}	D_{o3}	N_1	N_2	N_3	ω_1	ω_3

Computed Values

D_1	D_2	D_3	N_1/N_3	D_1/D_3	ω_3/ω_1

Fig. 7-5
The Data Table

EXPERIMENT 8

Date: ____________________

Name ____________________

Class ____________ Instructor ____________

Gear Parameters

Gear No.	N	D_o	D
1			
2			
3			
4			

Shaft Speeds

Qty.	$E_{DC} \approx 24V$	$E_{DC} \approx 21V$	$E_{DC} \approx 18V$
ω_1			
ω'_1			
ω_4			
ω'_4			

Fig. 8-7 The Data Table

EXPERIMENT 9

Name ______________________

Date: ______________ Class __________ Instructor ______________

Gear No	OD	N	D	P
1				
2				
3				
4				

θ_1	θ_2	θ_1/θ_2
Ave. Value		

N_2/N_1	N_4/N_3	Overall Gear Ratio

Center Distance

Measured at Input	
Measured at Output	
Computed at Input	
Computed at Output	

Fig. 9-3 The Data Table

EXPERIMENT 10 ______________ Name ______________________

Date: ______________________ Class ________ Instructor ______________

θ_p	θ_g	θ_g/θ_p
Ave. Value		

D_i	d_o	n	N

d	D	p	P

C	C′	n/N	d/D

Fig. 10-7 The Data Table

EXPERIMENT 11

Date: ______________ Name ______________________

Class __________ Instructor ______________

	ID or OD	N	D_p
Ring Gear			
Planet #1			
Planet #2			
Planet #3			
Sun Gear			

Sun Gear Locked				Ring Gear Locked				Carrier Locked				Unlocked			
θ_c	θ_r	θ_c/θ_r	ω_c/ω_r	θ_c	θ_s	θ_s/θ_c	ω_s/ω_c	θ_s	θ_r	θ_s/θ_r	ω_s/ω_r	θ'_c	θ'_r	θ'_s	θ''_s

Fig. 11-6 The Data Table

EXPERIMENT 12

Date: ______________________

Name ______________________

Class ____________ Instructor ______________________

n (R.H.)	n (L.H.)	N (R.H.)	N (L.H.)	ω_g	ω_p	f_y	ω'_g	ω'_p	f'_y

n/N	ω_g/ω_p	ω'_g/ω'_p

Fig. 12-9 The Data Table

EXPERIMENT 13

Date: ____________________ Name ____________________

Class __________ Instructor ____________________

Voltage	n	N	n/N	ω_p	ω_g	ω_g/ω_p	% Diff.
24 V							
18 V							
14 V							

Fig. 13-8 The Data Table

EXPERIMENT 14

Date: ____________ Name ____________

Class ________ Instructor ____________

θ (Degrees)	S_r (Meas.)	S_r (Comp.)	% Diff.

d_o	n	d	r_ℓ

f_r	f_p	T_p (Meas.)	T_p (Comp.)	% Diff.

Fig. 14-6 The Data Tables

EXPERIMENT 15

Date: ____________________ Name ____________________

Class ____________ Instructor ____________________

K	N	θ_w/θ_g	ω_w/ω_g (Ave.)	% Diff. (Step 17)	N/K	% Diff. (Step 19)

ω_w	ω_g	ω_w/ω_g

Fig. 15-6 The Data Tables

EXPERIMENT 16 ______________ Name ______________

Date: ______________ Class ________ Instructor ______________

k	N_s	ℓ	P_s	S_b (Ave.)	S_b (Comp.)	% Diff.

Screw Turns	S_b	S_b Per Rev.

Fig. 16-4 The Data Tables

EXPERIMENT 17 ____________ Name ____________________

Date: ____________________ Class __________ Instructor __________

Gear No. 1	Gear No. 2	Gear No. 3	Gear No. 4	Worm	Wheel	Screw

θ_1	S_b	(Comp.) S_b/rev	(Ave.) S_b/rev	% Diff.

Fig. 17-5 The Data Tables

EXPERIMENT 18 ____________ Name ______________________

Date: ______________________ Class ____________ Instructor ____________

N_1	N_2	N_3	N_1/N_3	N_2/N_3	N_1/N_2

θ_1	θ_2	θ_3	θ_1/θ_3	θ_2/θ_3	θ_1/θ_2

ω_1	ω_2	ω_3	ω_1/ω_3	ω_2/ω_3	ω_1/ω_2

Fig. 18-5 The Data Tables

RATIO	θ_1/θ_3	θ_2/θ_3	θ_1/θ_2	ω_1/ω_3	ω_2/ω_3	ω_1/ω_2
Ave. Value						
% Diff. (N)						

EXPERIMENT 19 ______ Name ______

Date: ______ Class ______ Instructor ______

Input 2 Fixed

θ_1	θ_2	θ_s	θ_s (Comp.)

Input 1 Fixed

θ_1	θ_2	θ_s	θ_s (Comp.)

Output 1 Fixed

θ_1	θ_2	θ_s	θ_s (Comp.)

Fig. 19-5 The Data Tables

Inputs Rotate Opposite Directions

θ_1	θ_2	θ_s	θ_s (Comp.)

Inputs Rotate Same Direction

θ_1	θ_2	θ_s	θ_s (Comp.)

EXPERIMENT 20 ______________ Name ______________________________

Date: ______________________ Class ____________ Instructor ______________

First Spring

d	OD	D	C	L_T	L'_T	ΔL	F_L	F'_L	ΔF	K

Second Spring

d	OD	D	C	L_T	L'_T	ΔL	F_L	F'_L	ΔF	K

Series Combination

K_T = ____________

(Computed Value)

Measured Values

L_T	L'_T	ΔL	F_L	F'_L	ΔF	K_T

Parallel Combination

K_T = ____________

(Computed Value)

Measured Values

L_T	L'_T	ΔL	F_L	F'_L	ΔF	K_T

Fig. 20-7 The Data Table

EXPERIMENT 21 ______ Name ______

Date: ______ Class ______ Instructor ______

W_1	W_2	W_3	D_1	D_2	D_3

Trial	t	KE
1		
2		
3		
4		
5		
6		
7		

Fig. 21-4 The Data Table

EXPERIMENT 22 ______________ Name ______________________

Date: ______________________ Class __________ Instructor ______________

Motor Voltage (Approx.)	OPEN BELT		CROSSED BELT	
	ω_s	ω_L	ω_s	ω_L
28				
26				
24				
22				
20				
18				
16				
14				
12				
10				

d = ____________ D = ____________

Fig. 22-10 The Data Table

EXPERIMENT 23 ______________ Name ______________________________

Date: ______________________ Class ____________ Instructor ______________

Two-Pulley Block					Three-Pulley Block					Four-Pulley Block				
F_L	f_e	$\frac{F_L}{f_e}$	N	η	F_L	f_e	$\frac{F_L}{f_e}$	N	η	F_L	f_e	$\frac{F_L}{f_e}$	N	η

Fig. 23-6 The Data Tables

EXPERIMENT 24 ______________ Name ______________________

Date: ______________________ Class __________ Instructor ______________

F_L	f_e	$\frac{F_L}{f_e}$	η

D	d_1	d_2	S_e	S_L	$\frac{S_e}{S_L}$	$\frac{2D}{d_1-d_2}$

Fig. 24-5 The Data Table

EXPERIMENT 25 ______ Name ______
Date: ______ Class ______ Instructor ______

P	n	N	$\frac{N}{n}$

c	d	D	$\frac{D}{d}$

L Meas.	L Comp.

ω_s Meas.	ω_L Meas.	$\frac{\omega_s}{\omega_L}$
Average $\frac{\omega_s}{\omega_L}$ →		

Fig. 25-8 The Data Table

EXPERIMENT 26 ______________ Name ______________________

Date: ______________________ Class __________ Instructor ____________

d_o	D_o	D_{ol}	η	N	N_I	L	C

d	D	D_I	ω_s	ω_L	ω_I	L′
Second Run						

$\frac{d}{D}$	$\frac{\omega_s}{\omega_L}$	% Diff.	$\frac{d}{D_I}$	$\frac{\omega_s}{\omega_I}$	% Diff.	$\frac{D_I}{D}$	$\frac{\omega_I}{\omega_L}$	% Diff.

Fig. 26-7 The Data Table

EXPERIMENT 27 ______________ Name ______________

Date: ______________ Class ______________ Instructor ______________

		$r_s =$			
$r_L =$		$r_L =$		$r_L =$	
ω_s	ω_L	ω_s	ω_L	ω_s	ω_L
$\frac{r_L}{r_s} =$		$\frac{r_L}{r_s} =$		$\frac{r_L}{r_s} =$	

Fig. 27-7 The Data Table

EXPERIMENT 28 ______________ Name ______________________________

Date: ______________________ Class ____________ Instructor ______________

	ω_1	ω_2
First Tension Setting		
Second Tension Setting		
Third Tension Setting		

Fig. 28-8 The Data Table

EXPERIMENT 29 ____________ Name ________________________

Date: ____________________ Class ____________ Instructor ____________

α =		
Input θ_1	Single U-Joint θ_2	Double U-Joint θ_2
0		
15		
30		
45		
60		
75		
90		
105		
120		
135		
150		
165		
180		
195		

α =		
Input θ_1	Single U-Joint θ_2	Double U-Joint θ_2
210		
225		
240		
255		
240		
255		
270		
285		
300		
315		
330		
345		
360		

Fig. 29-6 The Data Table

EXPERIMENT 30 ____________ Name ____________

Date: ____________ Class ____________ Instructor ____________

n (R.H.)	n (L.H.)	N (R.H.)	N (L.H.)

n/N	ω_g/ω_p	ω'_g/ω'_p

ω_g	V (Volts)	ω'_p	ω'_g	ω'_p
	10			
	12			
	14			
	16			

Fig. 30-8 The Data Table